Consuming and Appropriating Practical Mathematics and the Mixed Mathematical Fields, Or Being "Influenced" by Them: The Case of the Young Descartes

John A. Schuster

Unit for History and Philosophy of Science &
Sydney Centre for the Foundations of Science
University of Sydney

and

Campion College
Sydney, Australia

 Originally written for inclusion in Lesley Cormack ed. *MATHEMATICAL PRACTITIONERS AND THE TRANSFORMATION OF NATURAL KNOWLEDGE IN EARLY MODERN EUROPE*. This planned volume arose from a double symposium on this topic at the International Congress of the History of Science, Beijing, 2005. The project, pursued in conjunction with University of Chicago Press, fell through and was placed on indefinite hold in 2012. Despite that outcome, this paper is cited in my *Descartes Agonistes: Physico-Mathematics, Method and Mechanism 1618-33* (Springer, Dordrecht, 2012)

Externalist Narrative, the New Historians of Practical Mathematics and the Category of Natural Philosophy

Since I am not an historian of practical mathematics, I have no intention of adding to the substantive deliberations of the distinguished historians of practical mathematics brought together in this volume. Rather, as an historian of the Scientific Revolution, my orientation is toward the general question of *'what role[s] did practical mathematics and mathematicians play in the Scientific Revolution'?* My concerns reside with the culture and dynamics of what early modern actors called 'natural philosophy', as well as the specialist disciplines those actors held to be subordinate to natural philosophising, especially the fields of mixed mathematics, such as geometrical astronomy, optics, statics, and music theory. Hence, I ask, *"What did practical mathematics and mathematicians have to do with changes in early modern natural philosophy and its subordinate disciplines, and why and* *how did this happen'?* And, I do this not by looking in from practical mathematics toward the Scientific Revolution'; but rather by looking out from the culture of natural philosophy to see how practical mathematics and its resources—technical, theoretical and rhetorical—were received and appropriated by innovative natural philosophers of the period.

Three case studies are presented, of which the first and largest examines Descartes' early work in hydrostatics and geometrical optics, and his appropriation of that work into the construction of his brand of mechanical philosophy. Attention is paid to the way he practised the mixed mathematical sciences, which most Aristotelians held to be subordinate to natural philosophy in the sense of being of instrumental value only, incapable of treating questions pertaining to matter and cause. Descartes tried to render the mixed mathematical fields more 'natural philosophical' in character—or, as he would have said in his early years, more 'physico-mathematical'. His view of practical mathematics was implicated in these developments, hence this case illuminates the young Descartes' transactions with practical mathematics, in the service of what we may term 'the physicalisation of the mixed mathematical sciences'. The paper also makes a number of historiographical suggestions regarding the explanation of the 'Scientific Revolution'; the relevance of the practical mathematics tradition to that problem; and the avoidance of pitfalls in approaching these issues. This is done in the more historiographical sections of the paper, as well as through two shorter case studies, dealing with natural philosophers' appropriation of the sixteenth century mechanics tradition, and Descartes' complicated transactions regarding his lens grinding machine.

Before we examine those case studies or arrive at any new historiographical insights, we must first review our inherited starting point for thinking about 'practical mathematics and the Scientific Revolution'. This, it turns out, is a special case of traditional externalist narrative of the Scientific Revolution. By unpacking the traditional externalist problematic, we shall be better placed to appreciate the approach I am advocating, whilst still perceiving its continuities with the older externalist impulse. When relating practical mathematics to the Scientific Revolution, historians of practical mathematics usually see mathematical practitioners as agents of change, and that what they changed were the method and ideology of science: The method becomes mathematical and instrumental, whilst the ideology values material practice and social utility. This is perfectly consistent with the problematic of traditional externalism in the historiography of science, as promoted by Hessen, Zilsel, Needham and others. They variously argued that practical mathematics (often taken as part of a larger movement of the practical arts) had played the seminal role in the establishment of modern science, according to the following externalist emplotment: Modern science, product of the Scientific Revolution, was a goal, possessing an essence, which consisted in mathematicised theory, proper method, and the values of utility and social progress. Theory meant correct, definitive theory of a mathematicised nature, in mechanism, Copernicanism or Newtonianism; proper method conjoined mathematics with experiment.

Practical mathematics supplied DNA for that essence. And practical mathematics was itself powered by new economic demands and technical problems arising there from.[1]

There are modern versions of this emplotment. In Paolo Rossi's compelling story, the practical arts in general play the lead. From the mid to late sixteenth century they expressed, in the elite end of their literatures, the values which Bacon and the early mechanists later implanted in high cultural natural philosophising, to precipitate the essence of the new science.[2] Similarly Jim Bennett, doyen of the new history of practical mathematics, has followed a similar emplotment on those occasions when he has provided a master narrative: Practical mathematics finally had its pay-off in the emergence of the mechanical philosophy, whose essence consists in experimental practice, instrument deployment, mathematical formulation and mechanistic explanation, all DNA borrowed from practical mathematics.[3] In sum, the old externalism haunts our historical imaginations, threatening to materialise whenever we attempt big pictures of the relation of practical mathematics to the rise of modern science, so that, unless we are careful, we intone something that amounts to:

[practical mathematics] ➔[causes/shapes]➔ [modern science]

Now, since the business of this volume is to ask again, "What was the role of practical mathematics in the Scientific Revolution?", we need to think through our inherited externalist emplotment at a generic historiographical level, so that we can, at a general level, move beyond it.

Too Many Targets, Too Many Sources, Too many Modes of Causation

Externalist talk may be analysed under categories I term 'source', 'mode of causation' and 'target', defined as follows:

---

1 See John Schuster, "Internalist and Externalist Historiographies of the Scientific Revolution"; Schuster, "The Scientific Revolution", pp. 218-222; and Stephen Shapin, "Discipline and Bounding: The History and Sociology of Science As Seen Through the Externalism-Internalism Debate".

2 Paolo Rossi, *Philosophy, Technology and Arts in Early Modern Europe* (New York: Harper and Row, 1970)

3 Jim Bennett, "The Mechanics' Philosophy and the Mechanical Philosophy"; Bennett, "The Challenge of Practical Mathematics"; and Bennett, "Practical Geometry and Operative Knowledge". There is more to Bennett's historiography, and we shall later return to his very fruitful, less mundanely externalist emplotments.

Target: What is the 'thing' being shaped, influenced, brought into existence—Science; Mechanical Philosophy, scientific method, or new scientific values?

Source: Is it the practical arts in general, or some particular sector of the practical arts: sixteenth century mechanics; practical mathematics (or some part thereof, such as geography, algebra, or instruments); or, is it the rhetoric of men of practice; their social habituses and values?

