Physico-Mathematics and the Search for Causes in Descartes' Optics—1619-37

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[Abstract] One of the chief concerns of the young Descartes was with what he, and others, termed "physico-mathematics". This signalled a questioning of the Scholastic Aristotelian view of the mixed mathematical sciences as subordinate to natural philosophy, non explanatory, and merely instrumental. Somehow, the mixed mathematical disciplines were now to become intimately related to natural philosophical issues of matter and cause. That is, they were to become more 'physicalised', more closely intertwined with natural philosophising, regardless of which species of natural philosophy one advocated. A curious, short-lived yet portentous epistemological conceit lay at the core of Descartes' physico-mathematics—the belief that solid geometrical results in the mixed mathematical sciences literally offered windows into the realm of natural philosophical causation—that in such cases one could literally "see the causes". Optics took pride of place within Descartes' physico-mathematics project, because he believed it offered unique possibilities for the successful vision of causes. This paper traces Descartes' early physico-mathematical program in optics, its origins, pitfalls and its successes, which were crucial in providing Descartes resources for his later work in systematic natural philosophy. It explores how Descartes exploited his discovery of the law of refraction of light—an achievement well within the bounds of traditional mixed mathematical optics—in order to derive—in the manner of physico-mathematics—causal knowledge about light, and indeed insight about the principles of a "dynamics" that would provide the laws of corpuscular motion and tendency to motion in his natural philosophical system. [467]

1.0 "There are very few physico-mathematicians"

Thus in late 1618 Isaac Beeckman referred to a set of exercises prepared by René Descartes for or with him, regarding physico-mathematical questions. [1] Beeckman, the mentor of the twenty-two year old Descartes in corpuscular-mechanical natural philosophy and physico-mathematics, also wrote around this time,

> [Descartes] says he has never met anyone other than me who pursues enquiry in the way I do, combining Physics and Mathematics in an exact way; and I for my part, I have never spoken with anyone other than him who does the same. [2]

Descartes was entering the first stage of his intellectual career, and until he turned to concerted work in systematic natural philosophy after 1628, he was to pursue that career largely in the guise of a *physico-mathematicus*.[3]

By the term physico-mathematics Beeckman and Descartes broadly signalled a questioning of the Scholastic Aristotelian view of the mixed mathematical sciences as subordinate to natural philosophy, non explanatory, and merely instrumental.[4] [468] Somehow, the mixed mathematical disciplines were now to become

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1 Physico-mathematici paucissimi… Beeckman and Descartes were congratulating themselves on being virtually the only "physico-mathematici" in Europe. AT X 52. What they meant was that only they unified the mathematical study of nature with the search for true corpuscular-mechanical causes. In this regard Beeckman was to note in 1628 that his own work was deeper than that of Bacon on the one hand and Stevin on the other just for this very reason. Beeckman (1939-53) iii. 51-2, "Crediderim enim Verulamium [Francis Bacon] in mathesi cum physica conjugenda non satis exercitatum fuisse; Simon Stevin vero meo judico nimis addictus fuit mathematicae ac rarius physicam ei adjunxit." The physico mathematical exercises of 1618-9 are preserved in a document, '*Physico–mathematica*' found in volume 10 of the Adam Tannery edition of Descartes.

2 Isaac Beeckman, *Journal tenu par Isaac Beeckman de 1604 à 1634*, 4 vols., C. de Waard, ed. (The Hague: Martinus Nijhoff, 1939-53), vol I, 244.

3 See Gaukroger and Schuster (2002); Schuster (2005).

4 My use of the terms mixed mathematical sciences and subordinate sciences follows that stated in Gaukroger and Schuster (2002) p.537, which introduced a discussion of the young Descartes' enterprise in physico-mathematical hydrostatics: "The term 'mixed mathematics' had been framed by Aristotle to refer to a group of disciplines intermediate between natural philosophy, which dealt with those things that change and exist independently of us, and mathematics, which deals with those things that do not change but have no existence independently of us, since numbers and geometrical figures have (contra Plato) an existence only in our minds. (Aristotle, *Metaphysics* Book E.) A physical account of something — such as why celestial bodies are spherical — is an explanation that works in terms of the fundamental principles of the subject matter of physics, that is, it captures the phenomena in terms of what is changing and has an independent existence, whereas a mathematical account of something — such as the relation between the surface area and the volume of a sphere — requires a wholly different kind of explanation, one that invokes principles commensurate with the kinds of things that mathematical entities are. (Aristotle *Posterior Analytics*, 75a28-38; Cf. 76a23ff and *De caelo* 306a9-12.) In the *De caelo*, 297a9ff, for example, we are offered a *physical* proof of the sphericity of the earth, not a mathematical one, because we are dealing with the

intimately related to natural philosophical issues of matter and cause. That is, they were to become more 'physicalised', more closely intertwined with natural philosophising, regardless of which species of natural philosophy one advocated. A curious, short-lived yet portentous epistemological conceit lay at the core of this physico-mathematics: the belief that solid geometrical results in the mixed mathematical sciences offered windows into the realm of natural philosophical causation in the sense that one could read natural philosophical causes out of geometrical representations of such mixed mathematical results, and hence, in a way, "see the causes".[5]

Optics came to take pride of place within the young Descartes' physico-mathematics project, because he believed it offered unique possibilities for the successful determination of causes. Allowing for considerable differences in underlying natural philosophical commitments, he shared this strategy in optics with Kepler. This paper traces Descartes' early physico-mathematical program in optics, its origins, pitfalls and especially its successes, which were crucial in providing Descartes resources for his later work in systematic natural philosophy. In particular we examine how Descartes exploited his discovery of the law of refraction of light, an achievement well within the bounds of traditional mixed mathematical optics, in order to derive—in the manner of physico-mathematics—causal knowledge about light, and indeed insight about the principles of a "dynamics of corpuscles" that would provide the laws of corpuscular motion and tendency to motion in his natural philosophical system. This dynamics of corpuscles marks the ultimate achievement of Descartes' ambition of eliciting knowledge of natural philosophical causes out of solid mixed mathematical results, hence we shall have more to

properties of a *physical* object. In short, distinct subject matters require distinct principles, and natural philosophy and mathematics are distinct subject matters. However, Aristotle also recognises subordinate or mixed sciences, telling us in the *Posterior Analytics,* 75b14-16, that 'the theorem of one science cannot be demonstrated by means of another science, except where these theorems are related as subordinate to superior: for example, as optical theorems to geometry, or harmonic theorems to arithmetic.' Whereas physical optics — the investigation of the nature of light and its physical properties — falls straightforwardly under natural philosophy, for example, geometrical optics 'investigates mathematical lines, but *qua* physical, not *qua* mathematical.' (*Physics*, 194a10.) The question of the relation between mixed mathematics, on the one hand, and the 'superior' disciplines of mathematics and natural philosophy, which did the real explanatory work on this conception, remained a vexed one throughout the Middle Ages and the Renaissance, but so long as the former remained marginal to the enterprise of natural philosophy the problems were not especially evident."

[5] Here and elsewhere in this paper, the phrase 'see the causes' and its cognates should be construed in the following way: From our analyst's viewpoint evidence will be presented to the effect that the young Descartes was trying to read knowledge of (corpuscular-mechanical) natural philosophical causes out of what he deemed to be reliable and well formed results in mixed mathematics, which took the form of geometrical figures. From Descartes' perspective, operating under his particular understanding of the project of physico-mathematics, he may very well have assumed that such causes can literally be seen by inspecting such diagrams. We are not joining Descartes in this belief, but merely reporting his likely self-understanding of procedures we understand as necessarily heavily laden with assumptions and highly interpretative on his part.

say about it throughout the argument, tracing its gradual crystallisaton in and through Descartes' physico-mathematical program. [469]

2.0 Physico-mathematics and the dream of "seeing" the causes

When Beeckman and Descartes congratulated themselves on being amongst the few physico-mathematicians in Europe, they were in one sense quite correct. Physico–mathematics was engaging only a small sub-set of those both mathematically and natural philosophically literate. Nevertheless, the posturing of Beeckman and Descartes was also misleading. Of course others were operating in similar ways. Beeckman and Descartes marked merely one position in a wider domain of physico–mathematics. It contained meanings and implications reaching beyond the ken of any given practitioner, but which can be discussed by our shaping physico–mathematics as an historiographical category.

As noted above, physico–mathematics had to do with the relation between mixed mathematics, on the one hand, and the 'superior', explanatory discipline of natural philosophy on the other hand. Strict Aristotelians did not grant any natural philosophical relevance to the findings of the mixed mathematical sciences, and this is precisely where physico-mathematics enters the picture, because increasingly in the generations around 1600 not everyone followed the dictates of Aristotelianism about the nature and relevance of the mixed mathematical sciences. Taking a radical approach to the natural philosophical legitimacy of the mixed mathematical fields, physico–mathematics meant the old mixed mathematical fields would be explained in natural philosophical terms and therefore would not be subordinate to, but rather proper domains of, one's favoured natural philosophy. Conversely, it meant that novel findings in mixed mathematical sciences could bespeak new insights into whatever natural philosophy the physico–mathematician in question favoured. This led to the curious behaviour of Descartes (and Kepler amongst others) trying to "see the natural philosophical causes" of phenomena which were well represented and grasped in mixed mathematical, geometrical form. More generally, therefore, it should also be clear that physico–mathematics was not about the mathematisation of natural philosophy. Rather, physico–mathematical gambits envisioned the *physicalisation* of parts of mixed mathematics, whereby some natural philosophers aimed to render the mixed mathematical fields more physical, more about matter and cause discourse within one's favoured natural philosophy. [6] It is of course true that the protagonists were

[6] This conception of 'physicalisation of the mixed sciences rather than mathematisation of natural philosophy' was tolerably clear, if not perfectly literally stated, in Gaukroger and Schuster (2002), and also mooted in Schuster (2002). In 2006 I used the expression explicitly in a paper for the first workshop of the Baroque Science Project at the Unit for History and Philoso-

saying that mathematics could be natural philosophically explanatory, but the point is that mixed mathematical disciplines already existed; it was the physicalisation of their contents, rendering them about or relevant to issues of matter and cause, that was at stake. It follows, furthermore, that the rise of a broad and various interest in physico-mathematics did not constitute an invasion of natural philosophy by mathematicians to destroy or displace it. The players involved were *mathematically adept natural philosophers/natural philosophically literate, and aggressive mathematicians*. Such people constituted one, small, [470] intersectional sub-set of all European mathematicians and natural philosophers. No circulation or displacement of block elites took place.

Physico–mathematics was not a coherent, self-conscious intellectual movement, but a diffuse set of gambits and agendas sitting loosely inside the field of natural philosophising. Within the wide category of physico-mathematics there was a range of approaches, from conservative to radical in terms of attitudes toward Aristotelian natural philosophy and the type of 'natural philosophication' of the mixed mathematical sciences envisioned. The heightened natural philosophical contestation of the early seventeenth century intensified the proliferation, and competition of physico-mathematical gambits. As Peter Dear established in first drawing serious attention to the issue of physico-mathematics in the Scientific Revolution, some leading Jesuit mathematicians pursued a 'conservative' physico–mathematical program. They wanted the mixed mathematical fields to enjoy a status 'separate but more or less equal' to natural philosophising, thus liberating the mixed sciences from Aristotelian constraints, but handicapping their ability to enrich natural philosophical discourse of matter and cause.[7] More radical were Galileo's still rather piecemeal physico-mathematical excursions, including his construction of a *sui generis* new science of motion.[8] Most radical, perhaps, were Kepler and Descartes, with their strategies of trying directly to read natural philosophical causes out of representations of significant results in a physico-

phy of Science, University of Sydney, and its Director, Ofer Gal later quizzed me about it, bringing it to more full awareness and expression. Acknowledging Dr Gal's insight, I now (and in my forthcoming monograph on the early career of Descartes, physico-mathematicus) reappropriate the notion and insist on its literal meaning.

