Cartesian Physics

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Introduction: Cartesian Physics or Natural Philosophy

By the term "physics", René Descartes and other educated men of his generation would have understood "natural philosophy", the attempt systematically to understand matter and cause in nature, as well as cosmic and terrestrial structure. This was to be accomplished either by means of one or another version of the institutionally dominant neo-Scholastic Aristotelianism of the day, or one of its increasingly popular alternatives, neo-Platonic, Stoic, or atomistic. Descartes certainly would not have identified the word 'physics' with the classical mechanics later thinkers discerned in embryonic form in the work of Galileo and which matured in the next two generations in the work of Huygens, Newton and others. Nevertheless, Descartes' natural philosophy marks a significant moment in the larger history of physics, as conceived in this volume. His system of natural philosophy was a novel, daring and intricate construction in that field, with two main sets of historical significances for later physics.

First of all Cartesian natural philosophy contained several novelties, for example, his treatment of collision, force of motion and rest and the attempted mechanisation of the theory of light and celestial mechanics, which had unintended technical significances in the formation of the Newtonian and Huygensian dispensations in mechanics. Secondly, the Cartesian system embodied in its own terms a number of values, goals and strategic positionings which it shared with other innovative natural philosophies of the day, and which during the course of the mid and later seventeenth century broadly facilitated and shaped the eventual emergence of classical mechanics. These included his concerted effort to establish the truth of a realist and multi-planetary system version of Copernicanism, and, albeit in his own *sui generis* and ultimately unsuccessful manner, his attempt to mathematicise natural philosophy.

This chapter thus aims to explicate in detail Descartes' physics in a way that also brings into relief these two sets of subtle, largely unintended, but nonetheless significant consequences for Newtonian physics. These goals can only be achieved by a careful genealogical anatomy of Descartes' natural philosophy, taking account not simply of its mature, systematised form, but also the little explored but highly revealing history of how and why it developed as it did. We undertake this in Part 1, before Part 2 examines the implications for classical mechanics of the Cartesian form of physics.

Part 1. The Developmental Anatomy of Cartesian Physics, 1618-1644

1.1. Successes, Failures and Fate Of Descartes' Early Physico-Mathematics Program

In November 1618 Descartes, then twenty-two, met and worked for two months with Isaac Beeckman, a Dutch scholar eight years his senior. Beeckman was one of the first supporters of a corpuscular–mechanical approach to natural philosophy. However, it was not simply corpuscular-mechanism that Beeckman advocated to Descartes. He also interested Descartes in what they called physico-mathematics. In late 1618 Beeckman wrote that, "There are very few physico-mathematicians," adding, "(Descartes) says he has never met anyone other than me who pursues enquiry the way I do, combining Physics and Mathematics in an exact way; and I have never spoken with anyone other than him who does the same." (Beeckman 1939-53, I 244) They were partly right. While there were not many physico-mathematicians, there were of course others, such as Kepler, Galileo and certain leading Jesuit mathematicians, trying to merge mathematics and natural philosophy. (Dear 1995)

Physico-mathematics, was an actors' term of the day, and taken generically, across the spectrum of different natural philosophers who used it, dealt with the way the traditional mixed mathematical disciplines, such as hydrostatics, statics, geometrical optics, geometrical astronomy and harmonics, were conceived to relate to the discipline of natural philosophy. (Gaukroger and Schuster, 2002) In Aristotelianism the mixed mathematical sciences were interpreted as intermediate between natural philosophy and mathematics and subordinate to them. Natural philosophical explanations were couched in terms of matter and cause, something mathematics could not offer, according to most Aristotelians. The mixed mathematical sciences used mathematics not in an explanatory way, but instrumentally for problem solving and practical aims. For example, in geometrical optics, one represented light as light rays. This might be useful but did not explore the underlying natural philosophical questions: "the physical nature of light" and "the causes of optical phenomena". In contrast physico-mathematics signalled a commitment to revising radically the Aristotelian view of the mixed mathematical sciences, which were to become more intimately related to natural philosophical issues of matter and cause. Paradoxically, the issue was not mathematisation. The mixed mathematical sciences, which were already mathematical, were to become more "physicalised," more closely integrated into whichever brand of natural philosophy an aspiring physico-mathematician endorsed.

In the case of Descartes and his mentor Beeckman, the preferred, if unsystematised natural philosophy of choice was a fragmented and embryonic variety of corpuscular-mechanism. Hence what Descartes and Beeckman meant by the program of physico-mathematics was that reliable geometrical results in the mixed mathematical sciences were to be explained by invoking an embryonic corpuscular-mechanical matter theory and a causal discourse concerning forces and tendencies to motion. Three of Descartes' early exercises in physico-mathematics survive. The most important and symptomatic was his attempt, at Beeckman's urging, to supply a corpuscular-mechanical explanation for the hydrostatic paradox, which had been rigorously derived in mixed mathematical fashion by Simon Stevin. (AT X 67-74, 228; Gaukroger and Schuster 2002) We shall examine this portentous work in some detail before turning briefly to the other two cases.

In 1586 Simon Stevin, the great Dutch maestro of the practical mathematical arts and mixed mathematical sciences, had demonstrated that a fluid filling two vessels of equal base area and height exerts the same total pressure on the base, irrespective of the shape of the vessel and hence, paradoxically, independently of the amount of fluid it contains. Stevin's mathematically rigorous proof applied a condition of static equilibrium to various volumes and weights of portions of the water. (Stevin 1955-66 I, 415-7)

In Descartes' treatment of the hydrostatic paradox (AT X 67-74) the key problem involves vessels B and D, which have equal areas at their bases, equal height and are of equal weight when empty. (**Figure 1**) Descartes proposes to show that, "the water in vessel B will weigh equally upon its base as the water in D upon its base"—Stevin's paradoxical hydrostatical result. (AT X 68-69)

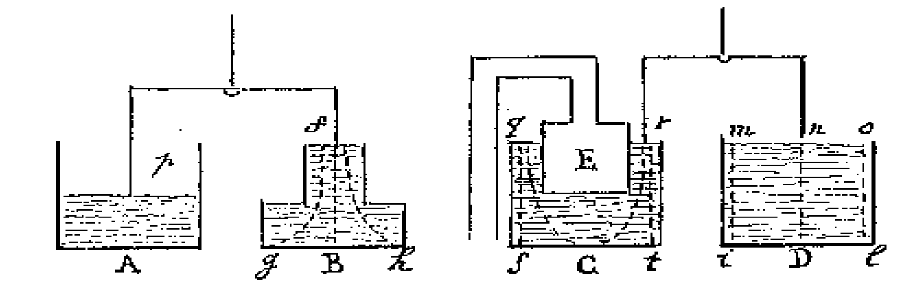

Figure 1. Descartes, “Hydrostatics Manuscript” (AT X 69)

First Descartes explicates the weight of the water on the bottom of a vessel as the total force of the water on the bottom, arising from the sum of the pressures exerted by the water on each unit area of the bottom. This “weighing down” is explained as “the force of motion by which a body is impelled in the first instant of its motion”, which, he insists, is not the same as the force of motion which “bears the body downward” during the actual course of its fall. (AT X 68)

Then, in contrast to Stevin’s rigorous proof, Descartes attempts to reduce the phenomenon to corpuscular–mechanics by showing that the force on each “point” of the bottoms of the basins B and D is equal, so that the total force is equal over the two equal areas, which is Stevin’s paradoxical result. He claims each “point” on the bottom of each vessel is serviced by a unique line of instantaneously exerted “tendency to motion” propagated by contact pressure from a point (particle) on the surface of the water through the intervening particles. But, while the surface of D is equal to and congruent with its base and posed directly above it, the surface of B is implied to have an area one-third its base. Hence exemplary points i, D and l in the base of D are each pressed by a unique, vertical line of tendency emanating respectively from corresponding points, m, n and o on the surface. In contrast point f on the surface of B is the source of three simultaneous lines of tendency, two being curved, servicing exemplary points g, B and h on the bottom of B. Descartes claims that all six exemplary points on the bottoms are pressed by an equal force, because they are each pressed by “imaginable lines of water of the same length” (AT X 70); that is, lines having the same vertical component of descent. Descartes smuggles the tendentious three-fold mapping from f into the discussion as an “example” but then argues that *given the mapping*, f can provide a three-fold force to g, B and h. (AT X 70-1)

The 'hydrostatic manuscript' thus shows the young Descartes articulating the program of physico-mathematics, by taking firmly established geometrical results in a mixed mathematical science as a basis for inferring in some way a natural philosophical explanation in terms of matter and cause, thus fusing together sound findings from mixed mathematics and the explanatory realm of his favoured brand of natural philosophising. Stevin's treatment of the hydrostatic paradox fell within the domain of mixed mathematics, rather than natural philosophy. It did not explain the phenomenon by identifying its causes. The account Descartes substitutes for it falls within the domain of natural philosophy, attempting to identify the material bodies and causes in play. Fluids are made up, on Descartes' account, of corpuscles whose instantaneously manifested movements or tendencies to movement are understood in terms of a theory of forces and tendencies. This should explain what causes the pressure exerted by a fluid on the floor of its containing vessel.

These moves implied a radically non–Aristotelian vision of the relation of the mixed mathematical sciences to Descartes' emergent form of corpuscular-mechanical natural philosophy. He aimed to shift hydrostatics from mixed mathematics into the realm of natural philosophy. Indeed, he seems to have believed that from crisp, simple geometrical representations of sound mixed mathematical results one could read out or 'see' the underlying corpuscular-mechanical causes. (Gaukroger and Schuster 2002, 549-550) This proclivity was to be displayed in other parts of his physico-mathematical work, most especially in his geometrical optics. More generally, although Descartes never again directly considered hydrostatical problems, his mature natural philosophy was to develop and rearticulate some of the explanatory resources he first glimpsed in this early work. Throughout his later career Descartes continued to use descendants of the concept of instantaneous tendency to motion analysable into its directional components (later termed "determinations"). These ideas became central to what may be termed his "dynamics", the concepts that govern the behaviour of micro-corpuscles in *Le Monde* (1629-33) and the *Principles of Philosophy* (1644, 1647) his two systematic treatises of natural philosophy.

Descartes and Beeckman's well known and frequently commented upon studies of accelerated free fall also belonged to their physico–mathematical project. (AT X 58-61, 74-78, 219-222; Jullien and Charrak, 2002) In contrast to the way this work has traditionally been interpreted, they were not simply striving for a 'Galileo–like' kinematic, or mixed mathematical law of fall, but also, as physico-mathematicians, they were looking for the causal explanation of such a law. This does not mean that the problem of mathematically describing fall was not important to Beeckman and Descartes. As physico-mathematicians they certainly wanted to find the descriptive law, if it existed. But simply to find such a law would have been to work superficially and without insight into natural philosophical issues of matter and cause, as they criticised Stevin for having done in hydrostatics. They did not fail to find candidate laws, nor did they fail to find speculative candidate causes. The problem was that there were too many possible and plausible regimes of natural philosophical causation, in continuously and discontinuously acting modes, and, there were too many resulting descriptive laws, which might well be impossible to determine one from another, even if measurement were possible. The problem of fall ended up looking like a poorly defined, or unsolvable puzzle in physico-mathematics. This was not hydrostatics, where as we have seen, Stevin's stunning and paradoxical results led Descartes, at least, to think he had made (quite radical) physico-mathematical capital; nor was it optics, where Descartes would eventually achieve profound physico-mathematical results.

This outcome undoubtedly contributed to Descartes' cool and sceptical response to Galileo's kinematics when it appeared eighteen years later. As early as 1619 Descartes could have begun to form the opinion that the highly idealised study of fall, in search of some sort of descriptive, mixed mathematical law, was of no natural philosophical, that is, physico-mathematical import. Of course, the search for and discovery of a law of falling bodies would be one of the key exemplars in the crystallisation of classical mechanics during the course of the seventeenth century. But the study of falling bodies would play no role in Descartes' formulation of the causal register, the dynamics, that would sit at the heart of his later system of corpuscular mechanics. That dynamics also made a contribution to classical mechanics, but as we shall see, it would be derived physico-mathematically from important work in optics, hence, we need to look at the third and final of Descartes' early physico-mathematical case studies, turning to his initial halting steps in rendering optics a physico-mathematical discipline.