In the literature on practical arts/practical mathematics and the Scientific Revolution, we find multiple sources for the same target: Geography supplies method, but so does algebra, or instrumental practice,[4] whilst for Needham it was the West's unique mixing of artisans, proto-methodologists, with scholars in need of a method fix.[5] Similarly, there are various sources accounting for the 'target', mechanical philosophy: For Rossi, it is the values and aims of practical artisans in general; for Bennett, the attitudes and modes of practice of practical mathematicians; for others it is sixteenth century mechanics, or reflections on clockwork and/or automata.[6]

Mode of causation: In externalist narratives, we often encounter appeals to the causal concept of 'influence', despite correct calls for its demise over the last generation by Quentin Skinner and colleagues, as well as leading sociologists of scientific knowledge.[7] In other species of externalism we meet either a kind of magical social structural imprinting upon the thoughts of cultural dopes like Descartes and Newton;[8] or, more convincingly, some kind of Zilselian causation via social proximity (which still leaves problems); or, more sophisticatedly still, in Biagiolian/Shapinian historiography, a displacement of social types: mathematicians (or experimenting gentlemen) replace/displace mere natural philosophers. [9]

---

[4] Lesley Cormack, "Geography"; David Livingston, "Geography"; Michael S. Mahoney, "The Beginnings of Algebraic Thought in the Seventeenth Century."

[5] Joseph Needham, *The Great Titration: Science and Society East and West*, pp.49-50; similarly for 'method' as the target, see Edgar Zilsel, "The Sociological Roots of Science"; or Boris Hessen, "The Social and Economic Roots of Newton's "Principia"; for natural law as the 'target' see Zilsel, "The Genesis of the Concept of Physical Law"; for Newtonian physics, see Hessen also; for the mechanical philosophy as target, see, for example, Franz Borkenau,. *Der Ubergang vom feudalen zum burgerlichen Weltbild. Studien zur Geschichte der Manufakturperiode.*

[6] Rossi, *Philosophy, Technology and Arts in Early Modern Europe;* Bennett, "The Mechanics' Philosophy and the Mechanical Philosophy"; Helen Hattab, "From Mechanics to Mechanism: The *Quaestiones Mechanicae* and Descartes' Physics."; Derek J de Solla Price, "Automata and the Origins of Mechanism and the Mechanistic Philosophy"; Otto Mayr, *Authority, Liberty and Automatic Machinery in Early Modern Europe.*

[7] Quentin Skinner, "Meaning and Understanding in the History of Ideas."; Jan Golinski, *Making Natural Knowledge: Constructivism and the History of Science*; Barry Barnes, *T.S.Kuhn and Social Science.*

[8] Mary Douglas, *Natural Symbols*, see pp.77-92 for the notorious group/grid theory which enjoyed a brief fad in historiography of science*;* David Bloor, *Knowledge and Social Imagery.*

[9] On the pitfalls of this last option, see John Schuster and Alan Taylor, "Blind Trust: The Gentlemanly Origins of Experimental Science".

Hence, there are problems across the board about target, source and mode of causation: We have multiple targets for the same source, and multiple sources for the same target, with little attempt to think through the modes in which the causes work, let alone consensus on how to approach them. Clearly, we need to eschew classical externalist talk, and to take stock of the multiplication of purported targets and sources. The way forward is through conceptual and historiographical house cleaning, and fortunately, the tools for this are at hand in other corners of the scholarship. To begin, we may learn from recent moves in another troubled area of Scientific Revolution historiography: the problem of science and religion. Margaret Osler has proposed replacing simplistic metaphors of conflict, separation and harmony with new metaphors of mutual appropriation and translation, designed to emphasise the interactions between theology and natural philosophy.[10] Accordingly, we should decide straightaway that talk of influencing, or shaping/imprinting must go. We should think, rather, of people borrowing, adapting and appropriating. But what? Well, obviously, material and discursive resources—and so the defining questions become, "Who were the borrowers and in what tradition, or field did they reside?" That is, if we get the 'target' group, the active agents,[11] right, causal mode sorts itself out as appropriating and translating, and the appropriators themselves will reveal their sources.

Natural Philosophising as Culture and Process

My key suggestion about the target group, the active agents, is that we employ the category 'natural philosophy' in preference to Science, Modern Science, new science, etc. 'Natural philosophy' is the appropriate historical category with which to think through our problem, because in the early modern period it was the central discipline for the study of nature.12 Early modern natural philosophy was a dy-

[10] M. Osler, "Mixing metaphors: Science and Religion or Natural Philosophy and Theology in Early Modern Science".

[11] There is no mistake here. Once we have corrected our explanatory categories, the natural philosophers who were the 'targets' of influence or imprinting stories become the agents in revised narratives, active appropriators and translators of cultural resources and artefacts.

[12] To place the evolution of natural philosophy, and its shifting patterns of relations to other enterprises and disciplines, at the centre of one's conception of the Scientific Revolution is not novel, but neither is it widely accepted in the scholarly community. Attempts to delineate the category of natural philosophy and deploy it in Scientific Revolution historiography include, Schuster, "The Scientific Revolution"; Schuster, "Descartes *Agonistes* New Tales of Cartesian Mechanism"; Schuster and Watchirs, "Natural Philosophy, Experiment and Discourse in the Eighteenth Century: Beyond the Kuhn/Bachelard Problematic"; Andrew Cunningham, "Getting the game Right: some Plain Words on the Identity and Invention of Science"; Cunningham, "How the *Principia* Got its Name; or, Taking Natural Philosophy Seriously."; Cunningham and Williams, "De-centring the 'Big Picture': *The Origins of Modern Science* and the Modern Origins of Science"; Peter Dear, "The Church and the New Philosophy."; Dear, "Religion, Science and Natural Philosophy: Thoughts on Cunningham's Thesis."; Peter Har-

namic, elite sub-culture and field of contestation. When one 'natural philosophised', one tried systematically to explain the nature of matter, the cosmological structuring of that matter, the principles of causation and the methodology for acquiring or justifying such natural knowledge. [**Fig. 1**] The dominant genus of natural philosophy was Aristotelianism in various neo-Scholastic species, but the term applied to alternatives of the various competing genera: neo-Platonic, Chemical, Magnetic, mechanistic or, later, Newtonian. Natural philosophers learnt the rules of natural philosophising at university whilst they studied the hegemonic Scholastic Aristotelianism. Because even alternative systems followed the rules of this game, all natural philosophers constituted one sub-culture in dynamic process over time.