[7] Dear (1995)

[8] Like more radical figures such as Kepler and Descartes, Galileo certainly made natural philosophical capital out of mixed mathematics. But, unlike them he did not pursue a *systematic natural philosophy*; rather, he tried to establish a realist Copernican cosmology and a strong anti-Aristotelian stance. Nevertheless, like Kepler and Descartes, Galileo was breaking the declaratory Scholastic rules about subordination of mixed mathematics, in pursuit of what amounted to gambits in the field of natural philosophising. Even more radical and symptomatic would be the remarkably innovative natural philosophically influential William Gilbert, a notable case, because he was trading off *practical mathematical* materials rather than *mixed mathematical* ones. Gilbert was articulating some bits of practical mathematics to his own novel natural philosophy, as Jim Bennet has observed, writing of how in Gilbert, "navigational magnetism" a practical mathematical node of instrumentation, theory and practice, "impinged on natural philosophy through the need to characterise and codify declination and inclination in their terrestrial distribution." (Bennet, 1998, p220)

mathematicised geometrical optics.[9] We turn now to the earliest of Descartes' endeavours in this direction.

3.0 Descartes Early Physico-Mathematical Project: 1618-20

3.1 Physico-mathematicising Stevin's Hydrostatics[10]

In 1586 Simon Stevin, Dutch maestro of practical mathematics, proved a special case of the hydrostatic paradox. Stevin demonstrated that a fluid filling two vessels [471] of equal base area and height exerts the same total pressure on the base, irrespective of the shape of the vessel and hence, paradoxically, independently of the amount of fluid it contains. Stevin's argument proceeds with Archimedean rigour on the macroscopic level of gross weights and volumes and depends upon the maintenance of a condition of static equilibrium.[11]

Stevin first proves that the weight of a fluid upon the horizontal bottom of its container is equal to the weight of the fluid contained in a volume given by the area of the bottom and the height of the fluid, measured by a normal from the bottom to the upper surface. He employs a reductio ad absurdum argument. **[Fig. 1]** ABCD is a container filled with water. GE and HF are normals dropped from the surface AB to the bottom DC, dividing the water into three portions, 1 [AGED], 2 [GHFE] and 3 [HBCF]. Stevin has to prove that on the bottom EF there rests a weight equal to the weight of the water of the prism 2. If there rests on the bottom EF more weight than that of the water 2, this will have to be due to the water beside it, that is water 1 and 3. But then, there will also rest on the bottom DE more weight than that of the water 1; and on the bottom FC also more weight than that of the water 3; and consequently on the entire bottom DC, "there will rest more weight than that of the whole water ABCD, which would be absurd." The same

[9] In one sense Isaac Beeckman, Descartes' mentor and initial inspiration in physico-mathematics, should be counted with Kepler and Descartes. However, below we will have cause to note that in the end Descartes' version of physico-mathematics involved tactics quite different from those of his mentor in physico–mathematics and corpuscular–mechanical natural philosophy.

[10] Material in this section derives from Gaukroger and Schuster (2002); Gaukroger (1995) pp.84-9 ; and Schuster (1980) pp.41-55.

[11] Simon Stevin, "De Beghinselen des Waterwichts" (Leiden, 1586) in *The Principal Works of Simon Stevin*. Vol. 1, pp.415-17.

argument applies to the case of a weight of water less than 2 weighing upon bottom EF.[12]

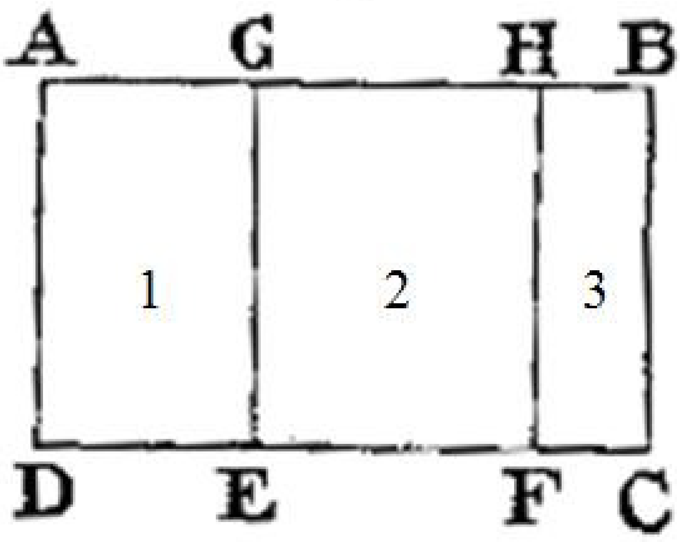

Fig. 1 Stevin, *Elements of Hydrostatics* (1586) in *Principal Works of Simon Stevin* vol I p.415.

Stevin then ingeniously argued that portions of the water can be notionally solidified, replaced by a solid of the same density as water. This permits the construction of irregularly shaped volumes of water, such as IKFELM, to which, paradoxically, the theorem can still be applied. **[Fig. 2]**

> Let there again be put in the water ABCD a solid body, or several solid bodies of equal specific gravity to the water. I take this to be done in such a way that the only water left is that enclosed by IKFELM. This being so, these bodies do not weight or lighten the base EF any more than the water first did. Therefore we still say, according to the proposition, that against the bottom EF there rests a weight equal to the gravity of the water having the same volume as the prism whose base is EF and whose height is the vertical GE, from the plane AB through the water's upper surface MI to the base EF.[13] [472]

That is, on bottom EF there actually rests a weight equal to that of a volume of water whose bottom is EF and whose height is GE. Stevin then applies these findings to the sides of containing vessels.

[12] Ibid, i. 415.
[13] Ibid, i. 417.

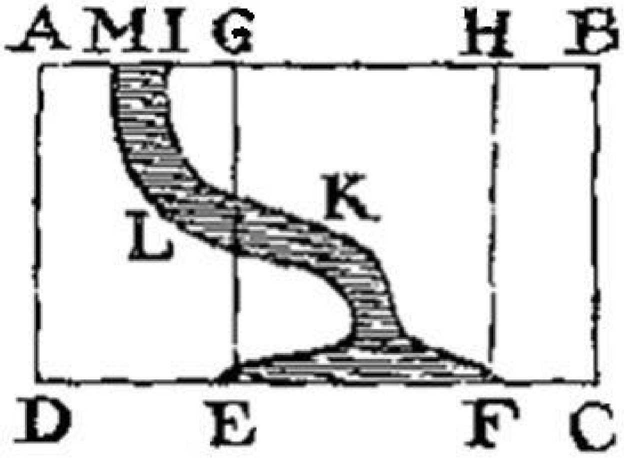

Fig. 2. Stevin, *Elements of Hydrostatics* (1586) in *Principal Works of Simon Stevin* vol I p.417.

In early 1619 the twenty-two year old Descartes and his thirty year old Dutch mentor, Isaac Beeckman, tried to turn Stevin's result into physico-mathematics, and in so doing uncover the causes of the phenomenon; that is, provide a natural philosophical explanation of it.[14] In the key example, Descartes considers two containers **[Fig. 3]**: B and D, which have equal areas at their bases, equal height and are of equal weight when empty, and are filled to their tops. Descartes proposes to show that, "the water in vessel B will weigh equally upon its base as the water in D upon its base"—Stevin's paradoxical hydrostatical result.[15] While Stevin's approach is geometrical, Descartes' analysis and explanation are based on an attempt to reduce the phenomenon to micro-mechanical terms. This was to be an exercise in Beeckman and Descartes' self-proclaimed physico-mathematics, the

[14] The text, *Aquae comprimentis in vase ratio reddita à D. Des Cartes* which derives from Isaac Beeckman's diary, is given in AT, X, pp. 67–74, as the first part of the *Physico-Mathematica*. See also the related manuscript in the *Cogitationes Privatæ*, AT, X, p. 228, introduced with, "Petijt è Stevino Isaacus Middelburgensis *quomodo aqua gravitet in fundo vasis* b…". Stephen Gaukroger and I have taken to calling this text "the hydrostatics manuscript" (Gaukroger and Schuster 2002).

[15] AT x. 68-9. "... the water in base B will weigh equally upon the base of the vase as does the water in D upon its base, and consequently each will weigh more heavily upon their bases than the water in A upon its base, and equally as much as the water in C upon its base." This is the second of the four puzzles posed in the text, the others are: "(First), the vase A along with the water it contains will weigh as much as vase B with the water it contains. … Third, vase D and its water together weigh neither more nor less than C and its water together, into which *embolus* E has been fixed. Fourth, vase C and its water together will weigh more than B and its water. Yesterday I was deceived on this point."

agenda of which demanded that macroscopic phenomena be explained through reduction to corpuscular-mechanical models, *and that the contours of the underlying natural philosophical causes would be revealed by inspection of correct geometrical figuring up of the phenomenon in question, in mixed mathematical terms.*

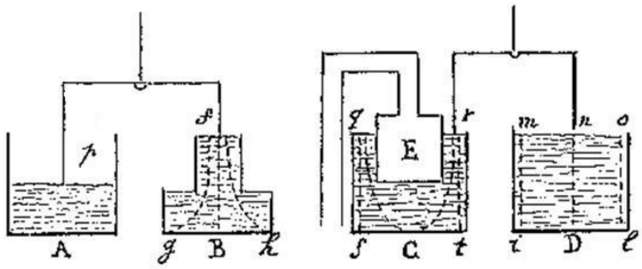

Fig. 3. Descartes, *Aquae comprimentis in vase ratio reddita à D. DesCartes*, AT X 69.

Descartes tells us that by the weight of the water on the bottom of the vessel he does not intend the gross weight of the quantity of water measured by weighing the filled vessel and subtracting the weight of the container itself. He means instead the total force of the water on the bottom arising from the sum of the pressures exerted [473] by the water on each unit area of the bottom. He explicates the term "to weigh down" as "the force of motion by which a body is impelled in the first instant of its motion". Descartes insists that this force of motion is not the same as the force of motion which "bears the body downward' during the actual course of its fall."[16]

Descartes attempts to reduce the phenomenon to micro-mechanics by showing that the force on each "point" or part of the bottoms of the basins B and D is equal, so that the total force is equal over the two equal areas. [17] He claims that each "point" on the bottom of B is, as it were, serviced by a unique line of "tendency to motion" propagated by contact pressure from a point (particle) on the surface of the water through the intervening particles. [See Fig. 3] He takes points g, B, h; in the base of B, and points i, D, l, in the base of D. He claims that all these points are pressed by an equal force, because they are each pressed by "imaginable lines of water of the same length"; that is, the same vertical component of descent—a nice piece of Stevinite thinking, by the way.

[16] AT x. 68. In the Cogitationes Privatae (AT x. 228) the inclination to motion is described as being evaluated 'in ultimo instanti ante motum'.

[17] Descartes consistently fails to distinguish between "points" and finite parts. But he does tend to assimilate "points" to the finite spaces occupied by atoms or corpuscles. Throughout we shall assume that Descartes intended his points to be finite and did not want his 'proofs' to succumb to the paradoxes of the infinitesimal.