In 1620 Descartes attempted in a physico–mathematical manner to find the mathematical law of refraction of light by considering the geometrical representation of its likely causes. He based the endeavour on some passages and diagrams in Kepler's optical masterpiece, *Paralipomena ad vitellionem* (1604), where Kepler had suggested that light moves with more force in denser optical media and "hence" is bent toward the normal in moving from a less to a more dense medium. (AT X 242-3; Schuster 2000, 278-285, 287-9) Descartes explored Kepler's speculation, using his newly acquired physico-mathematical style of 'reading' geometrical diagrams representing phenomena for their underlying message about the causal principles at work. On this occasion Descartes found neither a law of refraction nor its natural philosophical causes. However, seven years later, whilst working with the mathematician Claude Mydorge, he discovered the long sought law of refraction. This discovery was just one key step in a sustained course of inquiry in optics which brought Descartes' physico-mathematics to a climax and established the principles and exemplars for his emerging corpuscular-mechanical natural philosophy. Descartes' foray in optics in the late 1620s and early 1630s is thus one pivot of his natural philosophical career, and it repays careful study in any exploration of his physics. (Schuster, 2012)

1.2. The Optical Triumph of the 1620s: From Physico-Mathematical Optics to the Principles of Corpuscular-Dynamics

The trajectory of Descartes' work in optics between 1620 and the publication of his *Dioptrique* in 1637 has often been ignored in studies of his optics and physics. Recently more light has been thrown on his project in optics, mixed and physico-mathematical: Around 1627, he found, by traditional mixed mathematical means, a simple (cosecant) version of the law of refraction. He immediately set to work attempting, in a physico-mathematical manner, to exploit his discovery by reading out of his results causal principles on the natural philosophical level. This proceeds in two steps. First, from his key geometrical diagram for the new law, he tried to read out the principles of a mechanical theory of light as an instantaneous impulse. As early as 1628 he was attempting to demonstrate his law of refraction, using these mechanical conceptions of light which he had obtained by reflecting, physico-mathematically, upon the new law. Then, secondly, as he became involved in writing *Le Monde* from late 1629, he tried to extract further physico-mathematical causal 'insight' from the optical work. Using the principles of his mechanical theory of light elicited from the work on refraction, he reformulated and polished the central concepts of his dynamics of corpuscles—the "causal register" of his emerging system of corpuscular-mechanism—whose earliest, embryonic manifestations we have seen in his 1619 physico-mathematisation of hydrostatics. This more mature elaboration is presented in *Le Monde* (1629-33), where the polished dynamics of corpuscles, itself a physico–mathematical product of the optical work, runs Descartes' vortex celestial mechanics and his corpuscular-mechanical theory of light in its cosmological setting. (Schuster 2005)

It would be ideal if this story could be explicated in detail in strict chronological order. Unfortunately, the materials for reconstructing Descartes' mixed and physico-mathematical optical project are few and scattered. This, combined with the curious and opaque presentation of the law of refraction in the *Dioptrique*, as well as the need to factor in Descartes' physico-mathematical proclivities, dictates a different strategy must be used to unpack the details of his evolution from physico-mathematical optics to a dynamics of corpuscles. One must start from the end point—the *Dioptrique*, published in 1637 as one of the three '*Essais'* supporting the *Discours de la Méthode*—working back through scattered earlier hints and clues to uncover the genealogy of the discovery of the law of refraction, and its physico-mathematical exploitation, leading to "seeing the causes" in a mechanistic theory of light and corpuscular-mechanical natural philosophy.

However, on its surface the *Dioptrique* does not reveal the trajectory of Descartes' struggles in mixed and physico-mathematical optics. Indeed it has traditionally raised its own problems. For example, Descartes deduces the laws of reflection and refraction from a model involving the motion of some very curious tennis balls. Descartes' contemporaries tended not to see any cogency in this model, nor did they grasp the theory of motion (actually his corpuscular dynamics) upon which it is based. These problems fueled the question of how Descartes had arrived at the law of refraction. Accusations arose that Descartes had plagiarized the law from Willebrord Snel, although, it has long been well established that this is quite unlikely. (Kramer 1882; Korteweg 1896) Accordingly, in decoding the *Dioptrique,* one must first grasp how the tennis ball model for refraction coherently links to Descartes' theory of light as a mechanical impulse *through* his dynamics of micro-corpuscles. And, in order to do that, one must understand Descartes' mature dynamics of corpuscles, as first inscribed in *Le Monde* between 1629 and 1633.

Descartes' system of natural philosophy in *Le Monde* was concerned with the nature and mechanical properties of microscopic corpuscles and a causal discourse, consisting of a theory of motion and impact, explicated through key concepts of the "force of motion" and directionally exerted "tendencies to motion" or "determinations". It is this "causal register" within Descartes' natural philosophical discourse which scholars increasingly term his "dynamics", as noted earlier. (Gaukroger and Schuster, 2002) We saw that the rudiments of this dynamics of instantaneously exerted forces and determinations dates back to Descartes' work on the physico-mathematics of hydrostatics. In *Le Monde* Descartes teaches that bodies in motion, or tending to motion, are characterised from moment to moment by the possession of two sorts of dynamical quantity: (1) the absolute quantity of the "force of motion"—conserved in the universe according to *Le Monde's* first rule of nature, and (2) the directional modes of that quantity of force, the directional components along which the force or parts of the force act, introduced in *Le Monde's* third rule of nature. (Schuster 2000, 258-61) It is these Descartes termed actions, tendencies, or most often determinations.[1] As corpuscles undergo instantaneous collisions with each other, their quantities of force of motion and determinations are adjusted according to certain universal laws of nature, rules of collision. Therefore Descartes' analysis focuses on instantaneous tendencies to motion, rather than finite translations in space and time.

Descartes' exemplar for applying these concepts is the dynamics of a stone rotated in a sling. (AT XI 45-6, 85; G 30, 54-5) (**Figure. 2**) Descartes analyses the dynamical condition of the stone at the instant that it passes point A. The instantaneously exerted force of motion of the stone is directed along the tangent AG. If the stone were released and no other hindrances affected its trajectory, it would move along ACG at a uniform speed reflective of the conservation of its quantity of force of motion. However, the sling continuously constrains what we may call the privileged, 'principal' determination of the stone and, acting over time, de-

1 The understanding of determination used here develops work of A.I.Sabra (1967) p.118-121; Gabbey (1980), pp.230-320; S. Gaukroger (1995); O. Knudsen and K.M Pedersen (1968) pp.183-186; Prendergast (1975),pp 453-62; McLaughlin (2000) and Schuster (2000, 2005).

flects its motion along the circle AF.[2] Descartes decomposes the principal determination into two components: one along AE completely opposed by the sling—so no actual centrifugal translation can occur—only a tendency to centrifugal motion; the other, he says, is "that part of the tendency along AC which the sling does not hinder" (AT XI 85), which over time manifests itself as actual translation in a circle. The choice of components of determination is dictated by the configuration of mechanical constraints on the system.

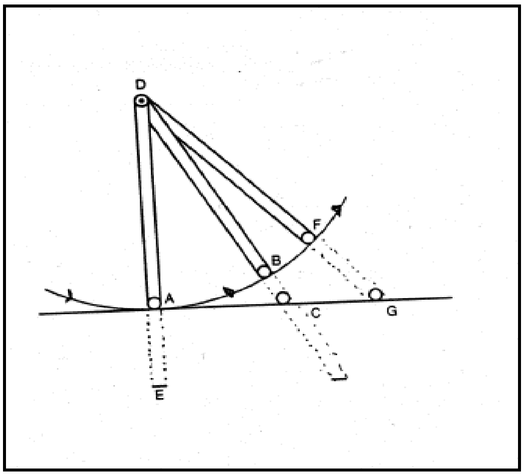

Figure 2. Descartes' Dynamics of the Sling in *Le Monde*

Finally, in approaching Descartes' treatment of the law of refraction, we need to bear in mind his mechanistic theory of light as presented in its natural philosophical context in *Le Monde*. Leaving aside Descartes' theory of elements and his cosmology, his *basic* theory of light within his natural philosophy is that light is a tendency to motion, an impulse, propagated instantaneously through continuous optical media. So, light is or has a principle determination—a directional quantity of force of motion—which can be further analysed into components as and when necessary.[3] Note that in Descartes' theory the propagation of light is instantaneous, but the magnitude of the force conveyed by the tendency to motion constituting a light ray can vary—there can be stronger and weaker light rays, all propagated instantly. (Schuster 2000, 261)

2 The term principal determination was coined by Schuster (2000, 260). It is meant to underscore this important concept, and differentiate this aspect of determination from the other determinations that can be attributed to a body in motion, or tending to motion, at any given moment.

3 In both *Le Monde* and the *Principia Philosophiae*, Descartes explained light mechanically in its full cosmological and matter theoretical context by the centrifugal tendency of the spherical particles of second element constituting each stellar vortex—light consisting in the instantaneous passage through a vortex of lines of tendency to centrifugal motion. (AT XI 87-90; G 55-8; AT VIII1 108-116; M 111-118)

Using Descartes' theory of light, and his corpuscular dynamics, one can analyse both his published 'proof' of the law of refraction of light, and its underlying rationale in terms of his real theory of light as instantaneous tendency to motion transmitted through the spherical particles of second element (*boules*) which make up his vortices.[4] In the *Dioptrique* of 1637 (**Figure 3**) Descartes demonstrated his law of refraction using the model of a tennis ball struck by a racket along AB towards refracting surface CBE, which is taken to be a perfectly flat, vanishingly thin cloth. (AT VI 97-8; CSM 1, 158-9) The tennis ball's weight and volume are ignored. It moves without air resistance in empty geometrical space on either side of the cloth and in breaking through it, the ball loses, independently of its angle of incidence, a certain fraction—one half—of its total quantity of force of motion. Descartes applies two conditions to the motion of the ball: [a] the new quantity of force of motion is conserved during motion below the cloth; and [b] the parallel component of the force of motion, the parallel determination, is unaffected by the encounter with the cloth. Drawing a circle of radius AB around B, he assumes the ball took time *t* to traverse AB prior to impact. After impact, losing half of its force of motion, hence half its speed, it must take *2t* to traverse a distance equal to AB, arriving somewhere on the circle after *2t*. This represents condition [a]. Descartes writes that prior to impact the parallel determination "caused" the body to move towards the right between lines AC and HBG. For condition [b] he considers that after impact, the ball takes *2t* to move to the circle's circumference, so its unchanged parallel determination has twice as much time in which to act to "cause" the ball to move toward the right. He sets FEI parallel to HBG so as to represent that doubled parallel travel. At time *2t* after impact the ball will be at I, the intersection of FEI and the circle point below the cloth. It follows that (sin i/sin r) = (AH/KI) or 0.5 for all angles of incidence.

4 Everything we are about to say concerning Descartes' dynamics underlying the demonstration of the law of refraction in the *Dioptrique* also applies to his preceding demonstration of the law of reflection. (Schuster 2000, 261-3)

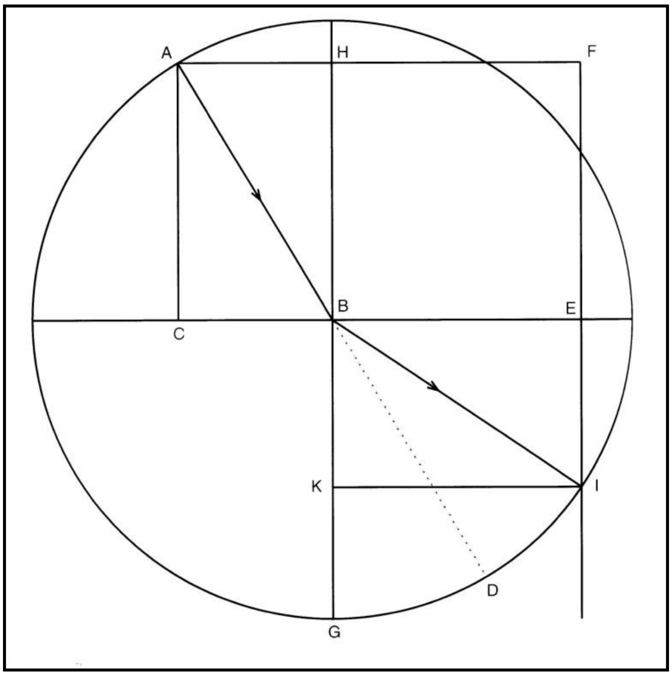

Figure 3. Descartes' Diagram for Refraction of Light (Tennis Ball) in *Dioptrique*