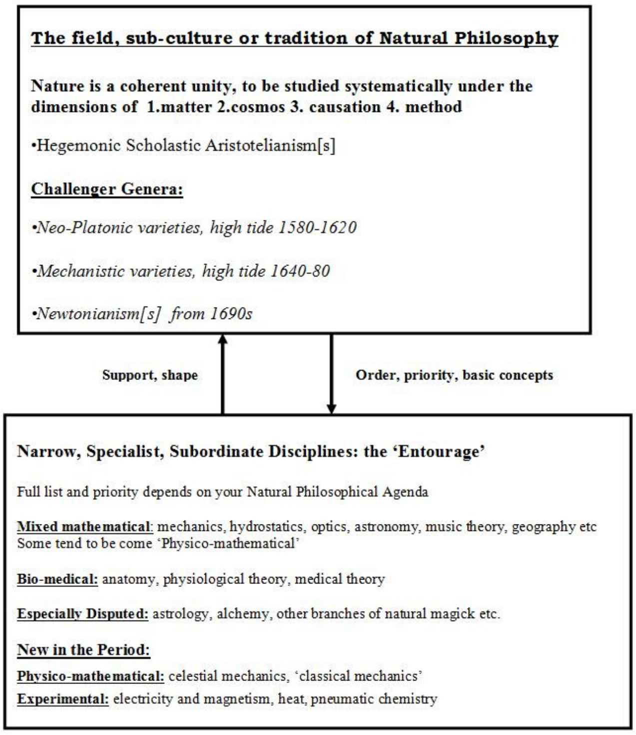

Fig. 1 Generic Structure of Natural Philosophy and Possible Entourage of Sub-ordinate Fields: In a given system of natural philosophy: (1) the particular entourage of subordinate disciplines lends support to and can even shape the system; while (2) the system determines the selection of and priority amongst entourage members, and imposes core concepts deployed within them.

rison, "The Influence of Cartesian Cosmology in England."; "Voluntarism and Early Modern science"; Harrison, "Physico-Theology and the Mixed Sciences: The Role of Theology in Early Modern Natural Philosophy" and John Henry, *The Scientific Revolution and the Origins of Modern Science*.

Therefore, we should not identify natural philosophy with Scholastic Aristotelianism only; nor should we imagine that natural philosophy died and was rupturally replaced by an essentially different activity, Science. The 'Scientific Revolution' largely consisted in a set of transformations inside the seething, contested culture of natural philosophising. Under internal contestation, and external drivers, natural philosophy evolved, and eventually fragmented, into more modern looking, science-like, disciplines and domains over a period of approximately 150 years from 1650.[13] This evolving complex is the 'target' in my 'source, mode and target' schema.

When focusing on natural philosophising as a contested field in process, our attention is drawn to how players constructed and positioned their competing claims in relation to other enterprises and concerns. These were taken either to be superior to natural philosophy (such as theology); or cognate with it (other branches of philosophy, such as ethics or mathematics); or subordinate to it (as in the dominant Aristotelian evaluation of the mixed mathematical sciences, such as astronomy, optics and mechanics); or simply of some claimed relevance to it, as for example pedagogy or the practical arts, including practical mathematics. We may assume that the positioning of natural philosophical claims in relation to other enterprises always involved two routine maneuvers: the drawing or enforcing of boundaries and the making or defending of particular linkages (including efforts to undermine others' attempts at bounding and linking).[14] This constitutes the analytical space where we locate players appropriating and translating resources from mixed and practical mathematics.

One may think of the subordinate disciplines as an ***entourage*** of more narrow traditions of science-like practice: **[Fig. 1]** These included the subordinate mixed mathematical sciences, as well as the bio-medical domains, such as anatomy, medical theorizing and proto-physiology in the manner of Galen. In the seventeenth century, some members of this entourage were disputed; some were created; some were changed; for example, as suggested, some mixed mathematical disciplines became more physico-mathematical. Natural philosophers, competing to co-opt the subordinate disciplines, had different interests and skills within the ***entourage***. Each natural philosopher had to prioritize entourage members, and conceptually articulate them to his natural philosophy, thereby affecting the practice of the subordinate sciences under his genre of natural philosophising.

Finally, what about causation—how did 'external stuff' come to affect the evolving field of natural philosophy? Again, not by influence or imprinting, but rather by members inside the domain appropriating and translating discursive and material resources, instruments, problems and agendas into their natural philosophising. Thus, I conceptualize natural philosophy as a sub-culture in process, defined over time by the resultant of its players' combats over claims, where some of

---

[13] Schuster, "L'Aristotelismo e le sue Alternative"; also Schuster and Watchirs, "Natural Philosophy, Experiment and Discourse in the Eighteenth Century: Beyond the Kuhn/Bachelard Problematic"; and Schuster, "The Scientific Revolution."

[14] Cf. Peter Anstey and John A. Schuster, "Introduction."

those claims involved responses to contextual forces, threats and opportunities. I see natural philosophical 'natives' adapting to challenges and opportunities by their own culturally specific moves, and not by being imprinted, influenced, or put out of business by ‘Science’. Moreover, these moves were not determined by a universal logic; could express considerable novelty; but, remained specific to the (evolving) culture.[15] I term this a *cultural process model* of the ‘mode of causation’.[16] Returning to our theme, the ‘role[s] of practical mathematics in the Scientific Revolution’, we now have a way to envision the ‘*target’*, natural philosophy, and the ‘*modes’* by which its mutually competing players appropriated, translated and redeployed what they perceived as relevant and useful in one of its main external ‘*sources’*, practical mathematics.

Practical Mathematics Was Also a Tradition in Process

We can now think through the relations of practical mathematics to natural philosophy, provided we realise that practical mathematics was also a field in process, and hence that appropriation and translation occurred in both directions. For example, Jim Bennett has provided a number of partial definitions of practical mathematics as an internally complex, dynamic and contested field or tradition.[17] He writes of a “domain of practical geometry”, containing sub-domains such as practical astronomy, surveying, perspective, cartography, architecture, fortification, engineering and machines, the art of war, navigation, and dialing. The larger

[15] Attentive readers will note the debt my model owes to theoretical insights about cultural dynamics pioneered by the anthropologist Marshall Sahlins, "Goodbye to *Tristes Tropes:* Ethnography in the Context of Modern World History". He models cultures as dynamic historical entities, focusing on their mechanisms of adaptation to exogenous and endogenous challenges over time. He argues that cultures display specificity of response to outside impingement; they are not simply imprinted upon or pushed around. The dynamics of response, *over time*, characterises the culture (ibid. p.25). Shapin, "Discipline and Bounding”, speaks in analogous ways of the various science**s** as cultures in process.