Despite that, Descartes' overall effort is distinctly odd. For example the mappings of lines of tendency are tendentious and not subject to any rule.18 But, young René [474] was quite pleased with himself. He continued to use descendants of these concepts the rest of his career as he continued to develop what we have termed his "dynamics of corpuscles": We have the key concept of instantaneous tendency to motion, and an example of its analysis into what he would subsequently term its component "determinations". Descartes' later mechanistic optics and natural philosophy would depend on the analysis of instantaneous tendencies to motion, rather than finite translations. Often Descartes will consider multiple tendencies to motion which a body possesses at any given instant, depending on its mechanical circumstances. There is evidence that even in 1619 Descartes was considering trying to systematise this set of new dynamical concepts to apply to corpuscular explanations, as he speaks in this "hydrostatics manuscript" and surrounding correspondence of a treatise of "Mechanics" he is planning to write.[19]

Descartes was not denying the rigour or correctness of Stevin's strictly mathematical, Archimedean account. What he was after was proper explanation, meaning explanation in terms of natural philosophy. Stevin's treatment of the hydrostatic paradox fell within the domain of mixed mathematics. The account Descartes substitutes for it falls within the domain of natural philosophy: the concern is to identify what causes material bodies to behave in the way they do, thus

18 AT X 70-1. Descartes can perhaps be taken to imply that when the upper and lower surfaces of the water are similar, equal and posed one directly above the other, then unique normal lines of tendency will be mapped from each point on the surface to a corresponding point directly below on the bottom. But, when these conditions do not hold, i.e. when the upper surface of the water differs from the lower in respect to size and/or shape, or when it is not directly posed above the bottom, then some other unstated rules of mapping come into play. It would seem that in the present case the area of the surface at f in the basin B is precisely one-third that of the bottom, so that each point or part on f must be taken to service three points or parts of the bottom. The problem, of course, is that no explicit criteria or rules for mapping are, or can be, given. Descartes makes no attempt to justify the three-fold mapping from f. He merely slips it into the discussion as an 'example' and then proceeds to argue that *given the mapping*, f can indeed provide a three-fold force to g, B and h. Indeed, the argument continues solely as a justification of the three-fold efficacy of f, rather than as a general demonstration of the problem, such as we might expect. Descartes writes, "It must be demonstrated, however, that point f alone presses g, B, h with a force equal to that by which m, n, o press the other three i, D, l. This is done by means of this syllogism: Heavy bodies press with an equal force all neighbouring bodies, by the removal of which the heavy body would be allowed to occupy a lower position with equal ease. But, if the three points g, B, h could be expelled, point f alone would occupy a lower position with as equal a facility as would the three points m, n, o, if the three other points i, D, l were expelled. Therefore, point f alone presses the three points simultaneously with a force equal to that by which the three discrete points press the other three i, D, l. Therefore, the force by which point f alone presses the lower [points] is equal to the force of the points m, n, o taken together."

19 AT X p.72 and in correspondence with Beeckman early in 1619, AT X pp. 159, 162. For more discussion see Gaukroger and Schuster (2002)

implying a radically non-Aristotelian vision of the relation of the mixed mathematical sciences to his emergent form of corpuscular-mechanical natural philosophising. Indeed the particular species of physico-mathematics he envisioned was very radical even by the standards of other physico-mathematical innovators of the day. It certainly was a far cry from the conservative physico-mathematics of the Jesuit Aristotelian mathematicians, discussed above.[20] He was even distancing himself from his physico-mathematical mentor, Beeckman, because the version of hydrostatics from which he starts is that of Stevin—mathematically rigorous, and rigorously *statical* in the Archimedean style—and hence unpromising as a basis for finding key *dynamical* concepts for a corpuscular-mechanism. The seemingly more promising approach, as followed by the young Galileo, Beeckman and others, sought physico-mathematical capital in the dynamical approach to statical problems found in the pseudo-Aristotelian *Mechanica*. Instead, in his hyper radical approach, Descartes starts from a mathematically rigorous [475] hydrostatics of all things, fleshing it out in terms of the corpuscularian model he learned from Beeckman, in order to adduce insights concerning 'forces or tendencies to motion', a dynamics of corpuscles, by in some sense 'reading' them out of putatively favoured geometrical representations. For Descartes novel findings in mixed mathematical sciences would now be taken directly to bespeak insights into the realm of corpuscular-mechanical explanation.

In physico-mathematics one ideally wants a crisp, clean geometrical result at the 'empirical' level in the relevant mixed mathematical discipline, so that the natural philosophical causes of that result can be discerned by reading out from (or into) the diagrams in question. In the case of hydrostatics, this was obvious to Descartes, as we have just seen. In the case of his ultimately successful optical work of the later 1620s, Descartes was to do the same thing; he would take a geometrical representation of an arguably well established mixed mathematical result—a law of refraction—and read back from certain of the parameters of the representation to knowledge of the underlying causes of the phenomenon. But, before we investigate that development, we first need to look at his earliest and somewhat abortive foray into physico-mathematical optics.

3.2 An Early Exploration in Physico-Mathematical Optics: 1620

We turn now to the earliest case of Descartes' physico-mathematics involving optics and an attempt to see natural philosophical causes through what he took to be the best available representations of phenomena in mixed mathematical terms. It is

[20] Firstly the mixed mathematical field in question is not to be severed from natural philosophy in order to secure it 'scientific status'; rather, it becomes coextensive with natural philosophical issues of matter and cause. Secondly, the species of natural philosophising in question no longer is neo-Scholastic Aristotelianism, but proto-mechanism.

a fragment on optics and theory of light found in the *Cogitationes Privatae* and datable from about 1620. It reads in part,

> Because light can only be produced in matter, where there is more matter there it is more easily generated; therefore, it more easily penetrates a denser medium than a rarer one. Whence, it happens that refraction occurs in the rarer medium from the perpendicular, in the denser medium toward the perpendicular. [21]

Close analysis shows that Descartes was studying Kepler's optical masterpiece, the *Ad Vitellionem paralipomena* [1604] and that this text is a physico-mathematical 'reading' of a set of texts and figures in Kepler's work. [22] Descartes construed Kepler the way he had read Stevin. Seeking grist for the physico-mathematical mill, he attempted to elicit a physical theory of light, and perhaps the law of refraction, from a set of compelling geometrical diagrams and texts about refraction. [476]

The most important passage occurs in Chapter IV of *Ad Vitellionem*, where Kepler attempts to discover a simple law of refraction by means of an analysis of its putative physical causes. Kepler asserts that there are two fundamental physical factors which any adequate theory of refraction must take into account: the inclination of the incident rays, and the densities of the media.[23] He offers a geometrical construction representing these factors **[Fig. 4].**

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21 AT X pp.242-3: "Lux quia non nisi in materia potuit generari, ubi plus est materiae, ibi facilius generatur, caeteris paribus; ergo facilius penetrat per medium densius quam per rarius. Unde fit ut refractio fiat in hoc a perpendiculari, in alio ad perpendicularem."

22 A full justification for this textual critical conclusion will be offered in my forthcoming monograph, *Descartes Agonistes: Physico-Mathematics, Method and Mechanism 1618-33*, Chapter 3. The 1620 optics fragment is little studied, apart from A.I Sabra's interesting speculation that it contains premises adequate for Descartes to have deduced from them his sine law of refraction of light, first published seventeen years later in the *Dioptrique* of 1637. [Sabra (1967) pp.97-100, 105-6, 116]. Reasons to reject Sabra's speculation will emerge below [See below, Notes 29 and 30 and texts thereto. See also Schuster (2000) pp.277-285]

23 It is useful to bear in mind what we might call Kepler's 'official' theory of refraction, keyed to his natural philosophical theory of light, presented elsewhere in *Ad Vitellionem*: Kepler held that light is an immaterial emanation propagated spherically in an instant from each point of a luminous object. Refraction, he maintained, is a surface phenomenon, occurring at the interface between media. The movement of the expanding surface of light is affected by the surface of the refracting medium, because, according to Kepler, like affects like, hence surface can only affect surface, and the surface of the refracting medium "partakes" in the density of the medium. He analysed the effect of the refracting surface upon the incident light by decomposing its motion into components normal and parallel to the surface. The surface of a denser medium weakens the parallel component of the motion of the incident light, bending the light toward the normal; a rarer refracting medium facilitates or gives way more easily to the parallel component of the motion of the incident light, deflecting it away from the normal. (The normal component of the motion of light is also affected at the surface by the density of the refracting medium, weakening or facilitating its passage, but not contributing to the change of direction). *Ad Vitellionem Paralipomena,* Chap.1 Prop. 12, 13, 14, 20, in Kepler 1938ff, vol. II pp. 21-3, 26-7. The two physical assumptions in Kepler's model in the passage in question are consistent with this 'official' theory.

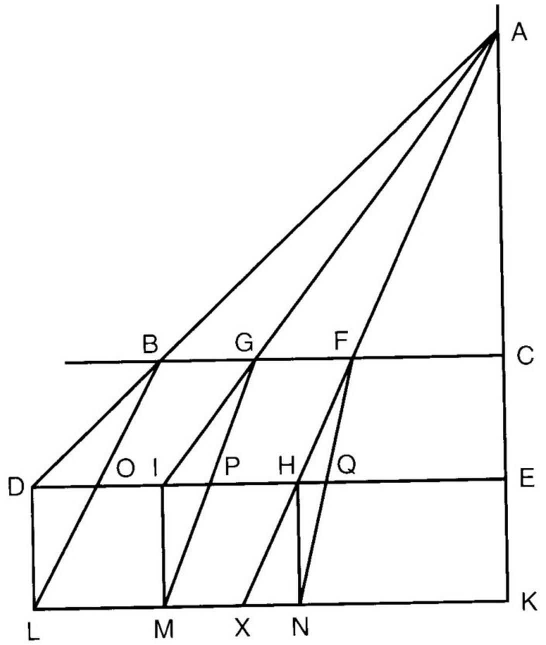

Fig. 4. Kepler's Physico-mathematical Analysis of Refraction

Take AG incident upon a basin of water. The density of water is said to be twice that of air. Kepler lowers the bottom of the basin DE to LK so that the new basin contains "as much matter in the rarer form of air as the old basin contained in the doubly dense form of water". Kepler then extends AG to I and drops a normal from I to LK. Connecting M and G gives the refracted ray GM. Its construction involves the [477] obliquity of incidence and densities of media.[24] Although Kepler then goes on to reject this construction on empirical grounds,[25] the question is, did this text speak to René Descartes, the budding physico-mathematician with aspirations toward geometrical optics?

Descartes, *physico-mathematicus*, would have been attracted to Kepler's approach. Kepler was trying to penetrate beyond the mere phenomenon of refraction, represented geometrically in mixed mathematical style, in order to identify, and work with, its physical causes. His procedure involved representing geometrically the action of these causes and building those representations into a technique for generating, by geometrical construction, the paths of refracted rays—which, if successful in empirical terms, would be tantamount to possessing both the sought for law of refraction of light and knowledge of its causes. Descartes, who had al-

24 *Ad Vitellionem Paralipomena,* in Kepler (1938ff, vol.II pp.81-5).

25 Kepler, *loc. cit. p.86.*

ready attempted to identify and geometrically represent the natural philosophical causes of the paradoxical statical behaviours of fluids, "superficially" examined by Stevin, probably saw Kepler's construction as a promising step toward the physico-mathematisation of the problems of explaining refraction in natural philosophical terms, and obtaining the true (rather than merely instrumentally useful) law of refraction (the best that mere mixed mathematical optics could provide).