Descartes' published proof is superficially kinematic and has the look of a Galilean type idealisation of bodies, motion and space. But if we consider Descartes' corpuscular dynamics and the fact that his tennis ball is virtually a mathematical point in motion, we can translate Descartes' proof into the terms of his actual theory of light as instantaneously propagated tendency to motion. (Schuster 2000, 265) (**Figure 4**) That is, beneath the kinematic and idealised veneer, Descartes is actually not talking about an idealised situation, but rather what he holds as a fully realistic picture of light and its modes of action in his plenist matter theory—a point to which we shall return below in Section 1.4. Consider a light ray incident upon refracting surface CBE. Let length AB represent the magnitude of the force of the light impulse. The *orientation* and *length* of AB represent the principal determination of the ray. The force of the ray is diminished by half in crossing the surface. So, to represent condition [a] we draw a semi-circle below the surface about B with a radius equal to one half of AB. As for condition [b], the unchanged parallel determination, we simply set out line FEI parallel to HBG and AC so that AH=½HF. The resulting intersection at I gives the new *orientation* and *magnitude* of the force of the ray of light, BI and the law follows, as a law of cosecants. The case of the light ray requires manipulation of unequal semi-circles, representing the ratio of the force of light in the two media. In the tennis ball case Descartes moved from the ratio of forces to ratio of speeds and hence differential times to cross *equal* circles. But, at the instant of impact, the same force and determination relations are attributed to the tennis ball and the light ray and hence Descartes' overt presentation of an idealised, kinematical situation is ultimately deeply misleading as to his actual physics and its principles.[5]

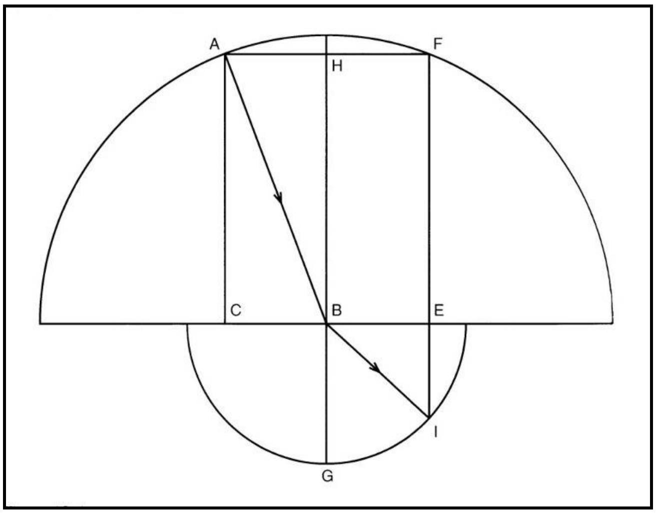

Figure 4. Refraction of Light Using Descartes' Dynamics & Real Theory of Light

In sum, we see that in both the tennis ball model and our reconstruction of Descartes' actual theory of light, the two conditions, [a] and [b], allowing derivation of the law of refraction, are actually dynamical premises—physico-mathematical principles—for a mechanical theory of light: [1] the absolute quantity of the force of motion of a light ray (or its tennis ball model) is conserved within any given medium, as is the ratio of those quantities of force of motion in refraction phenomena; [2] the component of the force of motion parallel to the refracting surface (parallel determination) of a light ray (or its tennis ball model) is unaffected by encountering the interface and is conserved in the second medium. These two dynamical premises of a theory of light may have permitted (from Descartes' perspective) a plausible deduction of the law of refraction, but they generated difficulties. Descartes' account in the *Dioptrique* has problems as soon as he discusses space filling media, or refraction toward the normal, and more generally with the question of how it happens that the alteration in the normal determination is variable, depending upon the angle of incidence. (Schuster 2000, 270-1) Moreover, the first dynamical assumption—path independent ratio of the force of light—seems to entail that optical media are isotropic, whilst the second dynamical as-

5 Some have claimed Descartes fell into a contradiction, because his theory of light states that light rays move instantaneously through any medium, whilst in the tennis ball model he deals with a ratio of finite speeds. This is mistaken: One simply must distinguish the *speed* of propagation of a light ray, which is instantaneous, from the *magnitude* of its force of propagation, which can take any finite positive value. The *speed* of Descartes' tennis ball corresponds not to the speed of propagation of light but to the intensity of the force of its propagation.

sumption—conservation of the parallel determination—seems to entail that they are not (Schuster 2000, 267-70).

All this suggests that Descartes did not obtain his two dynamical premises through a deep inquiry into the conceptual and empirical requirements of a mechanical theory of the propagation and refraction of light in actual media. It seems more plausible to associate the premises closely with the very geometry of the diagrams in which Descartes depicts and constructs the paths of refracted rays, once we understand the underlying dynamical rational of his proof. The question is: Were the dynamical premises post-facto glosses of geometrical constructions arrived at in some other way? Although Descartes dabbled with the physico-mathematics of refraction of light as early as 1620, recent work shows he discovered the law of refraction independently of any mechanical assumptions and through a process entirely within the bounds of a traditional mixed mathematics approach to optics. Then, the geometrical diagrams expressing his newly found law suggested to him (in physico-mathematical manner) the precise form and content of his two dynamical premises for a mechanistic theory of light and their mode of relation in explaining refraction.

In Paris in 1626/27 Descartes, collaborating with the mathematician Claude Mydorge, discovered the law of refraction of light. This was accomplished independently of, but in the same manner as Thomas Harriot, who had first discovered the law around 1598, using only traditional geometrical optics. (**Figure 5**)

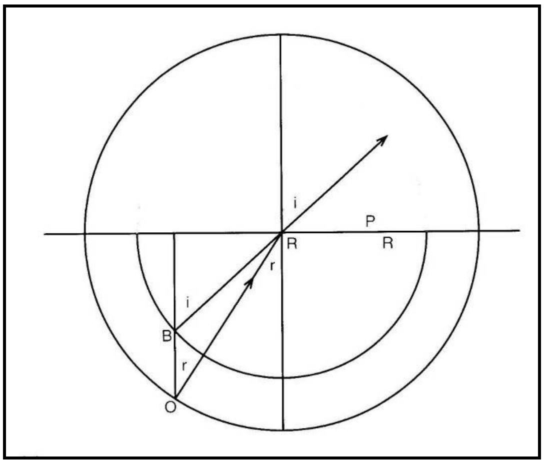

Figure. 5 Thomas Harriot's Key Diagram

Harriot used the traditional image locating rule to map the image locations of point sources taken on the lower circumference of a half submerged disk refractometer.[6] (Lohne 1963, Buchdahl 1972) This yielded a smaller semi-circle as the

6 The traditional image locating rule held that in Figure 5 the image of point 0, for example, which is immersed in water, will be seen at B, the intersection of the normal from 0 to the refracting interface, with the line RB, the extension of the path of the ray refracted into the air,

locus of image points and hence a cosecant law of refraction of light. In a letter, whose content dates from 1626/7, describing an identical cosecant form of the law, Mydorge presented a virtually identical diagram (**Figure 6**), but flipped the inner semi circle above the interface as a locus of point sources for the incident light. (Mersenne I 404-15; Schuster 2000, 272-5)

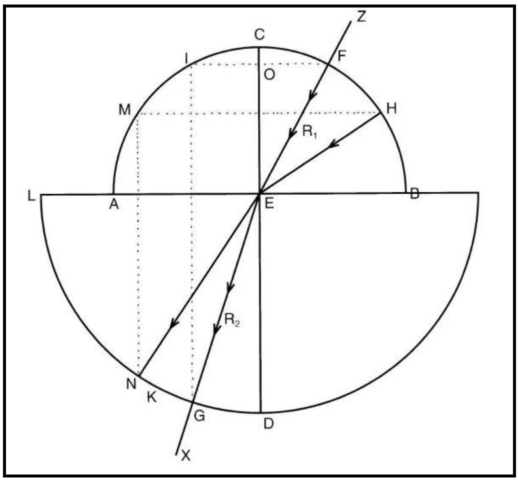

Figure 6. Mydorge's (and Descartes') Cosecant Form of the Law of Refraction

Figure 6 closely resembles Figure 4, the derivation of the law of refraction using Descartes' conditions from the *Dioptrique* and his theory of light as instantaneously propagated tendency to motion. It is the key to unpacking the co-evolution of Descartes' theory of light and his dynamics of corpuscles. (Schuster 2000, 275-277) After his discovery of the law of refraction by these purely geometrical optical means, issuing in the cosecant form of the law (Figure 6), Descartes sought to explain the law using a dynamics of corpuscles. Working in the style of his physico-mathematics, he transcribed into dynamical terms some of the geometrical parameters embodied in the cosecant representation. The resulting dynamical principles concerning the mechanical nature of light were, as we have seen: [1] the absolute quantity of the force of the ray is increased or decreased in a fixed proportion, whilst, [2] the parallel component (parallel determination) of the force of a light ray is unaffected by refraction. By October 1628 Descartes used these concepts to explain the law of refraction to his old friend Beeckman. Descartes revealed a remarkable analogy between the causes of the refraction of light and the behaviour of a bent arm balance beam whose arms are immersed in media of differing specific gravity. This analogy expresses precisely the two dynamical

back into the water. Willebrord Snel's initial construction of the law of refraction also followed the type of path indicated by the Lohne analysis. (Vollgraff 1913, 1936; deWaard 1935-6) principles later used behind the *Dioptrique* proof of the law of refraction. (AT X 336; Schuster 2000, 290-295)

Hence, Descartes' physico-mathematical trajectory explains the puzzle of why he was so focused on keeping the two dynamical premises, despite their dubiousness: It was not just because they allowed 'deduction' of the law of refraction. It was also because, physico-mathematically, they had come from the well grounded, mixed mathematical law. (Schuster, 2012) For Descartes this physico-mathematical interpretation of the cosecant law of refraction diagram arguably became the paradigmatic case of deriving natural philosophical causes from a well founded and geometrically clearly represented mixed mathematical result. Furthermore, his two mechanistic conditions for a theory of refraction in turn suggested the two central tenets of his mature dynamics as he composed *Le Monde* (1629-33). The first rule of nature in *Le Monde* asserts the conservation of the quantity of the instantaneously exerted force of motion of a body in the absence of external causes. (AT XI 38) This rule subsumed and generalised [1]. The third rule of nature defines what we above called the principal determination of the instantaneously exerted force of motion of a body, along the tangent to the path of motion at the instant under consideration. In the absence of external constraint, the principal determination would be conserved from instant to instant. Thus, this rule subsumed and generalised [2]. For Descartes the basic laws of light — itself an instantaneously transmitted mechanical impulse—immediately revealed the principles of the instant to instant dynamics of corpuscles. (Schuster 2000, 302-3)

In sum, then, when assessing the physics of Descartes from a developmental perspective, it is crucial to see that the principles of the dynamics of corpuscles governing that system arose from his physico-mathematical project. Epochal results in traditional geometrical optics allowed insight into the realm of causes in a mechanistic theory of light, and were then extrapolated as principles of the emergent corpuscular-mechanical system.

1.3 Descartes' Career Inflection 1628-33

In describing Descartes' elaboration in *Le Monde* of his corpuscular dynamics out of the results of his physico-mathematical optics, we have skipped over a crucial inflection point in his career, in the period 1628 to 1633. Descartes stopped being a practitioner of piecemeal physico-mathematics with corpuscular-mechanical leanings and turned into the designer of a systematic corpuscular-mechanical natural philosophy, which enjoyed certain theological and metaphysical groundings and displayed on the technical level residual physico-mathematical genes. Why and how did this happen?

1.3.1 The Failure of the Dream of Universal Method

Descartes in the years 1618-28 not only had physico-mathematical hydrostatics, optics and piecemeal corpuscular mechanism on his mind, but he also in 1618-20 had hit on a series of grand intellectual projects. In quick succession he envisioned two breathtaking projects reaching beyond physico-mathematics: universal mathematics and universal method. First, he imagined his universal mathematics encapsulating and transcending "mere" physico-mathematics. Then, in a peak of excitement later in 1619, around the time of his famous dreams on St. Martin's Eve, he envisioned his universal method, which was to absorb universal mathematics and move on much further. Traces of the universal mathematics project are embedded in the text of his *Rules for the direction of the mind*, the so- called rule 4B,[7] whilst an early statement of the method was inscribed in a text termed rule 4A which Descartes inserted into the *Rules*, along with most of the texts of rules 1-3 and 5-11, with some small exceptions. (Schuster 1980, 54) The young Descartes, therefore, was not just struggling to work out a physico-mathematics with possible corpuscular-mechanical bearings. He was also a dreamer of gigantic and seductive methodological fancies. (Schuster 1986) [8]

These endeavours came to a critical impasse in the later 1620s. After his optical breakthrough, and working partly in the shadow of Marin Mersenne's cultural battle against both radical scepticism and religiously heterodox natural philosophies, Descartes picked up universal mathematics and method again in detail, and tried to write a unified treatise about his earlier dream of a methodologically sound universal mathematics, the unfinished *Rules for the direction of the mind*. (Schuster 1980, 58-69) Systematic natural philosophy had no part in this vision. Natural philosophy, reduced to the content of solid but piecemeal physico-mathematical results—as in his physico-mathematical hydrostatics or optics—would become just one pursuit, within the total range of application of this universal mathematics run by a master method. Unfortunately, the renewed *Regulae* project of the late 1620s did not blossom into the intended magisterial work of method and universal mathematics. It collapsed under its own weight of self-generating mathematical, epistemological and ontological problems and contradictions, an outcome that shaped Descartes' next, decisive career moves. (Schuster 1980, 73-80)

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7 So-called by J-P Weber (1964) who first showed this. After Weber, others further articulated his findings, for example, Schuster (1980) 51-55, and Gaukroger (1995) 111ff, which builds on a synthesis of Weber and Schuster.