[16] This model holds for all types of contextual drivers or causes of natural philosophy asserted by externalists. Not merely practical mathematics, but quite macro entities—social structure, economic forces, political structures and processes—can be appropriately brought into play. The arguably objective existence of contextual structures and processes that historians need to model and explain did not cause, imprint or ‘influence’ thoughts about natural philosophy by natural philosophers. Rather, natural philosophers responded to challenges and forces and decided to bring them into play in the form of revised claims, skills, material practices and values in the field. To do that, the ‘things’ being brought in had to be represented to and by them (not us!) in appropriate form.

[17] Bennett explicitly endorses the attempt to construct such a concept of practical mathematics and apply it to historical inquiry and explanation. Bennett, “Practical Geometry and Operative Knowledge”, p.198 “Comparative accounts of what geometers *do* in the fifteenth and sixteenth centuries reveal a recognized, though not static, domain of practice with shared disciplinary assumptions, which should inform and illuminate our historical narratives.”

domain contained shared "disciplinary assumptions", "material resources and mathematical techniques,"[18] and "a recognised circle of practitioners and an understood, though expanding, domain of competence",[19] sharing a confidence in progress, and held together by a common legitimatory rhetoric.[20] Bennett's conception of this field or tradition focuses on groups sharing and developing particular instruments or geometrical techniques.[21] His and others' research shows that as individuals or groups pushed particular instruments, techniques and supporting rhetoric from one sub-domain to another, they tended to produce knock-on competitive effects. For example, just as navigation became for some a mathematical science, so elite practitioners attempted to push the trigonometry used in astronomy into surveying, leading to conflict between more theoretically oriented and more artisan-like practitioners.[22] Those commanding more sophisticated, theory-relevant techniques moved to displace more artisan types from work and reputation. Bennett also stresses the "rhetorical" dimension of instruments, involving the self image of the instrument's owner, the patron's status, the maker's ambition and his intended impression upon potential clients.[23]

Much more can be said about practical mathematics as a tradition in process; but, for present purposes, the following heuristic advice suffices.[24] First of all, one should focus on common artifacts, techniques, problem solutions and concepts, since these held the tradition together. The dynamics were then supplied by how tradition elements were transformed by players with different agendas, roles, access to patronage or other types of material support. Objectively determinable factors enter here: context specific distributions of university chairs; demands for types of instruction, and sites for their delivery; the distribution of sites for patronage; patterns of education and role expectation amongst the nobility, gentlemen and commercial classes.[25] Actions in the field depended upon perceptions of, and agendas regarding, the capture of these resources and roles.

---

[18] Ibid.

[19] Bennett, "Practical Geometry and Operative Knowledge", p. 219

[20] The field of geography, as described by Bennet, serves as an early exemplar of a dynamism understood by mathematical practitioners and their audiences. Bennett, "Practical Geometry and Operative Knowledge", pp.202-6.

[21] An example of a shared tool kit is projective geometry, used in perspective painting, cartography, and instrument design. Bennett, "Practical Geometry and Operative Knowledge", pp.198. For sixteenth century England similarly see Stephen Johnston, "Mathematical practitioners and instruments in Elizabethan England" and "The identity of the mathematical practitioner in sixteenth century England".

[22] Bennett, "Practical Geometry and Operative Knowledge" pp.206-7; and Bennet, "The Challenge of Practical Mathematics." pp.179-81.

[23] Bennett, "Practical Geometry and Operative Knowledge", pp.206-7.

[24] Material in this and the next paragraph arose through collaboration with Dr. Catherine Neal [Hill] and was first presented at the Quadrennial Joint HSS, BSHS and Canadian Society for the History and Philosophy of Science Conference, St Louis, Missouri, August 2000.

[25] For example, in the early seventeenth century, relatively centralised, monarchical France had fewer significant patronage sites than did Italy, but had many young gentlemen educated by

Returning to the relations between practical mathematics and natural philosophising, it is clear these were characterized by mutual articulation, not one way traffic, for practical mathematical work was occasionally affected by moves coming from natural philosophising. For instance, Napier's development of the logarithms shows the importance of concepts of uniform and non-uniform acceleration and velocity to his approach. Moreover, his aim was astronomical, so some of his tools and aims arose from the domain of natural philosophising. Similarly, as practitioners took parts of the mixed mathematical field of optics into the tradition of practical mathematics, their results in turn could be imported by natural philosophers and re-negotiated as part of their own trajectories in the natural philosophical contest.[26]

Two important insights emerge here: Firstly, the simple (but multifarious) externalist stories of source, target and cause, invoking practical mathematics, must be set aside, in favor of the study of the trajectories of both traditions—natural philosophy and practical mathematics—in their mutual articulations and internal contestations over time. Secondly, our modeling of both fields supports our earlier surmise that some of the most important action involving innovating natural philosophers and the realm of practical mathematics took place in the domain of mixed mathematics, which, according to the dominant neo-Scholastic Aristotelianism of the universities, was ambiguously placed and subordinate to, but not organically part of, natural philosophy. Accordingly, we next use our new models to ‘rectify’ the old domain of externalist explanation, in preparation for our case studies.

Rectifying the Terrain of Externalist Explanation

In this section we examine mixed mathematics as a contested borderland between natural philosophy and practical mathematics. We also reconsider practical mathematicians’ rhetoric concerning the utility and progressiveness of their domain, recalibrating how this element enters into revised narratives of ‘practical mathematics and natural philosophising’.

We begin with the question of the status of the mixed mathematical sciences, according to the dominant Scholastic Aristotelianism: Natural philosophy studies matter and cause and renders physical explanations. Mathematics deals with geo-

the Jesuits, and therefore indoctrinated into the value of practical mathematics for the ‘gentleman officer’, destined for service in the religio-political conflicts of the time.

[26] See on this Sven Dupré’s chapter in this volume. My point here was initially stimulated by Jim Bennett’s discussion of three cases of natural philosophical appropriation of practical mathematical resources—Tycho Brahe (practical astronomy), Leonard Digges (gunnery) and William Gilbert (navigational magnetism). Bennett, “Practical Geometry and Operative Knowledge”, p.220.

metrical figures and numbers, things that do not change and exist only in our minds. On this basis Aristotelians recognised the so-called ‘mixed’ mathematical sciences, such as planetary astronomy, geometrical optics, statics, music theory and mathematical geography, as sub-ordinate to natural philosophy. They give only instrumental mathematical descriptions, not causal explanations. For example, according to Aristotelians, the investigation of the physical nature of light falls under natural philosophy, involving principles of matter and cause. The mixed mathematical science of geometrical optics is subordinate to both natural philosophy and mathematics. Studying ray diagrams, where geometrical lines represent rays of light, it deals with phenomena such as the reflection and refraction of light in a descriptive, mathematical manner, and cannot provide causal explanations, based on the physical nature of light. Such was the dominant, “declaratory” neo–Scholastic view of how the mixed mathematical disciplines related to the ‘superior’ discipline of natural philosophy. [27] Subsequent debates started from this hegemonic base.