Kepler's construction technique does not focus upon, or work with, the parallel and normal components of the motion of the incident light or light ray.[26] In contrast, Descartes, in interpreting Kepler's passage, reintroduced the customary considerations about the parallel and normal components of the motion of the incident light, or of the ray that represents it.[27] Descartes' fragment introduces the concept of "generation"/"penetration" of light varying with density of media. Increased or decreased "penetration" (the product of greater or lesser density) causes refraction toward or away from the normal. Descartes, unlike Kepler, characterises the properties of the light or light ray itself, inserting the characterisation between the talk of "density" and of "refraction". Descartes' contention that the "penetration" of light varies with the density of the medium makes sense as a reading of Kepler's text, provided Descartes was thinking in terms of the comportment of the parallel and normal components of the motion of the incident light or of the incident ray. On this reading, Kepler's diagram and construction technique 'say' that the denser medium has the effect of increasing [478] the normal component (and perhaps less proportionately so the parallel component), hence causing refraction toward the normal.[28]

So, when Descartes writes of the "penetration"/"generation" of light being directly related to the density of the medium, he envisions the behaviour of the nor-

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[26] He directly represents the causally efficacious greater density of the lower medium and postulates a construction technique which uses that representation of density, and the obliquity of the incident ray, to manufacture a ray path bent toward the normal: the greater the obliquity of incidence and the farther the bottom DE has been lowered, the greater the resultant refraction toward the normal.

[27] Kepler, in other contexts in which he deals with refraction (and reflection), typically considers the comportment of these components, even though he does not always deduce changes in direction of light by (re-)composing altered components of its motion. For example, what we have termed Kepler's official theory of refraction dealt with the parallel and normal components of the motion of the light, asserting that both are weakened at the interface, whilst attributing the refraction to the alteration in the parallel component alone. In the traditional optical literature it was, of course, also thoroughly commonplace to attend to the comportment of the normal and parallel components of the motion of light when discussing its refraction and reflection.

[28] The refraction toward the normal certainly cannot be produced by a decrease in the parallel component, given the structure of Kepler's diagram and construction. Nor will proportionately greater increase in the parallel component over the normal component do the job. Descartes' likely reading boils down to the claim that the normal component of the motion of the incident light increases upon entering a denser medium, while the parallel component can remain constant, increase in appropriate proportion, or even decrease.

mal components of incident light rays. The magnitude of these components (the "penetration") varies with the density of the medium. Increase in the normal component (with conservation or appropriate alteration in the parallel component) will bend the refracted ray toward the normal; decrease in the normal component (with conservation or appropriate alteration in the parallel component) will bend the ray away from the normal. Importantly this also explains the entailment between the first and second sentences of the fragment, claimed by Descartes.[29] Descartes can assert that greater or lesser "penetration" causes refraction toward or away from the normal, because he identifies greater/lesser "penetration" with increase/decrease in the normal component, which can be represented in ray diagrams and used in the construction of refractions toward/away from the normal.

Needless to say, Descartes' strategy here is entirely consistent with his aspirations in physico-mathematics. He wants to read a possible geometrical construction and representation of refraction back to a knowledge of its (mathematically representable) causes. Kepler's figure and construction may not capture the law of refraction—Kepler admits as much—but some such inquiry of a similar type might. Descartes is working through what Kepler had done, presumably in order to articulate and correct it. That Descartes was looking at Kepler's figure can be confirmed by looking at a second set of passages in *Ad Vitellionem* which conditioned his thinking about the physico-mathematics of refraction. **[Fig. 5]**

Returning to Descartes' fragment, let us consider the sentence following on from the extract quoted above at the beginning of this section. Descartes continues, "Moreover the greatest refraction of all should be in the densest medium of all...".[30]

As it happens, in *Ad Vitellionem* Kepler twice considers the notion of "the most dense medium possible", pointing out on both occasions that any ray entering such a medium will be refracted into the normal direction. In the second of these passages he writes, "In the most dense medium of all refractions are performed toward the [479] perpendiculars themselves, and are equal in respect of (all) inclinations."[31] The context of these remarks is what we have called Kepler's official theory of refraction. The infinite density of the refracting medium destroys the parallel component of the motion of the light, leaving it only its normal component. When Descartes echoes these passages in his fragment, the context is not

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[29] First discerned and discussed in the literature by Sabra in his interesting analysis of part of this fragment. See above Note 22. Sabra noted Descartes signaling the entailment, but incorrectly interpreted Descartes' premise concerning "greater penetration in denser media" as applying independently of angle of incidence—thus allowing Sabra to deduce the sine law of refraction from the 'text'—whereas we have seen that Descartes' premise applied to the normal component of the force or motion of the incident ray. If Descartes carried out the resulting deduction, he would have arrived a tangent law of refraction.

[30] [AT x. 242-3] '…omnium autem maxima refractio esset densissimum, a quo iterum exiens radius egrederetur per eundem angulum.' In his analysis of the fragment, Sabra did not cite or discuss this remark; yet, it is of vital importance in understanding Descartes as a physico-mathematical reader and-interpreter of *Ad Vitellionem*. See Schuster (2000) pp.283-5.

[31] Kepler, *Ad Vitellionem*, *Gesammelte Werke* II p.107.

Kepler's official theory of refraction, but rather the first two sentences of his own 1620 text, as we have learned to read them. Clearly, Descartes intended to present the case of the "most dense medium" as a limiting case of the general proposition that *penetration varies with density and causes refraction to or from the normal*. And if Descartes drew his limiting case from Kepler, this lends extra weight to the claim that the first two sentences of the fragment constitute a physico-mathematical reading of the other passage in *Ad Vitellionem*.

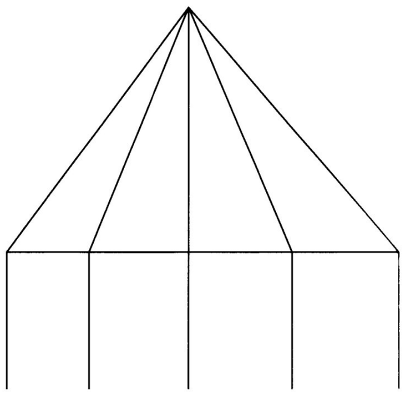

Fig. 5. Kepler: Refraction by the Most Dense Medium Possible

In sum, Descartes connected two lines of speculation present in *Ad Vitellionem* but not explicitly linked by Kepler: (1) The geometrical representation of the claim that "the greater the density, the greater the refraction toward the normal". And, (2) the claim that infinitely dense media would refract all incident rays into the normal. It was Descartes, not Kepler, who first related (2) to (1), using (2) to illustrate the limiting case of his own explicated version of (1) which related change in density to change in "penetration" (normal component) to change in direction.[32] [480]

In sum, then, Descartes' 1620 optical fragment actually is a piece of highly interesting, youthful Cartesian physico-mathematics. But what about the law of refraction? What did this physico-mathematical inquiry produce for Descartes? Well, we have now found that in the 1620 fragment Descartes embraced an as-

[32] All this is entirely consistent with the first of Kepler's passages as well: which occurs *loc. cit.* pp. 89-90: "...if you should ponder what ought to occur in the most dense medium (or medium of infinite density), you would comprehend from the analogy of other media that, if there could be such a medium, it is necessary that all rays falling from one point onto the surface would be fully refracted, that is, after refraction they would coincide with the perpendicular itself." ("…si perpendas, quid fieri debeat, medio existente plane densissimo (seu infinitae densitatis), deprehendes ex analogia mediorum caeterorum, oportere, si quod esset, omnes omnino radios ab uno puncto in superficiem huiusmodi illapsos, refringi plenarie, hoc est, coincidere post refractionem cum ipsos perpendicularis.")

sumption which would have hindered his deducing a sine law of refraction. He held that in two media the normal components of the force of light are in a constant ratio. Had he then assumed that the parallel components are constant, and then attempted a physico-mathematical deduction of the law of refraction, he would have been led to a law of tangents.[33]

Finally, we need to ask what sort of natural philosophical commitments—about matter and cause—were articulated to the physico-mathematical approach in the fragment. Descartes' physico-mathematics, after all, was meant on the one hand to 'natural philosophise' the mixed mathematical sciences—render them organic parts of natural philosophy, not subordinate, merely descriptive hangers on—and, on the other hand, it was supposed to produce from the terrain of 'to-be supplanted' mixed mathematical sciences, conclusions of natural philosophical relevance and import. Descartes' optical fragment of 1620 makes no direct reference to a corpuscular-mechanical ontology. Indeed it appears to take a quasi-Aristotelian view of the nature of light, with Descartes writing of the "generation" of light.[34] The generally Keplerian context of the fragment might suggest an underlying ontology of light as immaterial emanation. Yet, Descartes' apparent concern with quantifying the variation of "penetration" (normal component) with density might also bespeak an unarticulated theory of light as mechanical impulse or tendency to motion.[35]

In short it is not clear he had any definite view, and in any case Descartes seems less interested in precise natural philosophical matter and cause discourse about light, than with generally explaining refraction in physico–mathematical terms by relating density to generation/penetration (magnitude of normal component), and expressing the relation geometrically. In so far as Descartes sought to explain refraction by mathematicising the density-penetration relation (which could have various natural philosophical explications), he comported himself as a

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33 Had Descartes assumed that the parallel component varies either directly or inversely with the density, he would have again deduced "tangent laws" with slightly differing indices of refraction. There seems no way to proceed directly from the assumptions of 1620 to the sine law of refraction, unless one is prepared to introduce Newtonian complications about the variation in components as functions of the angle of incidence, a way of conceiving the problem foreign to Descartes in 1620, 1626, as well as 1637. Sabra, of course, assumed that penetration varied with density regardless of the angle of incidence, an assumption that does indeed yield the sine law when conjoined with the assumption that the parallel component of the motion, force or penetration of the incident ray is unaffected by refraction. Sabra's error consisted in his construal of the first premise: Descartes was envisioning that the normal component of penetration varied with density. We shall see below that the obstacle posed by the 1620 optical fragment is critically important in reconstructing Descartes' path to the sine law of refraction in the mid to late 1620s.

34 Although if taken literally this would imply light to be a substance rather the actualisation of a potential property of the medium, as Aristotle held.

35 For example, as we have already seen, in the hydrostatics manuscript of 1619, Descartes had explained gross weight as the product of summed corpuscular tendencies to (downward) motion, and he had analysed the 'weight-producing' normal components of those tendencies.

physico-mathematicus. The optical fragment is every bit as physico-mathematical as the hydrostatics manuscript, but it eschews a definite commitment to a specific natural philosophical approach, speaking a generic [481] matter/cause language of generation/penetration and density.[36] The 1620 fragment therefore demonstrates both that Descartes was interested in a physico-mathematical agenda in optics, and that matters were necessarily fluid and inconclusive at the level of technical accomplishment: that is, progress toward finding the law of refraction, and, in relation to specific natural philosophical aims and valencies.