8 Many modern scholars now hold that such grand, set-piece doctrines of scientific method, such as Descartes', cannot and do not control and guide living practice in any given field of research, let alone across the entire gamut of disciplines. Descartes' technical achievements in mathematics, the mixed sciences, and natural philosophy cannot and should not therefore be explained as applications of his method. A good example is the contrast between how Descartes actually discovered the law of refraction of light, and its mechanical rationale, as we have examined it above, and the fairy tale he tells about this in rule 8 of *Rules for the direction of the Mind*. (Schuster 1993, 201-203; 2000, 300-302)

Between late 1629 and 1633 Descartes was engaged in constructing his first system of natural philosophy, *Le Monde*. At the same time he was devising two doctrines meant to buttress parts of that natural philosophical system: [1] He worked on the first skeletal lines of his dualist metaphysics, with its strict ontological distinction between mind and matter, the latter construed as a completely space filling matter-extension; and [2] he took up and revised some elements of a voluntarist and creationist theology. He had never previously engaged in projects of this type. Work on the *Regulae* stopped. The method, the universal mathematics and the physico-mathematics supposedly contained within the *Regulae* were never again further articulated. Nor did Descartes ever again represent himself simply as a 'physico-mathematician' or practise the style of piecemeal, problem oriented physico-mathematics that had marked his earlier years. However, continuities persisted. The Descartes of *Le Monde* in 1633 was not the Descartes of the *Meditations* and *Principles of Philosophy* of the mid 1640s; yet, that later Descartes was indeed a systematic philosopher of nature and metaphysician, just what Descartes became between 1629 and 1633.[9] Similarly, the inflection of Descartes' agenda and identity between 1629 and 1633 had not completely erased his earlier practices or products. They lived on, sifted, revised and retranslated. His new systematic natural philosophy bore definite marks of his aims and results in physico-mathematics, and his physico-mathematical optics and the principles of the dynamics of corpuscles it suggested and supported, became central pillars of his system of mechanical philosophy. We therefore need to look at the work done in grounding Descartes' natural philosophical system by his voluntarist/creationist theology and his dualist metaphysical doctrine of matter extension.

1.3.2 Voluntarist/Creationist Theology and the Foundations of *Le Monde*

The work on voluntarist theology was initiated in 1630, in parallel with the natural philosophical moves leading directly toward *Le Monde*. It had a legitimatory role in the newly emergent natural philosophy, especially in regard to Descartes' new laws of corpuscular dynamics. Starting from the proposition that creation utterly depends upon God's will, Descartes elaborated a doctrine of continuous creation, or at least a doctrine of the necessity of God's continuous sustenance of creation from instant to instant. He stresses the necessity of God's acting at each moment both to conserve the existence of natural bodies and their modes (including force of motion and rest), and, to enforce the rules according to which natural change occurs. At the moment of a corpuscular impact, God instantaneously adjusts the

9 Although it is not as though Descartes mature intellectual persona emerged all at once in these years. His career after 1633 is marked by numerous further twists and reorientations, increasingly keyed to public debate and controversy. Works that make clear the continuing patterns of change in Descartes' career after 1633 include most notably Gaukroger (1995); Clarke (2006) and most recently Machamer and McGuire, (2009). Of these only Gaukroger gives sustained and useful attention as well to the earlier period, down to 1633.

quantities of force of motion and the determinations that will characterize the corpuscles concerned in the instant after the impact. God does this by following certain laws and rules of impact He has framed and 'ordinarily' follows. Consider Descartes' the first 'rule of nature' in *Le Monde*:

> Each part of matter always continues to exist in the same state as long as other bodies do not constrain it to change that state. If it has a certain size, it will never become smaller, unless other bodies divide it... if a body has stopped in a given place, it will never leave that place unless others force it out; and if it has once commenced to move, it will continue along with the same force, until other bodies stop or retard it. (AT XI 38)

While this seems to assert the conservation of the motion (or rest) of a body in the absence of external constraints, closer inspection reveals that Descartes is speaking of the instantaneously exerted "force of motion". Consistent with our analysis of the development of his optics and his principles of a dynamics of corpuscles, this is the quantity which is conserved. Here in *Le Monde* and later in the *Principles* Descartes is using the term in relation to his voluntarist, or more properly his creationist understanding of ontology: [10] God must continually support (or re-create) bodies and their attributes from moment to moment. This implies that in the final analysis a body in phenomenal translation, in motion, is really being recreated or continually supported at successive spatial points during successive temporal instants. In addition, in each of those instants of re- creation, it is characterized by the divine injection of a certain quantity of "force of motion". Similar reasoning, framed for presentation by theological concerns, arrives at the second law in *Le Monde*: the conservation of the total quantity of force of motion in collisions between bodies. At the instant of collision God conserves the total quantity of force of motion and redistributes it amongst the bodies involved.[11]

As regards the third law of nature in *Le Monde*, we have seen that it defines what we above called the principal determination of the instantaneously exerted force of motion of a body, along the tangent to the path of motion at the instant under consideration. Viewed in theological terms, it says that in the absence of external constraint, this particular directional quantity of force of motion would be conserved by God from instant to instant. The instantaneous determination of motion is rectilinear, *because* only a straight line can be grasped entirely in an instant without God having to calculate or observe the path of the body at one or more other instants past or future. A straight line can be defined in any instant of the body's motion through God's consideration of its present position and the implicit endpoint of the (straight) line along which one would point in saying that, 'At this instant the body is in the act of moving in ***that*** direction.' By contrast, to conserve

10 Harrison (2002, 69) has importantly pointed out that one should not simply conflate the notion that God's will is primary with the idea that nature is totally dependent upon God: "the doctrine that places God as the direct cause of what takes place in nature is thus independent of a voluntarism according to which the divine will is above reason."

11 This law, appearing as the third law of nature in the *Principles* will be significantly altered and articulated into a number of detailed rules of collision. See Section 2.1.2 below.

a curved determination God would have to recalculate the determination at each instant based on memory or prediction of one other point. Thus God would not conserve the body, as Descartes says, "precisely as it is at the same instant that He conserves it" (AT XI 44; G 30) but rather in a manner also dependent upon consideration of its past or future path.

In sum, since the corpuscular-mechanical universe of *Le Monde* was to be stripped of all Aristotelian forms, qualities and teleological processes, impact and pressure became the sole causes of natural change. Such a universe of perfectly hard particles foreclosed consideration of the problematical quality of 'elasticity'; but, by the same token, in such a universe impact and pressure were rendered inexplicable by natural causes. At just this point the recourse to theology became necessary in order to provide a rationale for the laws of nature, explicating how bodies interact at the mechanically opaque instant of collision. The answer on this legitimatory level is that material bodies do not in fact interact physically and causally with each other. They only appear to do so on a phenomenal level, this being the expression or effect of the rule bound ways in which God from moment to moment maintains or alters their forces and determinations of motion.

Descartes did not proceed in *Le Monde* to deal with these issues at a further level of philosophical or theological analysis. He was primarily interested in the reduction of the analysis of phenomenal translation to the consideration of divinely governed instants of time. This procedure was critically important to his enterprise of justifying the principles of his dynamics (which we know had always *in practice* been focused on the dynamical status of corpuscles in motion or tending to motion in terms of their instantaneous quantities and determinations of force). Descartes' discussion of the three laws of nature thus aimed to impart theological backing to his species of dynamics, and to this end he selectively employed resources of voluntarist theology articulated with special attention to the punctiform character of God's conserving concourse.

Of course, for the philosophically scrupulous, the conceptual entanglements of Descartes' voluntarist grounding for natural philosophy are obvious and began to show up in Descartes' thinking later with his most important published works, the *Meditations* and especially the *Principles of Philosophy*. In the latter work he articulated his voluntarist, punctiform dynamics of particles by reframing it within a more consistently deployed neo-Scholastic terminology of substance, mode and cause. This prompted debate about the correct Cartesian interpretations of these categories amongst his later seventeenth century followers and detractors, and still promotes discussion amongst leading historians of philosophy. (Schmaltz 2008; Machamer and McGuire 2009) Nevertheless, for the purposes of the history of physics, and the remainder of our discussion in this chapter, a possible way to comprehend how the voluntarist corpuscular dynamics was meant to work was provided by Gueroult (1980). It suggests a number of distinctions that can be applied to Descartes' voluntarist corpuscular dynamics in both *Le Monde* and in its scholastically articulated form in the *Principles*: One should distinguish between

[1] instantaneously exerted force of motion (causal), which is identified with God's own causal action and is discussed in the language of theology and metaphysics; this is equated with [2] an instantaneous force of motion (modal), which is the manifestation of that divine action in the material world, potentially observable in some of its effects, and taken by humans on the level of technical natural philosophical discourse of dynamics as a moment to moment possession of a body moving or tending to motion. Finally,[3] our commonsense notions of motion in space and time, unenlightened by either theology or natural philosophy, are seen to be merely appearances caused by God's law–like, moment to moment causal actions upon matter.

1.3.3 Dualist metaphysics and the Grounding Role of Matter-Extension

Descartes' project in dualist metaphysics differed from that in theology, in that he began work on it before the events of mid and late 1629 which crystallised into the project of *Le Monde*.[12] The dualism of mind and body was initially aimed at addressing problems emergent in the *Regulae*; but it soon came to play an important grounding role behind *Le Monde*, as that text next began to take shape. The essential matter theoretical finding of his metaphysics is that the notion of a void space is unclear and that a conceptual analysis of our ideas shows that every extended space is filled with matter, indeed is matter. The impossibility of any void spaces means that if a particle is to move, the 'space' it is about to vacate must be simultaneously filled by another particle of equivalent volume. This further implies that any motion at all entrains an instantaneous circuit of displacement, leading to the filling of the about to be voided space. Vortices became imaginable, and in turn invited detailed dynamical description. So, the vortex mechanics, central to the entire content and structure of *Le Monde* and the *Principles,* was elicited from the matter-extension plenum which itself followed from the initial metaphysical work. Descartes might well have thought of this as an admirably fruitful course of intellectual discovery, from the metaphysics down to the intricacies of the vortex mechanics. The emerging sophisticated vortex celestial mechanics in turn became the core exemplar for working out wide swathes of the natural philosophy—not just the celestial mechanics of planets, but of comets as well, plus the theory of light in cosmic setting, the theories of local (planetary) gravity, and tides, and the behaviour of planetary satellites. It is true that Descartes' embryonic dualist metaphysics eventually resided not so much inside the text of *Le Monde*, as immediately behind it. None of the detailed metaphysical argument presented later in the *Dis-*

12 In November 1630, he wrote to Mersenne that he had indeed begun work on what he termed 'a small treatise of metaphysics' early in the year, during his first few months in the United Provinces. The 'principle points' of the work he claimed to be to prove the existence of God and the immortality of the soul, when separated from the body. Descartes to Mersenne 25 November 1630, AT I 182.

cours, Meditations and *Principia* appear in *Le Monde*, and presumably the arguments were in only preliminary and very basic form. However, his metaphysically grounded commitment to a theory of matter-extension and a resulting plenist universe, is essential to *Le Monde*, as it explicitly was later to the *Principles*.

1.4. Vortex mechanics

The vortex celestial mechanics, as presented in the *Principles of Philosophy* and *Le Monde*, are the 'engine room' of Descartes' system of natural philosophy. As such the vortex mechanics exemplify what we may term a 'plenist–realist' style of explanation which permeates the entirety of Descartes natural philosophical project. This style prohibits mathematical abstraction or idealisation of the sort found in the traditional mixed mathematical sciences, and in the mechanics of Galileo which was to appear in 1638. Instead, Descartes' plenist realism favours explanations which, arguably, are inclusive or holistic about the factors taken into account, and in doing so supposedly reflect the 'real complexity' of phenomena in the plenist universe, and the 'real set' of causes in play, not some 'abstract' or 'fictitious' picture. Natural philosophical explanations thus need immediately and completely to grasp the tangle of causes and conditions in play behind any phenomenon in this plenum universe. Explanations must not abstract away from some or most of these causes, issuing in over simplified (strictly not real) models of phenomena under study. This, however, did not mean that physico-mathematical procedures or results were banned from *Le Monde* and the *Principia*. We have already seen this in Descartes' reliance on the belief that his physico-mathematical optics had cut to the core of that plenist reality, revealing, as far as he was concerned, the underlying dynamics of corpuscles that runs the cosmos.