One of the most attractive recent lines of inquiry looks to progressive Scholastics themselves, especially leading Jesuit mathematicians, for the decisive moves to liberate and more fully mathematicise these sciences. Peter Dear wove a sophisticated narrative along these lines, focusing upon previously neglected Scholastic mathematicians: Early in the seventeenth century some "Jesuit mathematical scientists"—astronomers and opticians—began to attempt “to justify these disciplines against criticism of their scientific status”.[28] Their strategic location in Jesuit colleges and universities amplified the import of these moves. Dear expertly followed a series of textbooks and debates amongst this group, which initiated the elaboration of a new, non-Aristotelian concept of singular, contrived and mathematically articulated ‘experience’. This represented a bid for the disciplinary autonomy of the mixed mathematical sciences from 'natural philosophy'. Dear argued correctly that for Jesuit mathematicians, such as Clavius, "Mathematical sciences that applied to the physical world were not taken to be in conflict with qualitative Aristotelian natural philosophy, but were typically seen as being about different things."[29] Clavius and others used this mathematics/natural philosophy distinction to preserve the integrity and certitude of mathematical pursuits, hence to legitimate the mixed mathematical disciplines as of explanatory and scientific status. Dear says this demarcation enhanced “their own pretensions to scientificity, and set the stage for a co-option of natural philosophy itself—the emergence of what Dear and his subjects termed "physico-mathematics".[30] This then fed into Dear’s larger story of the rise of modern (mathematico-experimental) science. Others, including Mersenne, Descartes and Beeckman, developed physico-

---

[27] I term the widely taught rule of subordination of mixed mathematics to natural philosophy ‘declaratory’ to denote that it was publicly proclaimed, but not necessarily binding or agreed to by relevant players.

[28] Peter Dear, *Discipline and Experience: The mathematical way in the Scientific Revolution*, p.6.

[29] Dear, *Discipline and Experience,* p.163.

[30] Dear, *Discipline and Experience,* p.163.

mathematics, and further mid century developments eventually led to Newton, who perfected the needed extra ingredient of the one-off 'event experiment' to arrive at the "spiritual core of modern science".[31]

Dear's elegant account has one unfortunate and unintended undertone, in that it resembles an origin tale: Embryonic modern science was hived off from 'natural philosophy' (equated with Scholastic Aristotelianism only), which conveniently died. The difficulty is that the key figures in the early physicalisation of the mixed mathematical sciences were not the Jesuit Aristotelian mathematicians, but the usual suspects in Scientific Revolution historiography, such as Galileo, Kepler, Descartes, Gilbert, Mersenne and Beeckman. Early in the seventeenth century, it was these natural philosophers who variously claimed that mathematics could play an explanatory role *in* natural philosophy, rejecting the declaratory Aristotelian position.[32] Moving between mixed mathematics and novel natural philosophising, they produced more 'physico-mathematical' versions of the old fields, supportive of their respective natural philosophical agendas. The origin of mathematicised science**s**, is really the emergence of more physicalised versions of the existing mixed mathematical sciences, and the construction of some new ones—all within the bubbling field of natural philosophising, as innovative natural philosophers competed to appropriate resources, technical and rhetorical, from a rich and dynamic practical mathematics tradition.[33] We shall touch on some technical matters later in our case studies. For the moment we concentrate on the rhetorical transactions involved.

Natural philosophical radicals, such as Descartes, Beeckman, Kepler, and Galileo, who were physicalising the mixed mathematical sciences, operated within a discursive framing of their enterprises, based on an already available rhetoric of the utility, intelligibility and cognitive value of the mechanical arts and practical

---

[31] This is not meant as a full summary of Dear's widely appreciated argument. We are interested here in the earliest stages of the story: [1] the tactics of the Jesuit mathematicians, and [2] the wider spectrum of meanings of physico-mathematics at the time.

[32] See Schuster and Taylor, "Seized by the spirit of Modern Science". We hold that the plays of Clavius and his colleagues were moves within the wider field of natural philosophising, and somewhat precious and unproductive ones. Moreover, theirs was not the only version of physico-mathematics on offer, as we learn below.

[33] All this serves to articulate the view of natural philosophy as a contested field in which players first learned the rules of claim-making through their neo-Scholastic Aristotelian educations, but could realise that these rules were 'negotiable', as the sociologists of scientific knowledge would say, and that their meanings were in the hands of successive waves of users. While some Aristotelians tried to bend the rules about the subordinate nature of the mixed mathematical disciplines, more radical natural philosophers, such as Kepler and Descartes, often ran right over them, forging new meanings and practices. (Kepler, however, still paid non trivial 'declaratory' allegiance to them in some contexts. Cf. Rhonda Martens, *Kepler's Philosophy and the New Philosophy,* Chapter 5 "The Aristotelian Kepler.) So, by the first third of the seventeenth century, the given rules of subordination of mixed mathematics were the subject of vexed debate. To bring resources from practical mathematics into this arena was, to radical players, a very attractive gambit.

mathematics.[34] Masters of the practical arts, including practical mathematicians, had spent a lot of time in the sixteenth century publicising the usefulness, and the knowledge-like character, of their enterprises. Our early seventeenth century natural philosophers picked up these messages, reformatted them for natural philosophical utterance and rebroadcast them as legitimations for new agendas in natural philosophy.[35] Such co-options were endemic, and perhaps cumulative; we find them all along the trajectory of interactions. Consequently, one certainly should not mistake any instance of a natural philosopher co-opting the rhetoric of the practical mathematicians for the 'foundation of the essence' of 'modern mathematical science' — a pitfall for the early externalists. Nevertheless, appropriation of practitioner's rhetoric was substantively important for innovative natural philosophers. It helped shape their self-understandings of their programs and it softened up audiences for their reception. As our case studies will show, technical developments have technical causes, but rhetorical transactions should be studied and woven into dense accounts of natural philosophical gambits.