4.0 Descartes' Project in Mixed and Physico-Mathematical Optics after 1620: Problems and Tactics of Study

Now we are ready to examine how Descartes' mixed and physico-mathematical project in optics developed in the 1620s. We shall see that he discovered the law of refraction of light using purely mixed mathematical concepts and techniques in optics. However, Descartes quickly tried to exploit this result in physico-mathematical terms, by reading out of his results, or "seeing in them", causal principles on the natural philosophical level. This proceeded in two successive steps. First Descartes elicited principles of a mechanistic theory of light as instantaneous impulse, deployed in attempted demonstrations of the law of refraction as early as 1628. Then, warming to the possibilities of further causal 'insight', he reformulated and polished the central concepts of his dynamics of corpuscles—the causal register of his emerging system of corpuscular-mechanism—whose earliest, embryonic manifestations we have seen in his 1619 physical-mathematisation of hydrostatics. This more mature elaboration first occurred in *Le Monde* (1629-33). In *Le Monde* this polished dynamics, themselves a physico–mathematical product of

[36] Two tentative reasons may be advanced as to why Descartes was so coy about specific natural philosophical claims in the 1620 optics fragment. The first relates to the likely shape of his physico-mathematical thinking and agenda at the time. Unlike the case of the hydrostatics paradox, Descartes did not have in hand a firmly established, mixed mathematical, geometrical representation of the phenomenon. A similar problem haunted, and helped stifle, Descartes' third major youthful physico-mathematical initiative, his study of the physico-mathematics of falling bodies with Beeckman. The matter is fully analysed in my forthcoming *Descartes Agonistes.* So, in this case, 'seeing the causes'; that is, reading back physico-mathematically from an agreed geometrical representation of a phenomenon to its true natural philosophical causes was not on the cards. The fragment is exploratory and preliminary, as though Descartes were playing with possibilities, hoping progress might be made in both representing the phenomenon (amounting to possession of the refraction in mixed mathematical terms) and in then 'seeing its causes' in that representation. The second possible reason has to do with his comparing Beeckman's corpuscular explanation of refraction unfavorably with that of Kepler. It is canvassed in Schuster (2000) p.288.

the optical work, ran Descartes' vortex celestial mechanics and his corpuscular-mechanical theory of light in its cosmological setting.[37] So, the optical project of the 1620s was on the one hand the culmination of the physico-mathematical agenda of the young Descartes, whilst on the other hand, it formed the basis for constructing parts of his mature, systematic natural philosophy. Nothing could be more important to understand about the early career of Descartes than this intertwining of mixed mathematical, physico-mathematical and natural philosophical enterprises; and nothing, with the exception of his fantasy of method,[38] has proven so resistant to critical study. [482]

The materials for reconstructing Descartes' mixed and physico-mathematical optical project are few and scattered. Hence, for reasons that will become clear, this inquiry takes the form of a detective story. We have to start from work published much later—the *Dioptrique*, published in 1637 as one of the three '*Essais'* supporting the *Discours de la Méthode*—working back through scattered earlier hints and clues to uncover the genealogy of the discovery of the law of refraction, and its physico-mathematical exploitation, leading to "seeing the causes" in a mechanistic theory of light and corpuscular-mechanical natural philosophy. We do have a head start, because we already know something about Descartes' physico-mathematics and embryonic corpuscular-mechanism, as well as his early interest in a physico-mathematised optics. We also know that in 1620 he not only did not have the law of refraction but was working within a set of physical assumptions likely to hinder rather than facilitate its discovery.[39] Little evidence survives between the 1620 optics fragment and the *Dioptrique* of 1637, and it only becomes useable for reconstructing Descartes' project once we have learned to decode the *Dioptrique* itself. And that is our first problem, because the *Dioptrique* is by no means a straightforward text.

On its surface the *Dioptrique* does not reveal the trajectory of Descartes' struggles in mixed and physico-mathematical optics. Indeed it has traditionally raised its own problems and even accusations. For example, Descartes deduces the laws of reflection and refraction from a model involving the motion of some very curious tennis balls. Descartes' contemporaries tended not see any cogency in this model, nor did they grasp the theory of motion (actually his dynamics) upon which it is based.[40] These problems further fueled the question of how Descartes had arrived at the law of refraction, if not through his dubious deduction. Suspicions were raised about whether Descartes had plagiarized the law from Wille-

[37] Schuster (2005)

[38] Schuster (1986, 1993)

[39] Additionally (cf. Note 36 above), we have seen that *in optics* Descartes was probably sceptical of kinematic-corpuscular models, and, under the stimulus of Kepler, leaning to instantaneous transmission (of an action or power).

[40] Henry and Tannery (1891-1912) ii. pp.108-9, 117-24, 485-9; Mouy (1934), p.55; Milhaud (1921), p.110.

brord Snel. If not, where had it come from?[41] To these problems, we may add our own. How did Descartes' physico-mathematical program in optics and corpuscular-mechanism unfold over these years, leading to the surviving texts in the *Dioptrique*? Fortunately, however, some of the puzzles about the *Dioptrique* can be resolved, and the resulting answers will show us how to use the surviving hints and clues to construct the genealogy of Descartes, physico-mathematical '*opticien'*, in the 1620s.

Our strategy in decoding the *Dioptrique* runs as follows: First I will show that the tennis ball model for reflection and refraction links quite coherently to Descartes' impulse theory of light *through* his dynamics of micro-corpuscles. Nevertheless, we shall also learn that the tennis ball model, even when given its proper dynamical basis, still poses a number of problems, acknowledged at the time by Descartes and his critics. Indeed, it is the very strengths *and* the weaknesses of the tennis ball model that will provide us with further clues toward our main goals. These are a reconstruction of [483] how, after 1620, Descartes' mixed- and physico-mathematical optics developed down to the discovery of the law of refraction in 1626/7; and how, after that discovery, he increasingly committed himself to mechanistic explanations of the law, instigating in effect a *physico-mathematical optics of firmly mechanistic tenor*, which, in turn, became exemplary for his emerging form of systematic corpuscular mechanism—with all this transpiring under the related proclivity toward reading causal insights out of well founded mixed mathematical results.

5.0 Decoding the Tennis Ball Model of Light in the Proofs of the Laws of Reflection and Refraction

Let's now unpack the demonstrations of the laws of reflection and refraction as presented using the tennis ball model in the *Dioptrique* of 1637. In order to do that we need to understand Descartes' mature dynamics of corpuscles, as inscribed in *Le Monde* between 1629 and 1633. Descartes' system of natural philosophy in *Le Monde* was concerned with the nature and 'mechanical' properties of microscopic corpuscles and a causal discourse, consisting of a theory of motion and impact, explicated through key concepts of the "force of motion" and directionally exerted "tendencies to motion" or "determinations". It is this "causal register" within Descartes' natural philosophical discourse which scholars increasingly term his "dy-

[41] It has long been well established that it is quite unlikely Descartes stole the law from Snel, as some contemporaries maintained. See Kramer (1882), 235-78; and Korteweg (1896), 489-501.

namics", as mooted earlier in this paper.[42] We have seen that the rudiments of this dynamics of instantaneously exerted forces and determinations date back to Descartes' earliest work in physico-mathematics. In *Le Monde* Descartes teaches that bodies in motion, or tending to motion, are characterised from moment to moment by the possession of two sorts of dynamical quantity: (1) the absolute quantity of the "force of motion"—conserved in the universe according to *Le Monde's* first rule of nature, and (2) the directional modes of that quantity of force, the directional components along which the force or parts of the force act, introduced in *Le Monde's* third rule of nature.[43] It is these Descartes termed actions, tendencies, or most often determinations.[44] As corpuscles undergo instantaneous collisions with each other, their quantities of force of motion and determinations are adjusted according to certain universal laws of nature, rules of collision. Therefore Descartes' analysis focuses on instantaneous tendencies to motion, rather than finite translations in space and time. Indeed, Descartes offers a metaphysical account of translation which dissolves it into a series of inclinations to motion exercised in consecutive instants of time at consecutive points in space. Additionally, [484] to understand Descartes' optical proofs, one must distinguish between what I term the "principal determination" of a body's force of motion at a given instant, bestowed upon it according to the first and third rules of nature, and the determinations into which it might be resolved according to the configuration of mechanical constraints acting upon the body at that instant. In the demonstrations of the optical laws in the *Dioptrique*, the reflecting or refracting surfaces dictate which components of the principal determination of a moving tennis ball come into play at the instant of collision. [45]

Finally, we need to bear in mind Descartes' mechanistic theory of light as presented in its natural philosophical context in *Le Monde*. Leaving aside Descartes' theory of elements and his cosmology, his *basic* theory of light within his natural philosophy is that light is a tendency to motion, an impulse, propagated instantaneously through continuous optical media. So, light is or has a determination—a directional quantity of force of motion. Note that light, as a tendency to motion,

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[42] On use of the terms "causal register" and "dynamics" see Gaukroger and Schuster (2002); and Chapter 3 of my forthcoming monograph on Descartes as a physico-mathematician. We have seen the rudiments appear in hydrostatic manuscript of 1619, and we shall see below that the key dynamical concepts probably did crystallise in Descartes' optical work of the 1620s, particularly his discovery of the law of refraction of light (Cf. Schuster [2000])

[43] On the explication of Descartes' first and third rules of nature to constitute his dynamics, see Schuster (2000) pp. 258-61 and especially Notes 7 and 8 thereto.

[44] The understanding of determination used here develops work of A.I.Sabra (1967) p.118-121; Gabbey (1980), pp.230-320; Mahoney, (1973); S. Gaukroger (1995); O. Knudsen and K.M Pedersen (1968) pp.183-186; Prendergast (1975),pp 453-62; McLaughlin (2000) and Schuster (2000), (2005).

[45] I coined the interpretative concept of "principal determination" (Schuster 2000) to underscore this important concept, and differentiate this aspect of determination from the other determinations that can be attributed to a body in motion, or tending to motion, at any given moment. I prefer this terminology to a perhaps too Whiggish concept of 'inertial' determination.

can have a greater or lesser quantity of force—we can have weak light impulses or strong ones—but the speed of propagation in any case is instantaneous. This distinction between the force of light and its instantaneous speed of propagation is central to our discovery of the physical theory and physico-mathematics underlying the 'tennis ball' proofs in the *Dioptrique*.

First reflection.[46] **[Fig. 6]** Descartes takes a tennis ball struck by a racket along AB towards surface CBE. Neglect the weight of the tennis ball, its volume, as well as air resistance. Consider the reflecting surface to be perfectly flat and perfectly hard: upon impact it does not absorb any of the force of motion of the ball. The tennis ball is now virtually a mathematical point in motion; it bears a certain quantity of force of motion, divisible into directional components, or determinations. The demonstration of the law of reflection is carried out as a geometrical locus problem. Descartes places two conditions upon the dynamical characterisation of the ball: First, the total quantity of its force of motion is conserved before and after impact—no force can be lost to the surface. Second, the component of the force of motion parallel to the surface, the 'parallel determination' is unaffected by the impact. Descartes expresses these conditions geometrically, and uses them to determine the quantity and direction of the force of motion of the ball after impact with the surface.