Descartes starts his vortex theory with an 'indefinitely' large chunk of divinely created matter-extension in which there are no void spaces whatsoever. When God injects motion into this matter-extension, it is shattered into micro-particles and myriads of 'circular' displacements ensue, forming large numbers of gigantic whirlpools or vortices. This process eventually produces three species of corpuscle, or elements, along with the birth of stars and planets. The large, irregularly shaped particles of third element form all solid and liquid bodies on all planets throughout the cosmos, including the earth. Interspersed in the pores of such planetary bodies are the spherical particles ("*boules*") of second element. The second element also makes up the bulk of every vortex, while the spaces between these spherical particles are filled by the first element, which also constitutes the stars, including our sun. Next, using his theory of the dynamics of corpuscles, Descartes introduces a vortex stability principle. In the early stages of vortex formation, before stars and elements have evolved, the then existing vortical particles become arranged so that their centrifugal tendency increases continuously with distance from the center. (AT XI 50-1; G 33) As each vortex settles out of the original chaos, the larger corpuscles are harder to move, resulting in the smaller ones acquiring higher speeds. Hence, in these early stages, the size of particles decreases and their speed increases from the center out. But, the speed of the particles increases proportionately faster, so that force of motion (size times speed) increases continuously. **Figure 7** shows the distribution of size and speed of the particles in any vortex before a central star and the three elements have formed.[13] (Schuster 2005, 46)

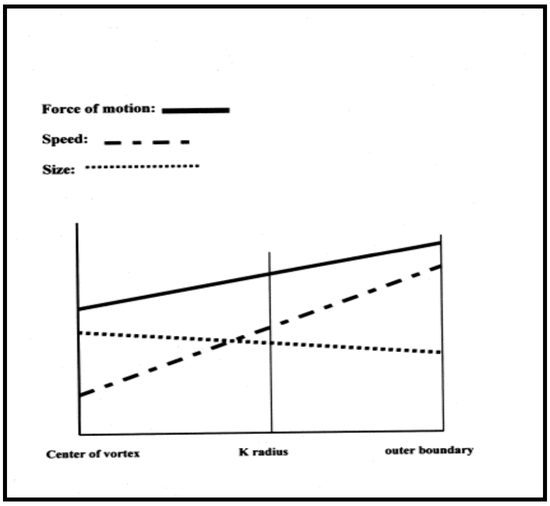

Figure 7. Size, Speed and Force of Motion Distribution of Particles of 2nd Element, Prior to Existence of Central Star

It is absolutely crucial to notice that according to Descartes the subsequent advent of a central star—a correlate of the formation of the three final elements—permanently and definitively alters the original size and speed distribution of particles in a vortex. It is the star's disturbing effect on the original size/speed distribution that allows the planets to maintain stable orbits and which also creates the limits of the trajectories of comets. A star is made of up the most agitated particles of first element. Their agitation, and the rotation of the star, communicate extra motion to *boules* of the vortex near the star's surface. This increment of agitation decreases with distance from the star and vanishes at that key radial distance, called K. (**Figure 8**)

13 In Figures 7, 8, 9 and 10 straight lines are used to represent the functional relations amongst *boules'* sizes, speeds and distances from the central star derived from the verbal expressions in Descartes' texts. It is not intended that Descartes necessarily or consistently entertained such linear relations. What is important is the general representation of the force-stability principle and how that relates to Descartes' claims about the size and speed distributions with distance.

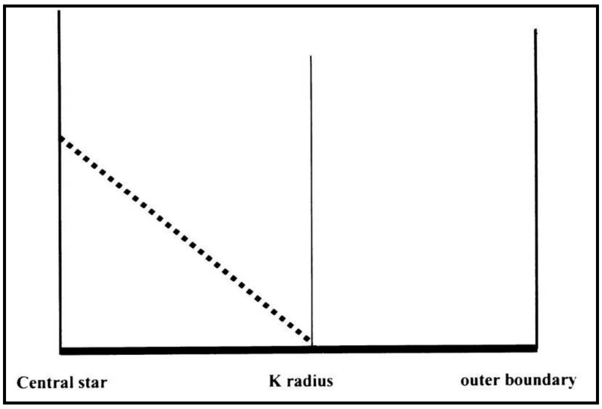

Figure 8. Agitation Due To Existence Of Central Star

This stellar effect alters the original size and speed distribution of the spheres of second element in the vortex, below the K layer. We now have greater corpuscular speeds close to the star than in the pre-star situation. But the vortical stability principle still holds, so the overall size/speed distribution must change, below the K layer. Descartes ends with the situation in **Figure 9**, with a crucial inflection point at K: Beyond K we have the old (pre-star formation) stable pattern of size/speed distribution; below K we have a new, (post–star formation) stable pattern of size/speed distribution. (Schuster 2000, 49) This new distribution turns a vortex into a machine that locks planets into appropriate orbits below K and extrudes comets into neighbouring vortices. Celestial vortices only behave this way because a star, made of first element, happens to inhabit the centre of each vortex, transforming its mechanical parameters and performance. This is Descartes' version of Kepler's emphasis (compared to Copernicus himself) on the physical-causal role of the sun in orbital mechanics.

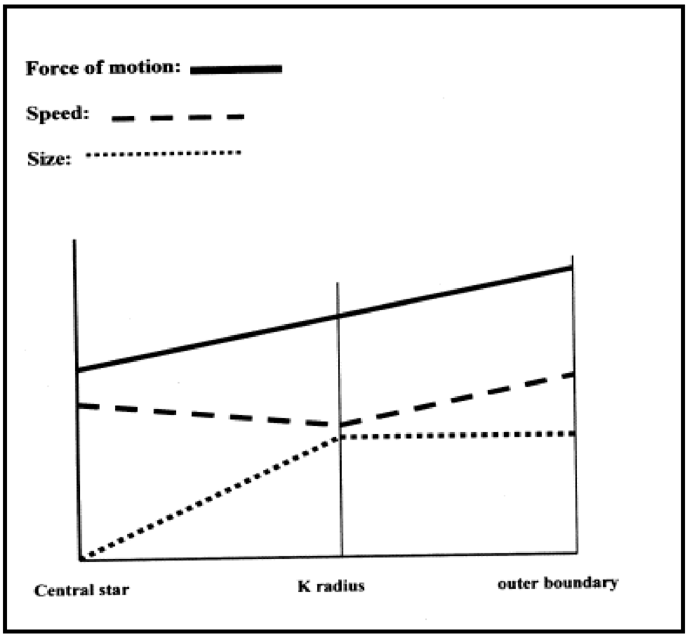

Figure 9. Size, Speed and Force of Motion Distribution Of Particles Of 2[nd] Element in a Stellar Vortex

The key question in Cartesian celestial mechanics thus becomes this, "When and why is the centrifugal tendency or orbiting bodies actualised as centrifugal motion, and when and why does that not happen?" The answer has two dimensions. First of all, and unsurprisingly, Descartes' sling exemplar for his dynamics plays a key role. In the vortex what constrains a planet into a curved path, on analogy to the sling, are the particles of second element, neighbouring it and superjacent to it. (Schuster 2005, 50-52) Secondly, as to why different planets maintain orbits at different distances from the central star, but all within the K radius, the key is Descartes' concept of the "massiveness" or "solidity" of a planet. By this he means its aggregate volume to surface ratio, which is indicative of its ability to retain acquired motion or to resist the impact of other bodies.

The resulting explanation runs as follows: The *boules* of second element making up a vortex also vary in volume to surface ratio, or massiveness, with distance from the central star. This may be gathered from Descartes' stipulations concerning the variation of the size of the *boules* with distance from the central star, illustrated in Figure 9. Figure 10 then represents what we may term the 'resistance curve', that is, the variation in massiveness of second element *boules* with distance from a star. A planet is locked into an orbit at a radial distance where a double balance is achieved amongst the relevant vortical mechanical parameters: On the one hand the centrifugal tendency of the planet, a function of its aggregate solidity, is balanced by the resistance to being displaced downward of the second element *boules* composing the vortex in the vicinity of the planet—that resistance similarly depending on the massiveness or surface to volume ratio of the those particular *boules*. On the other hand, simultaneously a balance is realized between the centrifugal force of the subjacent second element *boules* at that radius in the vortex and the resistance to being displaced downward offered by the planet (owing to its degree of massiveness). Hence the condition for a piece of third matter to be in stable orbit in the vortex can expressed as $\mathbf{F^m{}_b = R_{mu}}$ **and** $\mathbf{F^m{}_{ml} = R_b}$ Where $\mathbf{F^m{}_b}$ means force of motion of the orbiting body; $\mathbf{R_{mu}}$ means resistance of superjacent layer of *boules* (upper medium) to being extruded downward by the orbiting body; $\mathbf{F^m{}_{ml}}$ means force of motion of subjacent layer of *boules* (lower medium) and $\mathbf{R_b}$ means resistance of orbiting planet to being extruded downward by subjacent layer of *boules*, where both $\mathbf{F^m{}_b}$ and $\mathbf{R_b}$ are functions of the planet's solidity, while $\mathbf{R_{mu}}$ and $\mathbf{F^m{}_{ml}}$ are functions of the solidity of the superjacent and subjacent boules respectively.

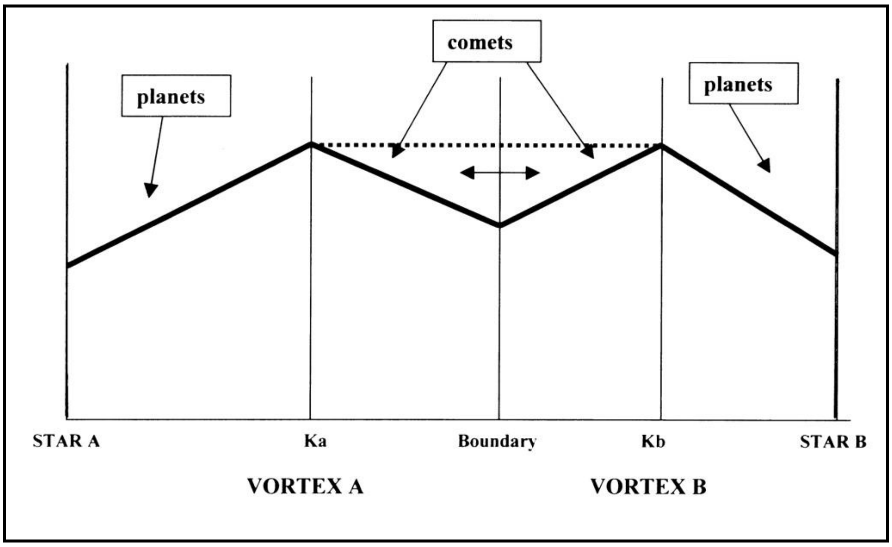

Figure 10. 'Resistance Curve': Derived from Volume/Surface Ratios Of Spheres Of 2nd Element

The most 'massive' or 'solid' planet in a star system will be closest to, but not beyond the K layer—as Saturn is in our planetary system. Comets are planets of such high solidity that they overcome the resistance of *boules* at all distances up to and including K. Such an object will pass beyond the K level, where it will meet *boules* with decreasing volume to surface ratios, hence less resistance, and be extruded out of the vortex into a neighbouring one. But, flung into the neighbouring vortex, the comet meets increasing resistance from its *boules* above that vortex's K distance. Picking up increments of orbital speed, the comet starts to generate centrifugal tendency again, eventually being flung back out of the second vortex.

To summarise, then, each vortex is a locking and extrusion device. Given Descartes' theory of the varying solidities of planets and comets, the variation of size and speed of vortical boules of second element with distance from the central star entails that planets are locked into orbits of differing radii. Comets are objects extruded from vortex to vortex, first 'falling' into a vortex and being extruded out.[14] The make–up and dynamical behaviour of the central stars are crucial, not to the bare existence of vortices, but to the creation of planet–locking/comet–extruding vortices. Otherwise extrusion would be the universal rule. Multiple vortices are conceptually necessary, as each vortex is set in a container made of contiguous vortices, exerting a kind of centripetal backwash at its boundary.