Because the mixed mathematical sciences formed a borderland between natural philosophising and the field of practical mathematics, one can "map" how mixed mathematics sat in relation to radical, anti-Aristotelian natural philosophising and to practical mathematics. One can envision a spectrum of players: [a] natural philosophers little concerned about mixed or practical mathematics; [b] natural philosophers actively interested in co-opting and using technical and rhetorical resources from mixed and/or practical mathematics; [c] elite practical mathematicians abstracting from lower level practical mathematics who might or might not link their activities to natural philosophising; and [d] lower level mathematical practitioners. The interesting action was in categories [b] and [c], where the physicalisation of the mixed mathematical sciences took place, and new physico-mathematical disciplines emerged.

For mathematically inclined, anti-Aristotelian natural philosophers, such as Kepler, Galileo, Beeckman and Descartes, the mixed mathematical sciences were ripe for co-optation into their innovative natural philosophical pursuits. For such players, practical mathematics tagged along with the outcome for mixed mathematical sciences. For example, geometrical optics was involved in a wide range of mathematical practices and artefacts, whilst it also articulated with high level natural philosophical theorising, and for some players, such as Kepler and Descartes, 'physicalised' versions of geometrical optics were a prime location for natural phi-

---

[34] Rossi, *Philosophy, Technology and Arts in Early Modern Europe,* has by far the best grasp of this process.

[35] Contemporary historians of practical mathematics, such as Jim Bennett, Catherine Neal [Hill], Stephen Johnson and Lesley Cormack, teach us that much conflict characterised the practical mathematical field. The common legitimatory 'front' about the value of the practical arts trumpeted by some natural philosophers, may therefore have had more to do with the natural philosophical *agon* than with any consensus amongst master mathematical practitioners. See, for example, Catherine Hill, "`Juglers or Schollers?': Negotiating the Role of a Mathematical Practitioner"; and Catherine Neal [Hill],"The Rhetoric of Utility: Avoiding Occult Associations for Mathematics Through Profitability and Pleasure."

losophical initiatives. Similarly, 'high' statics/hydrostatics was thought to ground understandings of simple machines, and through them, complex machines, and hence by extension, much of the 'mechanical arts'. This 'cultural fact' could be played upon from different directions by natural philosophers and practical mathematicians. For example, the young Galileo in his de Motu treated statics and hydrostatics dynamically in an (unsuccessful) attempt to extract from them anti-Aristotelian conclusions about natural and violent motion.[36] The view of the young Descartes, as we shall see, was that any rigorous result in the mixed mathematical sciences bespeaks the discovery of a deep physical truth, which can be reduced to corpuscular-mechanical terms. But, the mixed mathematical borderland could be appropriated in the other direction. The great Simon Stevin determinedly removed mixed mathematics and mathematical practice from the domain of natural philosophy (by which he understood Aristotelianism).[37]

Figure 2 looks more closely at conservative versus radical takes on the relation of mixed mathematics to natural philosophy, distinguishing those active in mixed mathematics from those not active, or merely talking about their classification and hence involved in rhetorical transactions only with practical mathematics.

	conservative	**radical**
Active in mixed maths	*Jesuit mathematicians, Simon Stevin (ultra conservative)*	*Kepler, Descartes, Harriot, Galileo (but not a systematic natural philosopher)*
Inactive in mixed maths	*Garden variety Scholastic Aristotelians*	*Some involved in making classifications of mixed sciences, especially mechanics, as dealing with matter and cause; Gilbert?*

Fig. 2 View of relation of mixed/practical mathematics to natural philosophy. A Classification of people talking about or practising the mixed mathematical sciences

[36] Stephen Gaukroger, "The Foundational Role of Hydrostatics and Statics in Descartes' Natural Philosophy.", and Gaukroger and Schuster, "The hydrostatic paradox and the origins of Cartesian dynamic."

[37] Stevin endeavoured to bring statics and hydrostatics, and the practices that follow from them, into an Archimedean, rigorous, mathematical context, thus rejecting the pseudo-Aristotelian Mechanical Questions with its dynamical approach to simple machines and statics. On Stevin see our first case study below and Note 47.

The latter, in the lower right hand quadrant, includes those sixteenth century Scholastic commentators on the status of mechanics who edged toward acknowledging its relevance to natural philosophy, but who did not technically practise mechanics. In the upper left-hand quadrant are Dear's Jesuit mathematicians, active in mixed mathematics but holding a conservative view of their relation to natural philosophy —disciplinarily separate but scientifically 'equal'. Joining them is Stevin, master of the mixed mathematical fields, holding a different conservative view of radical separation, and mutual irrelevance. Ordinary Aristotelians, adhering to the declaratory subordination rule, occupy the lower left-hand quadrant, whilst the 'usual suspects', radical natural philosophers seeking to "physicalise" the mixed mathematical fields, are in the upper right-hand quadrant.

Figure 3 asks of elite mathematical practitioners whether they tried to synthesise practical and mixed mathematics in any way beyond traditional understandings; and whether they linked such agendas to the field of natural philosophising.

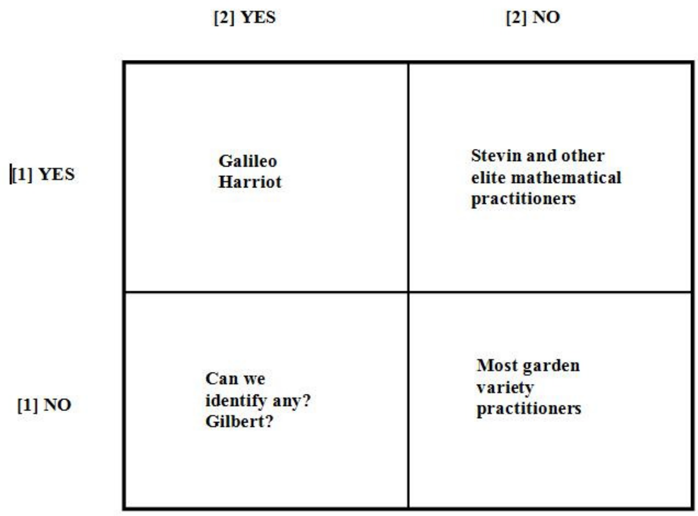

Fig. 3. Elite mathematical practitioners' agendas: [1] synthesize practical and mixed mathematics beyond traditional understandings yes/no [2] agenda articulated to the field of natural philosophy yes/no

Expert mathematical practitioners, such as Galileo and Harriot, pushed practical and mixed mathematics beyond traditional understandings to extract natural philosophical capital. Stevin, pursuing higher cultural status for mixed and practical mathematics, but also denying their relevance to natural philosophising, occupies the upper right hand quadrant. He is joined by advocates of the high status of practical mathematics, elite practitioners who, did not encroach into the domain of natural philosophy. Ordinary practitioners would be in the lower right-hand quadrant. The lower left-hand quadrant is reserved for those whose rhetoric or technical practise linked practical and mixed mathematics to natural philosophising, but who had little impact on contemporary practises or understandings of the mathematical fields. One inhabitant might be William Gilbert, an innovative natural philosopher and a consumer of others' mixed and practical mathematical work, but not a particularly innovative practitioner therein.