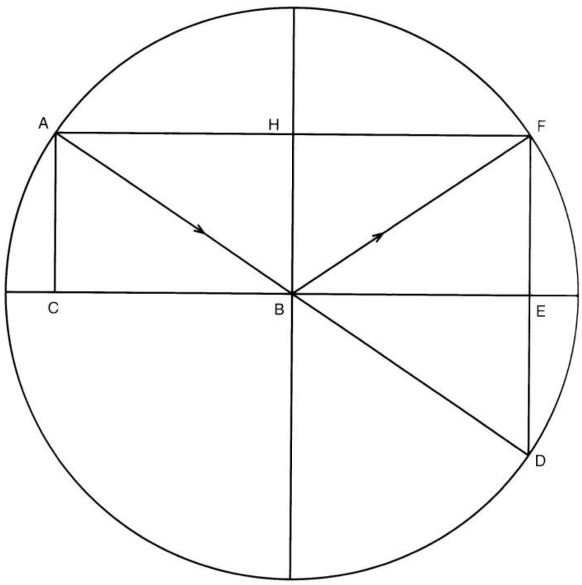

Fig. .6. **Descartes' Figure for Reflection of Light (Tennis Ball) in *Dioptrique***

[46] AT vi pp. 93-96

For the first condition, the conservation of the quantity of force of motion, we draw a circle of radius AB about B. Assume that prior to impact the ball took time *t* to travel along AB. Having lost no force of motion to the surface, the ball will, in an equal time *t* after impact, be located somewhere on the circle. The second condition is that the parallel determination, the component of force of motion along the surface, is unaffected by the collision. In time *t* before impact, while the ball traversed AB, Descartes says that the parallel determination "caused" the ball to traverse the horizontal distance between AC and HB. In an equal period of time *t* after impact, the unchanged parallel determination will "cause" the ball to move an equal distance toward the right. We represent this by drawing FED so that the distance between FED and HB equals [485] that between HB and AC. At time *t* after impact the tennis ball must lie somewhere on this line FED *and* it must also lie on the circle; that is, it must be at F or D. The surface is impenetrable, so at time *t* after impact the ball must be at F. Geometrical considerations immediately show that the angle of incidence is equal to the angle of reflection. This proof never takes into consideration the behaviour of the component of force of motion perpendicular to the surface, the 'normal determination' as we shall call it.

I now propose to do something Descartes refused to do in the *Dioptrique*, even though it is perfectly feasible in terms of dynamical concepts he then possessed, and follows easily in his overall natural philosophical perspective in *Le Monde*. I shall translate the tennis ball proof into the terms of Descartes' theory of light, using his dynamics, as both are presented in *Le Monde*. This is easy to do, because the tennis ball has already been stripped of all properties except location, force of motion and its determinations. It's already virtually a mechanical impulse, which is all a ray of light is in Descartes' theory. *So we can assert the same things about the tennis ball at the instant of impact as we would assert about a ray of light at the instant it meets a perfectly hard reflecting surface.* Consider, again in Figure 6, a light ray, AB, a line of tendency to motion, or determination, impacting the surface CBE at B. The surface is [486] perfectly hard, therefore the magnitude or intensity of the impulse is conserved. The parallel component of the impulse is unaffected by the collision.

The proof is again a locus problem. After impact, what are the orientation and magnitude of the force of the light impulse? The same two conditions apply: (1) unchanging total quantity of force of the ray; (2) conservation of the parallel component of the force of the ray. Represent (1) by a circle about A. Represent (2) by appropriate spacing of FED parallel to HB and AC. Combining our conditions gives BF as the representation of the unchanged magnitude of the force of the ray and its new orientation. The diagram of the tennis ball model is revealed as a diagram about forces and determinations. This is obvious, provided you attend to the very instant of impact, and you take the circle and lines to represent the quantity and principal determination of the force of motion of the ball, as they are instantaneously rearranged at the moment of impact. Descartes' vocabulary of 'forces', 'tendencies' and 'determinations' is already reading the diagram that way, and later correspondence supports this. In this "seeing the causes", the conceptual distance between the tennis ball model and the impulse theory of light virtually disappears.

Let's now turn to the tennis ball model for the refraction of light.[47] **[Fig. 7]** Again consider a tennis ball struck along AB toward surface CBE. In this case the surface is a vanishingly thin cloth. The weight, shape and bulk of the ball are again neglected. It is taken to move without air resistance in empty geometrical space on either side of the cloth. In breaking through the cloth, the ball loses a certain fraction of its total quantity of force of motion, say one half. This fractional loss is independent of the angle of approach.

Again two conditions are applied to the motion of the ball. First, the new quantity of force of motion (one half the initial amount) is conserved during motion below the sheet. Secondly, the parallel component of the force of motion, the parallel determination, is unaffected by the encounter with the cloth. Descartes takes the breaking through the cloth as an analogue to a surface collision, in which the parallel component is unaffected. We draw a circle about point B. Assume the ball took time *t* to traverse AB prior to impact. After impact it has lost one half of its force of motion, and hence one half of its speed. It therefore must take *2t* to traverse a distance equal to AB. It arrives somewhere on the circle after *2t*.

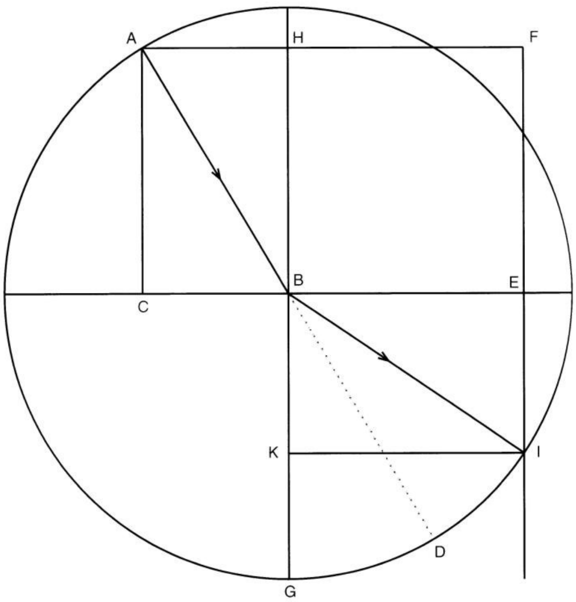

Fig. 7. Descartes' Figure for Refraction of Light (Tennis Ball) in *Dioptrique*

[47] AT vi pp.97-98.

Now, prior to impact the parallel determination "caused" the body to move towards the right between lines AC and HBG. But, after impact, the ball is taking *2t* to move to the circle's circumference, so its unchanged parallel determination has twice as much time in which to act to 'cause' the ball to move toward the right. Therefore set FEI parallel to HBG and AC, but make the distance between FEI and HBG twice as great as that between HBG and AC. At time *2t* after impact the ball will be on the circle and on line FEI; that is, at point I, their intersection point below the cloth. The sine of the angle of incidence AH is to the sine of the angle of refraction IK as one is to two; that is, as the force in lower medium is to the force in upper medium—which ratio is constant for all angles of incidence. [487]

Let's now sketch a proof of the law of refraction in the case of a light ray and Descartes' dynamics. **[Fig. 8]**

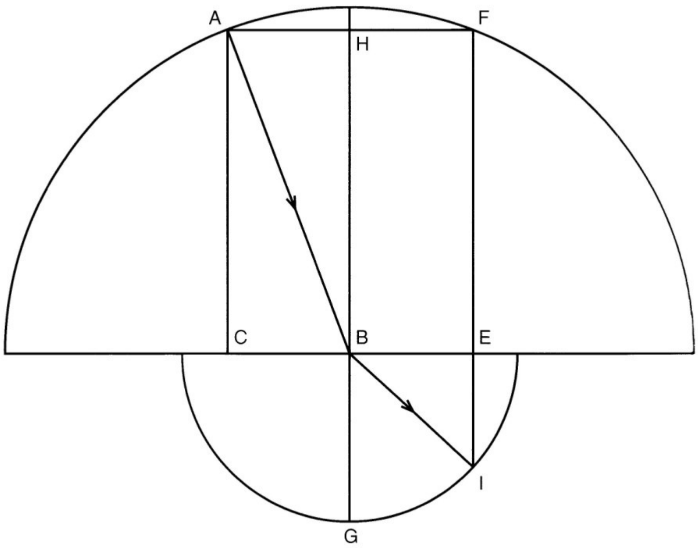

Fig.8 . Refraction of Light Using Descartes' Dynamics and Real Theory of Light

Consider a ray incident upon refracting surface CBE. Let length AB represent the magnitude of the force of the light impulse. The *orientation* and *length* of AB represent the principal determination of the ray. Now, the force of the ray is diminished by half in crossing the surface. So, we must draw a semi-circle below the surface about B with a radius equal to one half of AB. That is condition one: there is a constant, path independent ratio of the force of light in the upper and lower media. The parallel determination of the force of the ray is unchanged in crossing the surface. That is condition two. The distance between AC and HBG represents that parallel determination. Therefore, we must set out line FEI parallel to the two former lines and with the distance between FEI and HBG equal to that between HBG and AC. Again the intersection of the lower semi-circle and line FEI gives the new *orientation* and *magnitude* of the force of the ray of light, BI and the law of sines (actually a law of cosecants!) follows.

The case of the light ray requires manipulation of two unequal semi-circles. These directly represent the ratio of the force of light in the two media. In the tennis ball case we went from ratio of forces to ratio of speeds and hence differential times to [488] cross *equal* circles. *But, in both cases we are attributing the same type of force and determination relations to the ball, and to the light ray, at the instant of impact.*

Sometimes it is claimed that Descartes fell into a contradiction, because his theory of light states that light rays move instantaneously through any medium, whilst in the tennis ball model we must deal with a ratio of finite speeds. This is mistaken, as can be seen by factoring in Descartes' dynamics and theory of light: one simply must distinguish the *speed* of propagation of a light ray, which is instantaneous, from the *magnitude* of its force of propagation, which can take any finite positive value. The *speed* of Descartes' tennis ball corresponds not to the speed of propagation of light but to the intensity of the force of its propagation.

In sum, Descartes' two dynamical premises made good sense of the core aspects of these key proofs. But that is about all. They permitted a plausible deduction of the law of refraction, but they generated what seemed to some of his readers, and arguably to Descartes himself, to be crippling difficulties. Descartes' account has problems as soon as he discusses space filling media, or refraction toward the normal, and more generally with the question of how it happens that the alteration in the normal determination is variable, depending upon the angle of incidence.[48] And, at a fundamental level, the first dynamical assumption—path independent ratio of the force of light—seems to entail that optical media are isotropic, whilst the second dynamical assumption—conservation of the parallel determination—seems to entail that they are not. [49] Indeed, [489] virtually the only strength of Descartes' central assumptions resides in their pleasing ability to rationalise the geometrical steps in his construction of the path of a refracted ray or tennis ball. Descartes was willing to absorb accusations that the premises are empirically implausible, dynamically *ad hoc*, and, arguably, logically inconsistent, because (to him) the premises provided elegant and convincing rationalisations for the geometrical steps in his demonstration.

All this suggests that Descartes did not obtain his premises through a deep inquiry into the conceptual and empirical requirements of a mechanical theory of the propagation and refraction of light. It seems more plausible to associate the premises closely with the very geometry of the diagrams in which Descartes depicts and constructs the paths of refracted rays, as we have seen him doing here in the

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[48] Schuster (2000), pp.270-1

[49] Schuster (2000),pp.267-70. Additionally, using the dynamical premises we can relate the tennis ball model back to the real theory of light, but the real theory of light sits badly with his theory of colour, which depends upon *real translation* of the balls and their *spin to speed ratios*. See Note 64 below and Schuster (2000), pp.295-99.

Dioptrique, once we understand the underlying dynamical rational of his proofs. The issue then turns on whether the premises are post-facto glosses of geometrical constructions arrived at in some other way; or whether the diagrams themselves were invented to illustrate previously held dynamical principles concerning the behaviour of light. In the following section it will be suggested that the former hypothesis is the more likely: Although Descartes held a number of unsystematised ideas about the mechanics of light as early as 1620, he discovered the law of refraction independently of any mechanical assumptions and through a process entirely within the bounds of a traditional mixed mathematics approach to optics. It was the geometrical diagrams expressing his newly found law which suggested to him the precise form and content of his two dynamical premises and their mode of relation in explaining refraction. For Descartes this was to be the paradigmatic case of “seeing the causes”; that is, of reading natural philosophical causes out of a well founded and geometrically clearly represented mixed mathematical result.