Properly understood, Descartes' vortex mechanics reveals itself as a science of equilibrium, thus underscoring its sources in statical and hydrostatical exemplars, heavily refracted through the *sui generis* style of Descartes' physico-mathematics. The forces at work upon a planet can only be fully specified when orbital equilibrium has been attained, although, of course no actual measurements are involved. The rise or fall of a planet (or comet) results from the breakdown of equilibrium and cannot be defined mathematically. Hence celestial equilibrium and disequilibrium are analogous respectively to equilibrium and slippage in systems rigorously treated in the statics of Archimedes or Simon Stevin. However, Descartes' vortex celestial mechanics differs from classical statics. It deals with equilibrium conditions described not in terms of volumes, densities and specific weights, but rather in terms of Cartesian centrifugal tendency to motion, as well as relative 'solidity', a property defined by volume to surface relations aggregated over the constituent third matter corpuscles of a planet or comet.[15]

Descartes' vortex celestial mechanics is therefore intimately related to his youthful program of physico-mathematics**.** Descartes holds that at the descriptive level, in terms of appearances, orbital establishment and placement are 'statical phenomena'. We saw that in his hydrostatics work of 1619, he insisted that a macroscopic, statical regularity—the hydrostatic paradox—be reduced to corpuscular-mechanical terms. Similarly, in his mature celestial mechanics he insists that beneath the observable radial equilibrium of orbits there resides a corpuscular-mechanical reality, a micro machinery, whose behaviour is explained by his new dynamical concepts. As the corpuscular-mechanical and dynamical reduction of the hydrostatic paradox was an exercise in physico-mathematics, so too is the vortex mechanics. Taken as a whole, therefore, the vortex celestial mechanics are a

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14 The term 'falling' is chosen quite deliberately. Descartes makes it clear in discussing the placement of planetary orbits that a planet 'too high up' in the vortex for its particular solidity is extruded sun-ward, falling (and spiralling) down in the vortex to find its proper orbital distance. Descartes' theory of local fall, and theory of the orbital motion of the moon, when taken in their simplest and most charitable acceptations, both also make use of this notion of falling in a vortex until a proper orbital level is found (assuming no other circumstances prevent completion of the process, as they do in local fall of heavy terrestrial bodies near the surface of the earth). Ultimately, however, the interpretation of these two theories becomes more fraught, requiring additional interpretative attention, as Note 15 makes clear.

15 This point is very important and helps explain certain difficulties Descartes met in tyring to deal with the local fall of 'heavy' bodies—a phenomenon he tends to want to describe using the vocabulary of classical hydrostatics, which, however, simply will not mesh with his vortex mechanics, a science, at bottom, of 'solidity' or volume to surface relations.

hybrid entity: part physico-mathematical certainly, but also clearly a piece of generic natural philosophical discourse, playing the central role in this new corpuscular-mechanical system.

1.5. Explanatory Style of Mature Cartesian Physics

It is a mistake to take seriously Descartes' occasional claims to have been able to deduce—as if according to a mathematical ideal of "demonstration"—his entire system of natural philosophy from absolutely certain metaphysical principles. This folklore arose from the strictly deductivist tone of Descartes' method in both his formal and more offhand statements about it. In his mature work, after the demise of his detailed project on method in the *Regulae* in 1628, Descartes increasingly came to see that neither the details of particular explanatory models, nor the facts to be explained, could be deduced from metaphysics. *Le Monde*, with its tacit metaphysical grounding, began a process of reflection on the demands of corpuscular-mechanical explanation which is fully expressed at the end of the *Principles*. (AT VIII1, 325-9; M 285-88) Descartes tells the reader that when one constructs corpuscular-mechanical models to explain various phenomena, such as gravity, light, magnetism, sun spots, planetary motion, sensory perception and animal locomotion, nothing should be asserted in any particular model that contradicts any metaphysically derived certainties (concerning the existence of a material plenum, the laws of motion and presumably the existence of the three elements). Hence detailed corpuscular-mechanical explanatory models have a necessarily hypothetical character and can achieve at best only 'moral certainty'. (Clarke 1977; Buchdahl 1970, 97, 118-26 ; Sabra 1967, 21-45)

This is not simply a meta-commentary by Descartes on his physics. It reflects his intellectual and discursive processes in forming explanations, since, to reiterate, he had no 'method' for inscribing such models. (Schuster 1986, 62-6). It also highlights the crucial role of factual evidence in Descartes' mature physics. To design any particular middle level explanatory model, Descartes would have to select, weigh and appropriately deploy hard empirical evidence, which could include facts needing explanation, or facts lending credibility to the explanatory model offered. In the light of the 'evidence', and the metaphysical 'constraints' on hypothesis formation, a specific corpuscular-mechanical model for the phenomena would be constructed. Descartes' mature views on the central role of facts and evidence in the framing of explanations in natural philosophy helped inspire the probabilist and hypothetical-deductivist approach to explanation with a broad and loose commitment to a corpuscular-mechanical natural philosophy widely held in the later seventeenth century, at least until Newton began his campaign to install a methodological rhetoric of inductive certainty.

1.6. Descartes' Mature Physics: Pursuing Novel Facts and System-Binding Strategies

While *Le Monde* and *Principles of Philosophy* both contain Descartes' vortex mechanics, his mechanistic theory of light in cosmic setting and his theories of gravity, tides and satellite motion, the *Principles* are much more elaborate on the laws of collision and contain the explicit metaphysical grounding in dualism. However, looked at as systems of physics, the two treatises differ in two other ways which are often overlooked: First of all, the *Principles* deals with a much richer harvest of significant or novel matters of fact than one finds in *Le Monde*. Descartes' pays sustained and detailed attention to the phenomena of magnetism, sunspots, *novae* and variable stars. He co-opts and re-writes into mechanistic terms the experimental archive of William Gilbert's *De Magnete* (1600), which he ignored in *Le Monde*; he takes on board the existence and known properties of sunspots, a topic he avoided in *Le Monde*; and he deals extensively with variable stars, only recently discovered, and relates them theoretically to novae which, also, did not appear in *Le Monde*. Secondly, the *Principia* deals with these series of matters of fact in a strategic way entirely missing from *Le Monde*—indeed the *Principia* contains new system-binding strategies that virtually define the character of Descartes' mature physics.

In the *Principles* Descartes weaves his ranges of newly available, or newly chosen, matters of fact into explanatory and descriptive narratives which have a definite theme and aim: they are *cosmographical* in nature; that is, they have to do with how the relations between heavens and earth are theorised in a natural philosophy. (Biro 2009, Schuster and Brody 2012) Cosmography was an actor's category, used initially in geo-centric natural philosophies like Aristotelianism, where the 'relation' heavens /earth was not identity of matter and cause, but rather considerable difference. However, for Descartes and other realist Copernicans, the earth was a heavenly body and the traditional heavenly bodies were arguably 'like' the earth. That means cosmography became a contested space of great natural philosophical significance. The relation of 'the earth' to everything else, that is, 'the heavens', changed. It became the relation of any and all planets, their structures and geneses, to any and all stars, their nature and developmental patterns. Claims about the structure of the earth could now be exploited cosmographically, for realist Copernican ends. Gilbert had done this in his study of terrestrial magnetism; Galileo with his theory of the tides. Following this strategic lead, Descartes designed the final two Books of the *Principles* as an interrelated set of such radical, realist-Copernican cosmographical threads of narrative-cum-explanation. In these respects it is nothing like *Le Monde*. Descartes' cosmographical strategy has a foundation and a point of leverage: the foundation is his co-optation of Gilbert's claimed facts of magnetism linked to a denial of Gilbert's natural philosophical concepts and aims; the fulcrum is his strategy for handling sun spots.

William Gilbert's *De Magnete* (1600) was an impressive natural philosophy, grounded in experiments interpreting magnetism as an immaterial power. In his *Principles* Descartes accepted Gilbert's experiments, but explained magnetism mechanistically, based on the movements of two species, right– and left–handed, of 'channelled' or cylindrical screw shaped particles of his first element. Descartes claimed that magnetic bodies—naturally occurring lodestone, or magnetised iron or steel—have two sets of pores running axially between their magnetic poles: one set accepting only right handed channelled particles, with the other, directed oppositely, receiving only the left handed particles. Descartes re–explained Gilbert's experiments, including his use of a sphere of loadstone to demonstrate the properties of magnetised compass needles. (AT VIII1 275-311; M 242-272)

However, Descartes did more than appropriate and reinterpret Gilbert's 'laboratory' work. Gilbert called his sphere of lodestone a *terrella*, a 'little earth', arguing that because compass needles behave identically on the *terrella* as on the earth itself, the earth is, essentially, a magnet. Hence, according to his natural philosophy, the earth possesses a magnetic soul, capable of causing it to spin. Magnetic souls similarly cause the motions of other heavenly bodies. In his *Principles*, Descartes, aiming to displace Gilbert's natural philosophy, focuses on the 'cosmic' genesis and function of his channelled magnetic particles. Descartes argues that the spaces between the spherical corpuscles of second element, making up the vortices**,** are roughly triangular, so that some particles of the first element, constantly being forced through the interstices of second element spheres, are forged into relatively stable, longer, cylindrical forms, which are 'channelled' or 'grooved' with triangular cross–sections. All vortical interstitial first element corpuscles, including these bigger, longer channelled ones, tend to be flung by centrifugal tendency out of the equatorial regions of vortices and into neighbouring vortices along the north and south directions of their axes of rotation, where the large ones receive definitive opposite axial twists. The resulting left and right handed screw shaped first element particles penetrate into the polar regions of central stars and then bubble up toward their surfaces, and drift toward the equator, forming by accretion, Descartes claims, sun spots. (AT VIII1 142-8; M 132-6)

Two sets of consequences of the explanatory story so far are crucial to understanding Descartes' strategy in the *Principles*. Firstly, the sun spots formed on a star's surface out of accretions of the curious rimmed and handed first element particles are asserted to have become third matter. That is, they consist of large corpuscles, irregularly shaped and difficult to set in motion, which constitute the bulk of the matter in planets, comets and satellites in the cosmos, as we have seen in discussing the vortex celestial mechanics. In the matter theory of *Le Monde*, third matter corpuscles are said to exist from the moment of creation (AT XI 56-57; G 37), and in that treatise Descartes several times denies the possibility of any transformation of the elements. (AT XI 28, 29-30; G 19, 20) But here in the *Principia*, third matter does not exist from the creation, and only comes into existence by means of the cosmic process of sun spot formation. All terrestrial matter is formed on stars out of the stuff of magnetism.

Secondly, it also follows from Descartes' explanatory narrative that all stars are magnetic, as Gilbert maintained, but in a mechanistic sense, because they are all suited to reception of these oppositely handed, polar entrant first element particles. Moreover, for Descartes, planets are also magnetic, as Gilbert claimed, but again the explanation is mechanical. Descartes describes how a star may become totally encrusted by sun spots. This extinguishes the star, its vortex collapses and it is drawn into a neighbouring vortex to orbit its central star as a planet. (AT VIII1 195-96; M 171) This is the only way planets are formed, so all planets, including our earth, bear the magnetic imprint of their stellar origins, possessing axial channels between their magnetic poles accommodated to the right or left handed screw particles.

As for sunspots, once Descartes has theoretically re-constituted them on the surfaces of stars in terms of his corpuscular-mechanism and theory of vortices and magnetism, he re-derives in terms of his theory their consensually accepted properties:[16] Then in the pivotal move in this entire explanatory campaign, he uses this theory of sunspots further to explain novae and variable stars and, as just mentioned, the origin and nature of all planets, the earth included. Variable stars had only been recognized in the late 1630s, after *Le Monde* and just before Descartes started writing the *Principles*. Alternate creation and destruction of complete sunspot crusts on a star explain its variability. Novae, accepted facts amongst European astronomers since the late sixteenth century, he explains as a sub-class of variables: A nova is a star which has been in an occluded phase and never before observed by humans which then comes into view for the first time, as far as humans are concerned. Subsequently, it might continue to shine or quickly or slowly become occluded again. (AT VIII1 158-62; M 144-7)

Descartes' next move, expressing and completing the cosmographical intentions of his system, involves relating the Earth, and indeed every single planet in

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16 AT VIII1 148-50; M 136-9 [1] We see most sunspots in a belt near the equator and not at the poles, because by the time they have managed to stick together into a mass of third element particles big enough to be visible to our eyes they have covered a considerable distance from the poles. [2] Sunspots are just that, accretions of first matter magnetic particles into denser clumps of third matter—that is the only way third matter is made in the universe. [3] From the way they come into being it follows they have irregular shapes. [4] Being on the sun's surface, the spots are carried along by its rotation. [5] Spots sometimes have a dark nucleus surrounded by a lighter area, because at the lighter parts the accumulation of third element is thinner and lets some light pass through. [6] Bright areas or *faculae* can occur near sunspots: Sunspots restrict the movement of the sun's first element material which then tends to surge away at the edges of the spots, thus being more luminous, the central mass of a spot obstructs that tendency to motion; that is stops the light. [7] Sunspots can disappear. They get worn away by the rotating matter of the sun and disintegrate partially back into first element, partially into smaller bits of third element that then become an atmosphere around the sun. This he terms aether. It surrounds each star, and is inherited by a planet resulting from the death a star, becoming, its atmosphere, as on Earth.