In sum, we shall achieve better accounts of 'practical mathematics and the Scientific Revolution' if we think through the categories 'natural philosophy' and 'practical mathematics' in the ways suggested, and then follow the plays from each side into the 'marcher fiefdoms' of mixed mathematics. Therefore, we turn now to three case studies intended to illustrate and test these claims.

Case Study 1: Descartes, *physico-mathematicus*, forms the causal register of his mechanical philosophy, his dynamics, by appropriation and translation from [certain] mixed mathematical sciences

To understand the two episodes in this first case study, we first need to examine what Stephen Gaukroger and I term Descartes' dynamics, a set of concepts that supplied the doctrine of physical causation within Descartes' natural philosophy.[38] As already noted, a core aim of 'natural philosophising' was the identification of what causes material bodies to behave in particular ways. For example, in Aristotelianism, natural processes were explained primarily on the basis of causes identified with the nature or essence of the substance in question, while in neo-Platonic natural philosophies, brute matter was worked upon from the outside by various types of non-material causal agents. To theorise about matter and an associated 'causal register' was central to any genre of natural philosophy. Whatever disputes there might have been amongst Platonists, Aristotelians, Stoics, and atomists, there was consensus on what kind of theory provided the ultimate explanation of macroscopic physical phenomena, namely a theory of matter and causation. Descartes' mature natural philosophy was no exception, being concerned with the nature and 'mechanical' properties of microscopic corpuscles and a causal discourse, consisting of a theory of motion and impact, explicated through such key concepts as the 'force of motion' and 'tendencies to motion'. It is this causal register within Descartes' natural philosophy which we term his 'dynamics'.

---

[38] Gaukroger and Schuster, "The hydrostatic paradox and the origins of Cartesian dynamics", pp.557, 561, 568-70; Schuster, "'Waterworld': Descartes' Vortical Celestial Mechanics: A Gambit in the Natural Philosophical Contest of the Early Seventeenth Century", pp. 38-41.

In Descartes' mature corpuscular-mechanical natural philosophy, his carefully articulated theory of dynamics governed the behaviour of micro-particles. Bodies in motion, or tending to motion, are characterised from moment to moment by the possession of two sorts of dynamical quantity: (1) the absolute quantity of the 'force of motion'—conserved in the universe according to *Le Monde's* first rule of nature and (2) the directional modes of that quantity of force, which Descartes termed 'determinations', introduced in *Le Monde's* third rule of nature. Descartes' dynamics focused on instantaneous tendencies to motion, rather than finite translations in space and time. As corpuscles undergo instantaneous collisions with each other, their quantities of force of motion and determinations are adjusted according to certain universal laws of nature, rules of collision.

Descartes' exemplar for applying these concepts to light and celestial mechanics is the mechanics of a stone rotated in a sling.[39] **[Fig. 4]** He analyses the dynamical condition of the stone at the precise instant that it passes point A.

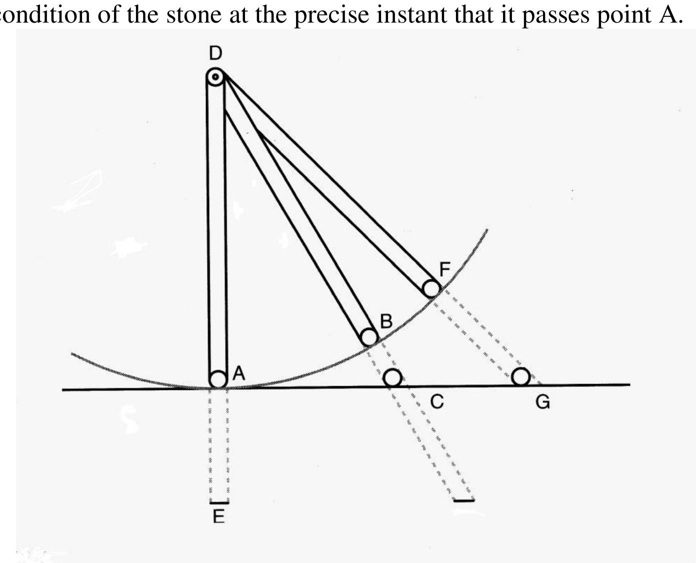

Fig. 4. After Descartes, *Le Monde*, AT XI p.45 and p.85

The instantaneously exerted force of motion of the stone is directed along the tangent AG. If the stone were released and no other hindrances affected its trajectory, it would move along ACG at a uniform speed reflective of the moment to moment conservation of its quantity of force of motion. However, the sling continuously constrains the privileged, principal determination of the stone and, acting over time, deflects its motion along the circle AF. Descartes decomposes the principal determination into two components: one along AE completely opposed by the

[39] Descartes, *Oeuvres* vol. XI pp.45-6, 85 [Hereafter cited as AT (for Adam and Tannery edition, roman numeral for volume, plus page.]; Descartes, *Descartes, The World and Other Writings*. Trans. Stephen Gaukroger, pp.30. 54-5.

sling—so no actual centrifugal translation can occur—only a tendency to centrifugal motion; the other, he says, is "that part of the tendency along AC which the sling does not hinder", which over time manifests itself as translation in a circle. The choice of components of determination is dictated by the configuration of mechanical constraints on the system.

1619—From Hydrostatics to Dynamics: From Mixed Mathematics to Corpuscular Mechanism [40]

In 1586 Simon Stevin, Dutch maestro of practical mathematics, proved a special case of the hydrostatic paradox. Stevin demonstrated that a fluid filling two vessels of equal base area and height exerts the same total pressure on the base, irrespective of the shape of the vessel and hence, paradoxically, independently of the amount of fluid it contains. Stevin's argument proceeds with Archimedean rigour on the macroscopic level of gross weights and volumes and depends upon the maintenance of a condition of static equilibrium.[41]

In 1619 the twenty-two year old Descartes and his thirty year old Dutch mentor, Isaac Beeckman, tried to provide a natural philosophical explanation for Stevin's result.[42] In the key example, Descartes considers two containers **[Fig. 5]**: B and D, which have equal areas at their bases, equal height and are of equal weight when empty, and are filled to their tops. Descartes proposes to show that, 'the water in vessel B will weigh equally upon its base as the water in D upon its base'—Stevin's paradoxical hydrostatical result.