In other words, having discovered the law of refraction at the level of a descriptive geometrical result in mixed mathematics, he worked back, in the style of physico-mathematics, extracting underlying natural philosophical causes out of features of the diagram geometrically expressing the new found law. In 1619 this is what he had done in hydrostatics and it was also what he had abortively explored in the physico-mathematics of refraction in 1620. Moreover, Descartes’ physico-mathematical trajectory explains the puzzle of why he was so focused on keeping the premises, despite their dubiousness: why, in short, he defended the premises at all costs. It was not just because they allowed ‘deduction’ of the law of refraction. It was also because, physico-mathematically, they had come from the well grounded, mixed mathematical law! That was their ultimate source; their warrant being that they had been “seen” in the geometry. Descartes knew well what his physico-mathematics was supposed to produce, where it had succeeded and where not. This is why seemingly obvious objections to his finished product were waved aside—after much trouble he had cracked a classical problem in a physico-mathematical way. [490]

6.0 Discovering the Law of Refraction Using Traditional Mixed Mathematical Optics

6.1 Evidence in the Mydorge Letter of 1626/27.

Turning therefore to the discovery of the law of refraction, we know the following: Thomas Harriot discovered the law in exact form around 1598 and Willebrord

Snel, who died in 1626, discovered it sometime after 1620. My claim here is that Descartes, working with Claude Mydorge, discovered it in 1626/27. The chief document we shall examine is a letter from Mydorge to Mersenne.[50] It is well known to students of seventeenth century optics, but not generally properly understood. That depends upon its dating, and the dating depends upon its content.[51]

Mydorge's first claim is that if he is 'Given the inclination and refraction of any one ray at the surface of any refracting medium' he can 'find the refraction of any other ray incident on the same surface'.

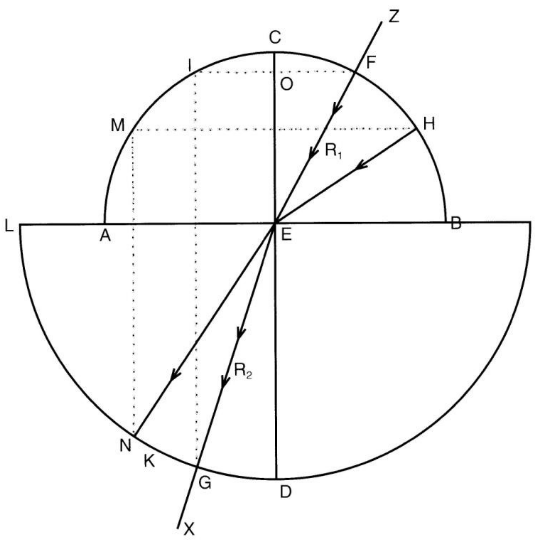

Fig. 9. Mydorge's Refraction Prediction Device

This is the procedure: **[Fig. 9]** Ray ZE is refracted at surface AEB, along EX. Draw a semi-circle above AEB cutting the ray at F. Draw FI parallel to the surface. From I, where FI intersects the semi-circle, drop IG perpendicular to the surface until it cuts the refracted ray at G. Then with radius EG draw another semi circle about E, this time below the surface. This gives a technique, in the manner of practical mathematics, for constructing the path of any other ray, say HE.[52]

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[50] Mersenne, (1932-88) I. 404-415.

[51] The letter consists of five propositions: We shall deal with the first proposition here, as it provides key evidence about the discovery of the law of refraction. The final four propositions deal with Descartes' early theory of lenses, and they, along with the first proposition provide important supporting evidence for the date of the content of the letter, and hence for this reconstruction. They will be treated in my forthcoming *Descartes Agonistes*,, Appendix 1 "Descartes, Mydorge and Beeckman: The Evolution of Cartesian Lens Theory 1627-1637."

[52] The procedure obviously is: Draw HM parallel to the surface cutting the upper semi-circle at M. Drop MN normal to the surface until it meets the lower semi-circle. Connect E and N, then EN is the refracted ray.

Mydorge observes that the law is given here as a law of cosecants. That is, taking the first ray:

$$\frac{\text{cosec i}}{\text{cosec r}} = \frac{\text{R1 / OF}}{\text{R2 / OI}}$$

since OF = OI, the cosecants are as the radius of the upper semi-circle is to the radius of the lower semi-circle.

Let's call this the cosecants or unequal radii form, compared to Descartes' *Dioptrique* form, which we shall term the sine form or equal radius form. We have seen this diagram before. It is identical to our Figure 8 for refraction using Descartes' theory of light as an instantaneous impulse. Mydorge uses two conditions to calculate the refracted ray. They are the same conditions that Descartes uses in his theory of light. The difference is that Mydorge states them only as rules of geometrical construction, while Descartes also gives them a dynamical rationale. The two conditions of course are: (1) The constant ratio of the radii of the upper and lower semi-circles for all angles [491] of incidence. This, in Descartes' theory, becomes the path independent constant ratio of force of light in the two media. (2) The equality of lines FO, OI, the parallel component of the line representing the ray. This later becomes the conservation of the parallel determination of the ray. Note that Mydorge's figure gives a clearer picture of Descartes' two dynamical assumptions than does Descartes' one circle diagram (Figure 7) in the *Dioptrique*. To find out why, we must provisionally date the *material* in the letter.

Descartes' earliest recorded statement of the *sine* law of refraction dates from a report to Isaac Beeckman in October 1628.[53] Descartes consistently identified 1626/27 as the crucial period for his optical studies.[54] He collaborated with Mydorge in that period, and Mydorge credited Descartes with the discovery of the law.[55] [492] In editing Mersenne's correspondence, De Waard dated this letter

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[53] AT X. pp.336ff; also Beeckman (1939-53) fol. 333v ff.

[54] Descartes repeatedly mentioned that during this period he recruited Mydorge and the master artisan Ferrier in an attempt to confirm the law and construct a plano-hyperbolic lens. Eg. Descartes to Golius, 2 February 1632, AT I. p.239; Descartes to C. Huygens, December 1635, AT I. pp.335-6.

[55] In addition to the material cited in the previous note, see Descartes to Ferrier, 8 October 1629, AT I. 32; 13 November 1629, AT I. 53ff; Ferrier to Descartes, 26 October 1629, AT I.38ff. In the mid 1620s Mydorge annotated Leurechon's *Récréations mathématiques*, a popular work dealing with mathematical tricks and fancies of a natural magical character. Leurechon's work was first published anonymously in 1624 and reprinted several times thereafter with additional notes, including those by Mydorge. I have consulted (Jacques Ozanam) *Les Récréations Mathématiques...Premierement revu par D. Henrion depuis par M. Mydorge* (Rouen, 1669). Mydorge notes concerning the nature of refraction "Ce noble sujet de refractions dont la nature n'est point esté cogneue n'y aux anciens, n'y aux modernes Philosophes et Mathematiciens iusque à present, doit maintenant l'honneur de sa découverte à un brave Gentilhomme de nos amis, autant admirable en scavoir et subilité d'esprit." p.157.

from 1626, but that was merely a conjecture based on this collateral evidence.[56] But, evidence in the letter concerning the presentation of the law and the development of lens theory, strongly suggests this *material* is from 1626/7, and is contemporary with the initial construction of the law and first articulation of lens theory. The key issues about Descartes' lens theory are these:[57] [1] In constructing his lens theory Mydorge begins with the cosecant form of the law and only finds a sine formulation in the course of elaborating the theory. [2] His synthetic proofs of the anaclastic properties of plano-hyperbolic and sphero-elliptical lenses are similar to, but clearly pre-date those offered by Descartes later in the *Dioptrique* of 1637. Moreover, [3] Descartes' own synthetic lens theory demonstrations in the *Dioptrique* differ from those of Mydorge in another historically revealing way, the matter turning on a technical and aesthetic issue which Descartes seems to have learned from Beeckman in October 1628. All these facts therefore suggest that the Mydorge letter contains Mydorge and Descartes' *earliest lens theory*, and arguably *their first form of the law*, the cosecant form. The *material in the letter,* if not the artefact itself, pre-dates October 1628, certainly predates composition of the *Dioptrique* and very plausibly is as early as 1626/7. So, this dating points to the cosecant form of the law as the first form Mydorge and Descartes possessed. This, it transpires, is the key to reconstructing how they obtained it, because the other independent discovers first obtained it in the same *unequal radius form*.[58] We can now return to that reconstruction.

6.2 A reconstruction of the discovery of the [cosecant] law of refraction of light

To reconstruct how Descartes found the law we start by following Johannes Lohne's analysis of how Thomas Harriot discovered it. Mydorge's letter provides evidence for an identical path of discovery. One obvious phenomenological expression of the behaviour of refracted rays is the displacement of images of objects viewed under refracting media. Traditional geometrical optics had a rule for constructing the image locations of such sources. Lohne supposed that Harriot attempted to discover a general relation between the incident and refracted rays using the image rule; and that the *cosecant* form of the law resulted from this strategy of research. Here is that traditional image placement rule as shown in **Fig. 10**. [493]

[56] DeWaard admits that the copy he examined dated from 1631 at the earliest, Mersenne (1932-88). I. p.404. Costabel, Shea and others date the letter from 1631 at the earliest. Shea (1991) p.243 n.38.

[57] Cf. Note 51 above: The following claims will be more fully documented in my forthcoming monograph, *Descartes Agonistes,* Appendix 1.

[58] Lohne (1959), (1963); Vollgraff (1913), (1936); deWaard (1935-36); Buchdahl (1972).

AB is a refracting interface; a normal has been dropped to AB at O, the point of incidence. E is a point source emitting ray EO, refracted at O to the eye at F. Experience teaches that E will not appear at E. Where does it *seem* to appear? The rule says that it will appear at I, which is the intersection point between the refracted ray FO drawn back into the first medium, and EG, which is the normal to the surface from E.