the universe, to a certain pattern of possible stellar development. As indicated above, occasionally, when a star becomes covered with sun spots, its vortex collapses and the defunct star is captured by another vortex, becoming a comet or a planet. Here we encounter what is usually termed Descartes' "Theory of the Earth". He famously explains, in the first forty-five articles of Book IV ("Of the Earth") of the *Principia*, how from a dead, encrusted star there results the formation of land masses, with mountains and declivities, the latter filled with water to form oceans subject to tides (another crucial cosmographical phenomenon for Descartes, as for Galileo). But clearly all planets undergo the same processes: The dynamic of spots encrusting and eventually destroying stars is what accounts in matter theoretical and structural terms for each and every planet found in the universe. So, on this breathtaking vision, every planetary object in the cosmos traces its genealogy to a pattern of events that in principle might befall any 'star-in-a-vortex-afflicted-with-sunspots'. This indeed is a grand cosmographical gambit, meant systematically to bind together his system and to establish his brand of realist Copernicanism of innumerable star and planetary systems, all worked by his corpuscular dynamics and vortex mechanics. (Schuster and Brody, 2012)

Inside the toils of his radical realist Copernican cosmographical explanations *cum* narratives, Descartes did not aim at linear, deductive explanations of each and every particular state of affairs he recognized as a reliably reported matter of fact. Descartes' laws of nature do not function as premises of deductive explanations. Rather, his laws of nature in the *Principia* function as human laws do in the making of legal arguments. The laws are woven, along with carefully selected matters of fact, into flows of argument, narrative lines of description-explanation, of the sort we have just canvassed.[17] Descartes proceeds by asserting a network of basic explanatory concepts involving matter theory, magnetism, vortices and sunspot formation/dissipation that in principle can explain, via *discursive causal story telling*, a spectrum of possible empirical outcomes. The causal stories are filled out according to the varieties of observed outcomes by appealing, loosely, to a variety of possible interactions amongst sunspots, vortices, the surfaces of stars, and the 'aether' of old dissipated sun spot material that floats in each stellar vortex near each star.[18] So, although when compared to *Le Monde*, Descartes' mature physics in the *Principia* values novel matters of fact, the system remained relatively closed

17 In the telling remark that ends Book III (AT VIII1 202; M 177), Descartes asserts that all inequalities of planetary motion can be sufficiently explained using the framework he has provided. Clearly he in no way intends that explanations will proceed by deductions from laws of motion, plus boundary conditions, leading to the exposure and study of various levels and types of perturbations. So, for example, it is not elliptical orbits, and their deviations that he wishes to study, leading to refinement of the relevant laws. Rather he offers a 'sufficient' (verbal and qualitative) explanation of orbital phenomena and the general facts that no orbit is perfectly circular, and that all orbits display variations over time.

18 Descartes introduces the section of Book III of the *Principles*, dealing with sunspots, novae and variable stars at Article 101 by stating: (AT VIII1, 151; M 139) "That the production and disintegration of spots depend upon causes which are very uncertain."

to novel, deep discoveries at the theoretical level, because unexpected observational outcomes were accounted for at the level of contingent narrative formation, rather than by considering modification to the structure of deep concepts.

Part 2: Cartesian Physics and Classical Mechanics: Technical Problems and Shared Goals and Values

This concludes our first task, the exploration of the development of Descartes' physics from a piecemeal physico-mathematics of corpuscular-mechanical leanings to a discursive, cosmographically focussed system of corpuscular-mechanical natural philosophy. Our analysis will be completed by investigating the two types of consequences Descartes' physics had for the emergence of classical mechanics: [1] unintended technical significances; and [2] certain values, goals and strategic positionings which broadly shaped the eventual emergence of classical mechanics, and which manifested themselves in the work of Descartes and other innovative natural philosophers. We explore [1] and [2] within the three core domains of Descartes' physical inquiry—mechanics, optics and celestial mechanics.

2.1. Descartes' Role in the Attempted Transformation of Traditional Mixed Mathematical Mechanics

In his attempt to transform the traditional mixed mathematical field of mechanics, Descartes was responding in his own way to aims and values characteristic of many natural philosophers of the time. When he spoke about 'mechanics', Descartes was appealing in the first instance to readers' understandings of mixed mathematical statics, hydrostatics and the study of the simple machines, fields in which Descartes was fully expert and with which he engaged throughout his career. But, as early as 1619 he twice told Beeckman that he was devising a "mechanics", which seems to have meant a set of rules about corpuscular behaviour derived from his physico-mathematical projects. (AT X 159, 162; Gaukroger and Schuster 2002, 566-7) That is, somehow the field of mechanics was to become part of the business of natural philosophising. Such a goal was by no means unprecedented, because from the early sixteenth century there had been attempts—expressed through classificatory arguments, rhetoric about values and aims, or downright technical gambits—to move mechanics into closer contact with natural philosophising. (Hattab 2005; Laird, 1986)

However, we have seen that Descartes' program in this regard was quite radical. He worked, over time, in a conceptually precise manner to establish a new set of mechanical principles, which would apply to the corpuscular realm and provide the causal register of his new natural philosophy. The resulting doctrine, which he twice terms a 'mechanics' in the *Principles*, (AT VIII1 185, 326; M 162, 285) and which we have dubbed his "dynamics of corpuscles", was novel, *sui generis* and had significant evidential basis in his early work in optics and hydrostatics. It also had three unintended relations to the classical mechanics to come:

[1] Descartes' dynamics of corpuscles by no means resembled what post-Newtonian thinkers would mean by classical mechanics. Descartes' "dynamics", as we have seen, was an essential dimension of his system of natural philosophy, not some free standing, emerging master-science of mechanics, intended to displace natural philosophy. Moreover, it did not involve the mathematical treatment of idealised bodies and motions, microscopic or macroscopic. Indeed it rejected that as a relevant concern in natural philosophy. Hence Descartes' dynamics, central as it was to his entire physics, must in the historical long run appear as just one of several daring attempts to transform mechanics and place it at the centre of natural philosophising.

[2] Nevertheless, Descartes' work on his version of 'mechanics' was shaped by the aim to exploit, enrich and shift the intellectual role, and evaluation, of traditional mechanics, goals which he shared with other innovators, from the sixteenth century master engineers such as Benedetti, Tartaglia and the young Galileo, through the mature work of Galileo and Kepler and right up to the origins of Newtonian mechanics. The shared aspirations and values, and the wide variety of technical projects and outcomes, are two sides of the same long term historical coin.

Finally, [3] Descartes' dealings with and about mechanics left technical resources and problems for later thinkers who were developing a more obviously classical mechanics. These concerned inertia, so-called; the laws of collision; and the problem of circular motion. Descartes' claims in these areas, and resulting problems for later physical theorists, are intertwined with one another. This is because they arise from the constraints upon his theorising exercised by his commitment to a plenum universe, the ubiquity of circular displacements therein, and his adherence to the apparent corpuscular-dynamical revelations of his optics when rearticulated as a theory of light in the plenum cosmos of vortices.

2.1.1 Why No Cartesian Law of Inertia?:

Descartes never stated a law of inertia of Newtonian type, despite occasional later claims that his first and third laws of nature in *Le Monde* (first and second in the *Principles*) conjointly amount to the law of inertia in classical mechanics. In fact Descartes' theory is quite different. In the first place we have seen that his physico-mathematical optical work formed the initial template and motivation for the formation of his first and third laws of nature in *Le Monde*. The laws of behaviour of light, understood by Descartes as an instantaneously transmitted directional quantity of force, provided him an exemplar for framing more general laws of nature, since they involve no complex, curved or time-dependent physical actions (Section 1.2 above). Upon this basis were superimposed certain conceptual constraints exercised by his emerging system of natural philosophy, including his voluntarist/creationist conception of God's relation to Nature, leading to the final form of the laws. (Section 1.3.2 above)

In the dead mechanical world of corpuscles, the continued existence of bodies and their properties is radically dependent upon God's moment to moment exercise of his freely willed conserving concourse. It follows that the laws of natural change must deal with divinely regulated instantaneous conservation or alteration of instantaneously exerted forces of motion possessed by bodies in phenomenal translation or merely tending to motion. Moreover, in Descartes' plenum universe no unhindered rectilinear translation is possible; corpuscular collisions occur at every instant; hence there is nothing to be gained by idealising the situation in the manner of mixed mathematics or Galilean kinematics. A further constraint arises from that fact that although collision involves perfectly inelastic basic particles, it cannot be allowed over time to run down the total quantity of force of motion in the universe. However, this stipulation cannot cover 'determinations', the directional manifestations of force of motion. It probably appeared to Descartes that determination cannot be conserved, since the (scalar) sum of it present in any system depends upon how the grid of components is applied to what we have above termed the 'principal' determination, denoted by the third rule of nature. If, because of systematic conceptual constraints, rectilinear tendency to motion could only be preserved in instantaneous terms, as the third law of *Le Monde* asserts, so by the same token, cosmic conservation of the total quantity of the force of motion could only be formulated in essentially scalar terms, as the first law of nature in *Le Monde* declares. Moreover, Descartes was in fact fully aware of a principle of inertia very similar to that later asserted in classical mechanics—the one formulated by his original mentor, Beeckman, as early as 1613/14. [19] (Beeckman 1939-53, I 24-5) But, it was precisely this principle which Descartes revised and rearticulated into his first and third laws in *Le Monde*, for the reasons just cited.

2.1.2 Corpuscular Collision and its Rules

The rules supposedly governing corpuscular collisions were central conceptual elements in any mechanistic philosophy of nature, such as Descartes'. But, how collision was theorised and what rules were asserted depended on the particular mechanistic system in question and its choice of physical exemplars. For example, Descartes' mentor in mechanism, Beeckman, did not believe in any internal moving force in the case of inertial motion, so collisions amongst his inelastic atoms threatened to run down the total quantity of motion in the cosmos over time. His

19 Beeckman believed that both rectilinear and circular inertial motion were possible.

exemplar for formulating laws of collision was the behaviour of the balance beam, interpreted dynamically in the tradition of the medieval science of weights and pseudo-Aristotelian *Mechanica*. (Gaukroger and Schuster 2002, 555-6) Descartes, for his part, paid little attention in *Le Monde* to articulating a collision theory, leaving the matter largely to his second law of nature, which stipulated the conservation of total scalar force of motion in collisions. His corpuscles might be inelastic, but the force of motion, whether looked at in terms of its causal or modal manifestations (Section 1.3.2), is conserved and no run down of cosmic force could occur.

In the *Principia* the collision theory is both enlarged and conceptually articulated, starting with the third law of nature, which corresponds to law two in *Le Monde*. Now Descartes adds to it a crucial, and seemingly odd, stipulation that a body "upon coming in contact with a stronger one, loses none of its motion" but "is turned aside in another direction" thus retaining its quantity of force of motion and changing only its determination. (AT VIII1 65; M 61). Collision has become a contest of 'stronger' and 'weaker' forces manifested by bodies, where these terms apply to the force of motion, or of rest (a new conception) possessed by the contending bodies. Seven further rules are elicited, the most telling and paradoxical being the fourth which asserts the following: If a body C entirely at rest is struck by body B, and body C is even slightly larger than B, the latter cannot move C no matter what its speed of impact; rather B will rebound off C in the opposite direction at the same speed with which it approached. Here we have a contest between forces of motion and rest, where the latter is in effect evaluated in terms of the force of motion B would possess, if, after collision, the total quantity of force of motion were shared out in proportion to the sizes. For in that case C would have more force than B, so in actuality its force of rest (thus evaluated) entirely overcomes the force of motion of B and causes its reversal of direction with its original quantity of force of motion In other words B's principle determination is reversed.[20]

What the modern critic should remember about this is that Descartes' rules do not apply in the phenomenal world of macroscopic bodies. Descartes was still elaborating a dynamics of corpuscles. Moreover, in his theory of light he had a perfectly good physical exemplar for this rule. The law–like behaviour of light, an instantaneously transmitted tendency to motion, displayed the underlying dynamics of corpuscles and its rules, as his exploitation of his discovery of the law of refraction had shown. The relevant phenomenon here was reflection of light. The law of reflection would not hold in Cartesian physics unless law three and its following rule 4 were true: the angle of reflection would not exactly equal the angle of incidence of a ray of light if the (inelastic) reflecting surface of corpuscular objects were to absorb any force of motion from the ray (remembering that in Des-

20 In correspondence Descartes later further rationalised this approach in terms of a supposed principle of least modal change in a collision (AT IV 185): in rule 4 only the determination of B changes and not the quantity of force of motion of B, or force of rest of C.

cartes' optics, transmission is instantaneous, but the strength of a ray can vary, being its instantaneously manifested quantity of force of motion). Hence the contest view is very far from any approach to collision which concentrates on equivalence of action and reaction or attends to the motion of the center of gravity of the system. (Although Descartes' manner of dealing with the force of rest may have influenced Newton's path toward his own second law of motion (Gabbey 1980). The contest view of collision was shaped by Descartes' neo-Scholastic conceptual language in the *Principles*, but also by the more theoretically compelling reason that his exemplar for physical interactions and laws of corpuscular dynamics remained what it had first become in the late 1620s, his own results in a physico-mathematical optics of corpuscular-mechanical tenor.