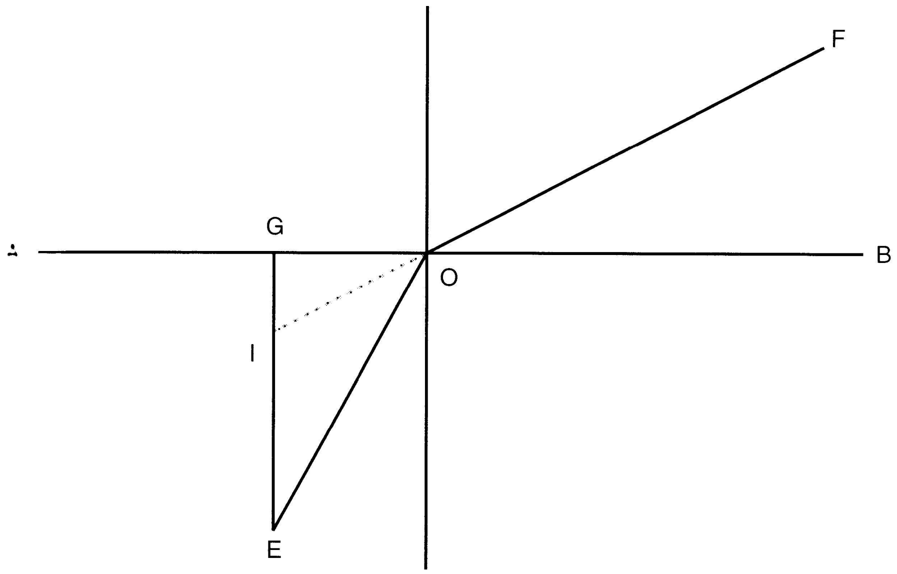

Fig. 10 Traditional Image Location Rule

Harriot used this rule in conjunction with observations made with a disk refractometer half immersed in water. Taking source points at 10 degree intervals around the lower circumference of the disk, he observed the corresponding angles of refraction, then constructed the image places for the source points, by applying the image rule. With the source points located around the circumference of the disk, he found the calculated image places lie roughly on a smaller, concentric circle. If you suspect the plot is really a circle, a little trigonometric analysis gives you the cosecant form of the law. Harriot's key diagram is indistinguishable from Mydorge's diagram.[59] **[Fig. 11]**

Mydorge and Descartes need not have made any such observations. They could have used Witelo's rather cooked data for water/air and glass/air interfaces, which data is good enough to give a strong suggestion of a semi-circular plot when used

[59] Schuster (2000)pp.275-7; Lohne (1959), pp.116-7; Lohne, (1963) p.160. Gerd Buchdahl (1972) p.284, provides a particularly clear statement of the methodological role played by the image principle in Harriot's discovery of the law. Willebrord Snel's initial construction of the law of refraction also followed the type of path indicated by the Lohne analysis. See Vollgraff, (1913) (1936); and deWaard (1935-6).

in Harriot's manner.[60] Trigonometricians of the power of Snel, Mydorge and Descartes need only have suspected the circular plot to seize upon it and explore it further. Thus the explanation of Mydorge's diagram is that he and Descartes proceeded in the same way as Harriot, leading to the same cosecant form of the law. Mydorge probably took [494] a diagram like Harriot's and then flipped the smaller semi-circle up above the surface to create the path predicting device in his first proposition in the letter.

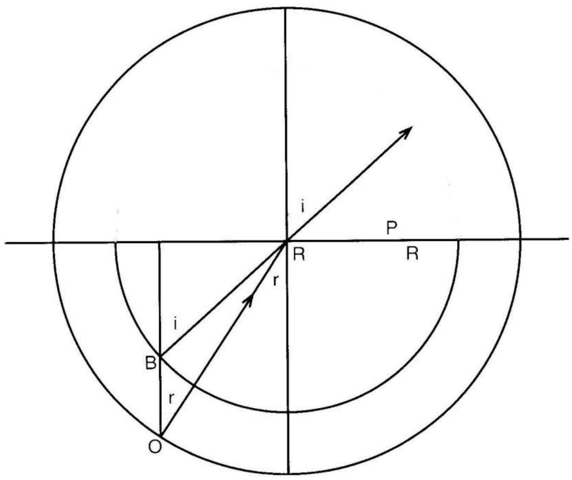

Fig. 11. Harriot's Key Diagram

The law was constructed by traditional means, using data and concepts familiar to skilled students of the mixed mathematical science of geometrical optics.[61] This

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[60] See Schuster (2000) p. 276, based on J. Bossha, (1908).

[61] The discerning reader will note a difficulty in this reconstruction. It has been argued that Descartes and Mydorge (as well as Snel) used the traditional image finding rule in their path of research leading to the law of refraction. But, unlike Harriot, the three later discoverers presumably were well aware of Kepler's new theory of vision which cast grave doubt on the use of the traditional rule. Descartes, after all, was working on a mechanistic version of Kepler's theory of vision around the same time he and Mydorge discovered the law, and his 1620 optical fragment indicates familiarity with Kepler's new work on vision. This fascinating issue cannot be addressed in full here. Suffice it to say that the problem is more Descartes' than our own. That is, there is evidence that Descartes suppressed discussion of his actual path of discovery for several reasons, one of which was the embarrassing point that his work depended upon an optical principle he could no longer accept. For example, his odd methodological story about how the law might be discovered, offered in rule 8 of the *Regulae ad directionem ingenii*, seems intended to occlude this fact, and to mythologise several of his other theoretical quandaries, under a cloak of persuasive, but necessarily vacuous 'method talk'. See J.A Schuster, 'Whatever Should We Do With Cartesian Method: Reclaiming Descartes for the

account involves nothing about the dynamics of light—let alone the dynamics of tennis balls—and it betrays no traces of the young Descartes' program for a physico-mathematical transformation of the mixed mathematical sciences. What then is the relation between the cosecant form of the law and Descartes' two dynamical assumptions? The answer, of course, resides in Descartes' tactic of trying physico–mathematically to read the natural philosophical causes out of well defined mixed mathematical results. [495]

7.0 'Seeing the Causes' in Physico-Mathematical Optics, and in Corpuscular-Mechanical Natural Philosophy in General

We know that in the early 1620s Descartes possessed some intriguing views about the dynamics of light, but that these ideas could not have directed him to the law of refraction. In his physico–mathematical optical fragment of 1620 Descartes held that in two media the normal components of the force of light are in a constant ratio. Had he then assumed that the parallel components are constant, he would have gotten a law of tangents![62] So, how did Descartes ever devise his two assumptions—and in particular why did he ever decide that the constant force ratio applies to media in a path independent manner?

My answer is that Descartes only formulated his assumptions *after* he had constructed the law in cosecant form, using traditional means—issuing in the Mydorge diagram. The Mydorge diagram—the cosecant form—gives you the two assumptions *if you are looking to read them out of the diagram.* And in 1626 Descartes, physico-mathematician, was *very* interested to read out of his ray diagram some mechanical theory of light explaining that diagram.[63] He did to the Mydorge diagram exactly what he earlier did to diagrams in Stevin and Kepler. He took a geometrical picture of a macroscopic phenomenon and read out of it the underlying causes. Viewed through such physico-mathematical spectacles, the Mydorge diagram was where the two dynamical assumptions were forged and coordinated. In short, the two dynamical premises were modelled upon, or "seen in", the Mydorge diagram, with Descartes realising that the geometry of that diagram clarified and modified his earlier, inefficacious, dynamical notions about refraction. He decoded the Mydorge diagram—representing the correct mixed mathematical form of the law—as a message concerning the causes of refraction. This also helps explain why Descartes embraced such problematical dynamical premises for explaining refraction—why he used dynamical premises which simultaneously and unfortunately entail that optical media both are and are not isotropic.

History of Science' in S. Voss (ed.), *Essays on the Philosophy and Science of René Descartes* (Oxford, 1993), pp. 195-223

62 Schuster (2000), p.285

63 Schuster (2000) pp.278-85, 287-89; Gaukroger and Schuster (2002), pp.558-63; 568-70.

Having formulated (or "seen") the premises by inspecting the geometry of the co-secant form of the law of refraction, he accepted and defended these premises because of their supreme value in grounding a deductive physical rationale for the law.[64] [496]

Following his initial physico-mathematical breakthrough in optics in 1627 Descartes moved quickly to further articulate his findings. The proof in the *Dioptrique*, devised in the early 1630s, depends upon both the dynamical ideas read out of the Mydorge diagram and his initial ideas about a dynamics of corpuscles, first emerging in 1619. Even before the *Dioptrique*, however, Descartes was moving further to articulate his dynamical rationale for the law of refraction. In 1628 he spelled out to Beeckman a remarkable analogy between the causes of the refraction of light and the behaviour of a bent arm balance beam whose arms are immersed in media of differing specific gravity. This balance beam analogy expresses precisely the two dynamical principles later used behind the *Dioptrique* proof of the law of refraction.[65] The analogy and slightly later *Dioptrique* proof display Cartesian physico-mathematics and its conceit of "seeing the causes" at the height of its achievement.

However, Descartes went even further, for in composing *Le Monde* between 1629 and 1633 he used his physico-mathematical theory of refraction as an exemplar in formulating his general dynamics of corpuscles.[66] We have seen that Descartes' mature dynamics of corpuscles distinguishes between the absolute quantity of a force of motion, and its directional manifestations as expressed respectively in the first and third rules of nature in *Le Monde*. These principles derived from a further generalisation of his original reading of the Mydorge diagram. Descartes first read the diagram for some basic principles of physico-mathematical optics, assumptions about the quantity and directional quantity of the force of light. But, how better to base the laws of nature than to use as an exemplar the dynamical principles revealed by successful optical research! Light, after all is just an im-

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[64] This, of course, still leaves the question of why Descartes introduced the tennis ball model into the *Dioptrique*. After all, it has always hampered comprehension of Descartes' optical proofs. We have only been able to decode those proofs by relating them intimately to Descartes' dynamics and his real theory of light as impulse. Why, then, did he bother with tennis balls? A quick partial answer resides in the demands of his theory of colour which figures prominently later in the *Dioptrique* and *Météores*. That requires the real spatial translation of balls or corpuscles, so that spin/speed ratios can account for colours: you cannot have a ratio of a tendency to spin to a tendency to move. This problem partially explains the tortuous history of the composition of the *Dioptrique* and *Le Monde*, and Descartes' characteristic reticence about colour theory at the level of the real theory and natural philosophical systematics. Using tennis balls at least allowed Descartes to finesse the problem in his 1637 texts. The tennis ball model directly linked to the real theory of light, and it could bear the weight of the colour theory. Unfortunately, his colour theory and real theory of light did not cohere. Descartes always knew this and struggled with the tensions it generated Schuster (2000), pp. 295-99.

[65] Beeckman preserved the report of this encounter in somewhat garbled form. Its content and import are explicated in detail in Schuster (2000),pp. 290-295.

[66] Schuster (2000), pp.302-3

pulse, so its behaviour clearly reveals the basic dynamics of forces and determinations.[67] Descartes would have had every reason to be confident that his optical exemplar was well chosen and correctly analysed; that is, the cause had been well "seen" within it. Hence he would have had every reason to think that his natural philosophical dynamics of force and determination could be premised upon his having cracked the code of the physico-mathematics of refraction.[68]

In conclusion, therefore, one can say that Descartes' optical researches marked the high point of his work as a physico-mathematician transforming the 'old' mixed mathematical sciences and co-opting the results into a corpuscular–mechanical natural philosophy: On the one hand his results realised his 1619 agenda of developing a [497] corpuscular ontology and a causal discourse, or dynamics of corpuscules, involving concepts of instantaneously exerted force of motion or tendency to motion, and directional determinations of that force. On the other hand, his results concretely advanced and shaped his concepts of light as an instantaneously transmitted mechanical tendency to motion, as well as the precise principles of his dynamics. And, running through the entire project was the curious, historically short-lived, yet fruitful epistemological conceit that physico-mathematically construed results in mixed mathematics make possible the (non-Aristotelian) natural philosopher's dream of unproblematically reading natural philosophical causes out of geometrical representations of solid mixed mathematics results—in effect "seeing the causes". [498]

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67 This claim requires more explication than space here allows. We need to consider that Descartes was constructing a *system* of natural philosophy, and one that required a plenist theory of matter. This helped shape his expression of the rules of nature. In addition he was taking into account and revising ideas about the laws of motion he had inherited from Beeckman. These factors and his experience in 'seeing the causes via optics'; that is reading causes out of mixed mathematical optical results, all shaped his utterances in *Le Monde* about the laws of nature/principles of the dynamics of corpuscles. This issue will be more fully developed in my forthcoming monograph on Descartes as a physico-mathematician.

68 One can go even further along these lines: I argued elsewhere that when Descartes' vortex celestial mechanics (which reside at the very center of his natural philosophical systematics) are carefully analysed in conceptual and genealogical terms, it becomes rather clear that they, too, carry conceptual chromosomes reflective of the project of physico-mathematics. (Schuster, 2005, pp.72, 76-77.)

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