2.1.3 Circular motion and the Failure to Focus upon Centripetal Force

Descartes' physics of ubiquitous vortical displacement circuits places virtually all its conceptual attention on the centrifugal tendencies of bodies actually moving in the plenum. The vortex celestial mechanics focuses on this property, and so does the theory of light in cosmic setting as lines of radial tendency to motion instantaneously propagated from each point on the surface of a star. Both theories are anchored in Descartes' analysis of the instantaneously manifested quantity of force of motion (and its determinations) of a stone in a sling. Of course, Descartes recognised that the sling physically constrains the stone (into its circular path); and that layers of second element *boules* resist the centrifugal tendency of planets, constraining them at given orbital distances from their stars. But, he in no way needed further to conceptualise in his dynamics what came to be called centripetal force. He concentrated on the force instantaneously present 'in' a body (in the 'causal' and 'modal' senses discussed earlier in Section 1.3.2) and not on the physical process of constraint into curved paths. (Gabbey 1980; Westfall 1972) Moreover, given the qualitative bent of his mature physics; its propensity to deal in a general but putatively 'law bound' manner with unavoidably complex situations, he never conceived of quantifying the centrifugal tendency, as Huygens, a mechanist of Cartesian leanings, was to do in the course of his brilliant studies of pendulum motion. When the young Newton first explored the deflection of inertial motion into polygonal approximations of circular paths, he already had to hand a principle of action equalling reaction in collisions and the germ of a generic notion of force as productive of, and measured by, change of momentum (including mere change of direction without alteration of scalar speed)—tools well beyond Descartes' grasp or interest. (Herivel 1965, 6-7; Westfall 1970, 353-5)

2.2 Descartes' Role in the Evolution from Mixed Mathematical Optics toward a Modern Discipline of Physical Optics

Optics was another of the traditional mixed mathematical sciences that evolved into something much more approximating modern form during the seventeenth century. Simple narratives about the field see Newton as the isolated founder of modern physical optics, displacing traditional mixed mathematical geometrical optics. In fact, optics first began to develop away from its ancient and medieval mixed mathematical form within the natural philosophical turbulence and competition of the early and mid seventeenth century. Like mixed mathematical mechanics, optics began to be re-shaped by attempts to render the discipline more physico-mathematical, with closer interaction between optical theorising and problem solving on the one hand, and natural philosophical explanation in terms of matter and cause on the other.

Descartes' work in optics was exemplary in this sense, his entire natural philosophical career being characterised by tight mutual articulations between moves in a physico-mathematical optics, and an increasingly systematised corpuscular-mechanistic physics. Yet, Descartes did not invent the physico-mathematical gambit in optics or any other mixed mathematical field (Dear 1995) and in optics he very much emulated and attempted to surpass and displace the work of Kepler. The latter had in effect pursued a physico-mathematical agenda, practising geometrical optics under, and in the service of, a neo-Platonic natural philosophy and conception of light, rather than a corpuscular-mechanical one, obtaining brilliant results in the theory of the camera obscura, theory of vision, and, to some degree, the theory of refraction and the telescope.

As a result of these sorts of early efforts, geometrical optics, the mixed mathematical science of the ancients and Scholastics, began to evolve into a much more obviously physico-mathematical discipline. In the latter half of the seventeenth century the process intensified in the physico-mathematical optical work of Huygens, Robert Hooke, Newton and a host of others. By 1700 there was a great density of new phenomena, instrumental/practical applications and new problems. Optics became a physico-mathematical discipline, increasingly independent of any particular natural philosophical system, and indeed relatively independent of the domain of natural philosophising as a whole. Descartes' physico-mathematical and natural philosophically relevant optics is therefore both illustrative of the process in its early stages, and also, taking a longer historical perspective, a contribution of the first order to it.

On the technical level, Descartes bequeathed a host of problems, claimed solutions and puzzles to the evolving field. They include his claim that light is propagated instantaneously; his fundamentally 'ray', rather than 'wave' version of a mechanistic theory of light (Shapiro 1974); his sine law, which required articulation and correction (Dijksterhuis 2004); and his views on telescope design and the fabrication of lenses (as planned on paper and as meant to be materially realised).

(Ribe 1997; Burnett 2005) His greatest achievement, long recognised but only recently explored in its full historical significance, was his explanation of the geometrical and chromatic properties of the rainbow. This was a stunning solution to a mixed mathematical and natural philosophical puzzle of ancient standing. However, it was also something else, portentous, but without further issue in Descartes' career. In the case of his researches on the rainbow, he achieved the only instance in his work where a corpuscular-mechanical model is applied and further articulated with relation to novel experiments which have quantitative implications. (Buchwald 2008) Not only does this prefigure the young Newton's optical work, but both achievements consist in just the kind of ongoing interaction between physical models and quantified, often novel experimental manipulations that characterize the explanatory and practical style, and content, of Newtonian and all subsequent physics.

2.3. The Ultimate Context and Driver: Bidding to Establish Realist Copernicanism

There is one other large context into which Descartes' intentions and strategic aims in physics should be set, with regard both to his technical initiatives and to the ways in which his work instantiated larger drivers of the emergence of classical mechanics. This context is the long process of the so-called Copernican revolution, taken as culminating in the emergence of Newtonian mechanics and celestial mechanics. It is obvious that Descartes' physics—like that of Kepler, Galileo and Newton—taken in its widest acceptation is set almost entirely within the problematic of bold, realist Copernicanism.

In the universities, under the hegemony of neo-Scholastic Aristotelianism, geometrical astronomy had, of course, always been viewed as a mixed mathematical science and routinely held to lack the explanatory power of natural philosophy, because it did not deal with the material and causal principles of planetary motion, but merely with appearance-saving geometrical models. However, when, in the later sixteenth and early seventeenth centuries, two generations after the death of its founder, Copernican astronomy came to be hotly debated, it was not as a set of new calculational fictions, but rather as a system with realistic claims about the physical structure and causal regime of the cosmos, implying the need for a framework of non-Aristotelian natural philosophy adequate to justifying its existence and explaining its physical mechanisms. This radicalized the usually accepted relations between mathematics and natural philosophical explanation, making the Copernican debate a notable hot-spot even within the desperate natural philosophical struggle of the early seventeenth century in which Descartes was a major player.

Accordingly, in the physics of Descartes, and that of his major contemporaries, we find nothing resembling a smooth, linear, progress toward the crystallization of Newtonian physics. Rather we see heightened contestation amongst natural philosophers. New doctrines of motion and cause were variously asserted, applied to the heavens or earth or both, leaving a wide field of problems, apparent technical achievements or failures (depending upon the views of various successors, including Newton), as well as targets of revision or replacement, such as Kepler's treatment of orbital motion under two sets of forces, radial and rotary, or, Descartes' vortex mechanics.

That vortex mechanics, as shown above, was a peculiar sort of science of equilibrium, grounded in Descartes' home grown dynamics of corpuscles, itself emergent from his physico-mathematical optics. For him, this dynamics-as-manifested-in-the-vortex mechanics was the key to a new physics, in the sense of a systematic natural philosophy, which remained purely discursive but was daringly and strategically constructed to win adherents. Cartesian physics was thus one bid amongst several to create a new physics, meant to sit inside, run and legitimate a Copernican cosmos. It was, like the physics of Galileo or Kepler, a *sui generis*, before the fact (of Newton) and in the end stillborn project. But these various attempts to establish a form of realist Copernicanism within a replacement natural philosophy involving a (hoped for) more integral use of mathematics display shared attitudes and goals, which characterise the intellectual landscape and conditions through which, in these earlier generations of the seventeenth century, classical mechanics was unintentionally taking shape. Newton shared these attitudes and goals, being himself also an innovative natural philosopher, seeking the more integral use of mathematics in natural philosophy, and aiming to refine the (now widely accepted) Copernican view, in order to claim the prize of having established its definitive winning form.

On the plane of technical consequences, it follows from all this that Descartes' vortex celestial mechanics had to become a target of refutation and displacement by Newton in Book II of his *Principia*, in the service of his own mechanics and celestial mechanics. Descartes' vortex fluid of second element *boules* and interstitial first element particles offers no resistance whatsoever to the motion of a planet or comet. (Gaukroger 2000) The business of the rotational motion of the Cartesian vortex is to move planets and comets in a rotary manner, leading to the generation of centrifugal tendencies that are the proper objects of analysis on the central question of orbital stability and placement. His focus is on specifying orbital equilibrium or its breakdown, not on solving what seemed to Newton, and later classical mechanics, as exemplary fluid mechanical problems. It is no wonder, then, that Newton found the Cartesian vortex theory so problematical, and in particular its supposed implications for a properly mathematical and idealised mechanics of vortical fluid motion so unsatisfactory.[21] It is well known, of course, that more

21 Descartes' vortical fluid mechanics raises other theoretical problems in relation to his theory of cosmic optics. To explain how the entire disk of the central star is visible to observers on a

highly articulated and quantitative vortex theories were advanced both at the time of Newton's work and on into the eighteenth century on a non-trivial basis. These far outstrip Descartes' earlier qualitative and discursive construction, and are the true competitors to Newtonian celestial mechanics in the eighteenth century. (Aiton 1972) However, it remains true that Newton initially had to confront and, on his own grounds, dispose of Descartes' version of vortex celestial mechanics as part of the making of his own *Principia*.

Conclusion—Mathematics, Natural Philosophy and the Path to Classical Physics

The conventional narrative about the emergence of classical mechanics assumes first of all a clear distinction at the time between the world of mathematics and mathematicians and the (merely scholastic) world of natural philosophy and natural philosophers. It then adduces causes for the birth of a mathematicised physics. That reason could be neo-Platonic metaphysics; the rise of a useful practical mathematics and its practitioners; the recovery of ancient mathematics, in particular Archimedes; or the increased cultivation of the mixed mathematical sciences of mechanics, optics, geometrical astronomy and the like. In contrast, Descartes' trajectory, like that of other physico-mathematically sensitive and ambitious, pro-Copernican, mathematician/natural philosophers shows that the real, but initially unintended growth point for classical mechanics was precisely where such thinkers explored and renegotiated the relations between mathematics and natural philosophy, especially between the now rapidly developing mixed mathematical sciences and the wider field of natural philosophising. The disciplinary and conceptual boundaries were not fixed, and the range of (shifting) initiatives and outcomes is well illustrated by Descartes' own career.

That is why we have had to look at Descartes' natural philosophy in developmental terms and with special attention to the aims and outcomes of his early physico-mathematical program and to his bold concerns with realist Copernicanism. It is also explains why we find little in Descartes' work that is in a direct path of development toward classical physics. He was a natural philosopher, a daring, innovative one, with special aptitudes and ambitions for the relevance of mathematics and the existing mixed mathematical fields inside a new system of natural

planet in its vortex, Descartes seems to imply that the radial tendencies of the *boules* lying next to the surface of the star can be considered to give rise to oblique lines of tendency which pass through the vortex in all directions above the tangent planes to each point of the surface. Once constituted, these lines of oblique tendency continue out into the vortex unaltered and without giving rise to any secondary diffusion of oblique tendencies at each intervening *boule*, lest the vortex become entirely filled with light. This theory hardly comports with a classical mechanics understanding of the propagation of pressure in a fluid (Shapiro 1974, 254-7, 265).

philosophy. His system of physics was one outstanding instance in a spectrum of physico-mathematically coloured, pro-Copernican natural philosophical initiatives of the time. In the long run, his work, along with that of others, in this genre of natural philosophy, had to be displaced in order for classical mechanics to emerge as the new master science and core of a new order of physical knowledge, involving an evolving but essential dialectic of mathematics and experimentation/instrumentation, But, his and other related versions of physics marked a stage in the evolution, and eventual dissipation of, natural philosophy, through which elite European knowledge of nature had to pass if there ever was to be a classical mechanics. That is where the texture of unintended consequences comes into play, with positive achievements as well as failures or dead ends being played upon later by Newton and others in the creation of classical mechanics and celestial mechanics.

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