Descartes attempts to reduce the phenomenon to micro-mechanics by showing that the force on each 'point' or part of the bottoms of the basins B and D is equal, so that the total force is equal over the two equal areas. He claims that each 'point' on the bottom of B is serviced by a unique line of 'tendency to motion' propagated by contact pressure from a point (particle) on the surface of the water through the intervening particles. **[See Fig.5]** He takes points g, B, h; in the base of B, and points i, D, l, in the base of D. He cleverly claims that all these points are pressed by an equal force, because they are each pressed by 'imaginable lines of water of the same length'; that is, the same vertical component of descent. Despite this, Descartes' overall effort is distinctly odd. For example the mappings of lines of

---

[40] Material in this section derives from Gaukroger and Schuster, "The hydrostatic paradox and the origins of Cartesian dynamics"; Gaukroger, *Descartes: An Intellectual Biography*, pp.84-9 *;* and Schuster, "Descartes' *Mathesis Universalis*, 1619-28", pp.41-55.

[41] Simon Stevin, "*De Beghinselen des Waterwichts*" (Leiden, 1586) in *The Principal Works of Simon Stevin.* Vol. 1, pp.415-17

[42] The text, *Aquae comprimentis in vase ratio reddita à D. Des Cartes* which derives from Isaac Beeckman's diary, is given in AT, X, pp. 67–74, as the first part of the *Physico-Mathematica*. See also the related manuscript in the *Cogitationes Privatæ*, AT, X, p. 228, introduced with, 'Petijt è Stevino Isaacus Middelburgensis *quomodo aqua gravitet in fundo vasis* b…'.

tendency are tendentious and not subject to any rule. Even so, for the rest of his career, Descartes continued to use descendants of these concepts of instantaneously exerted force of motion and its analysis into component 'determinations'.[43]

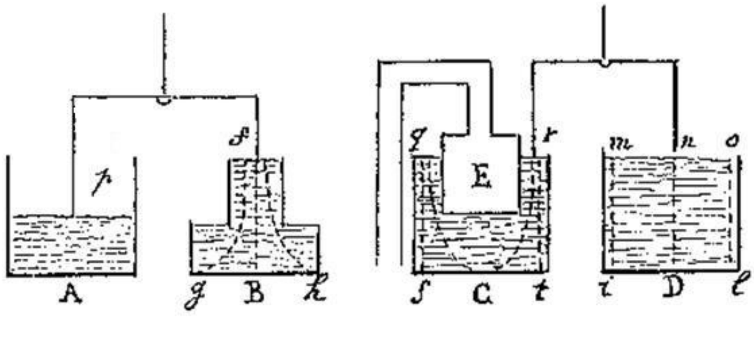

Fig. 5 Descartes, *Aquae comprimentis in vase ratio reddita à D. DesCartes*, AT X 69.

Descartes' manuscript signals that he no longer viewed hydrostatics as a discipline of mixed mathematics in the Aristotelian sense. Rather, he saw it as a domain of application of corpuscular-mechanical natural philosophy, because, to explain the key hydrostatical results, concepts of matter and cause of clear natural philosophical provenance had to be deployed. This anti-Aristotelian program Descartes termed 'physico-mathematics',[44] but, for several reasons, it was a far cry from the physico-mathematics of Dear's Jesuit Aristotelian mathematicians: Firstly, mixed mathematical hydrostatics is not severed from natural philosophy in order to secure it 'scientific status'; rather, it becomes coextensive with natural philosophical issues of matter and cause. Secondly, the species of natural philosophising in question no longer is neo-Scholastic Aristotelianism, but proto-mechanism. Finally, Descartes' approach was extremely radical, even within the small club of anti-Aristotelian physico-mathematical aspirants, because it was based in the rigorous style of Stevinite/Archimedean statics and hydrostatics, whereas most attempts to make anti-Aristotelian natural philosophical capital out of the mixed mathematical sciences depended on taking a dynamical approach to statics and the

---

43 Gaukroger and Schuster, "The hydrostatic paradox and the origins of Cartesian dynamics"; Schuster, "'Waterworld': Descartes' Vortical Celestial Mechanics: A Gambit in the Natural Philosophical Contest of the Early Seventeenth Century; Schuster, "Descartes *Opticien:* Descartes' Manufacture of the Law of Refraction and Construction of its Physical and Methodological Rationales 1618-1628"

44 Descartes' employed the term physico-mathematics following lead of Beeckman, "Physico-mathematici paucissimi": AT X. 52. They clearly prided themselves on being virtually only true physico-mathematicians in Europe. In this regard Beeckman was later to note in 1628 that his own work was deeper than that of Bacon on the one hand and Stevin on the other just for this very reason. Beeckman *Journal* Vol. 3.pp. 51-2.

simple machines, following the lead of the pseudo-Aristotelian Mechanical Questions.

In the Mechanical Questions one views equilibrium conditions on a lever or simple machine as a balance of forces, where force is defined as Weight times Speed. Equilibrium is a special case of the dynamic opposition of the bodies; and statics is simply a limiting case of a general dynamical theory of motion.[45] Stevin, Descartes' exemplar in this matter, had preferred pure Archimedean statics and so had rejected this approach: dynamical thinking could not explain systems in static equilibrium.[46] However, Stevin had been in a minority on this issue.[47] But, the young Descartes daringly followed Stevin, starting from a mathematically rigorous hydrostatics and then fleshing it out with Beeckman-esque corpuscles. The young Descartes' radical version of physico-mathematics involved reducing Stevin's hydrostatics to an embryonic corpuscular mechanism in which discourse concerning 'forces or tendencies to motion' would provide the basis for unifying the mathematical sciences, under a dynamics of corpuscles. His astonishing strategy was to appropriate practical and mixed mathematical materials, and creatively rework them through moves in the culture of natural philosophising.[48] To confirm this, let us consider Descartes' work on refraction and physical optics, which, I contend, was the climax of his early physico-mathematical program.[49]

---

[45] On the *Mechanical Questions* in this connection, see Gaukroger and Schuster, "The Hydrostatic Paradox", pp.544 note 19. More generally, see Henri Carteron, *La Notion de force dans la système d'Aristote*; Pierre Duhem, *Les origines de la statique;* Paul Lawrence Rose and Stillman Drake, "The Pseudo-Aristotelian *Questions of Mechanics* in Renaissance Culture"; W. R. Laird, "The Scope of Renaissance Mechanics."; and Helen Hattab, "From Mechanics to Mechanism: The *Quaestiones Mechanicae* and Descartes' Physics".

[46] Gaukroger and Schuster, "The Hydrostatic Paradox", pp. 540, 545-9; Stevin, "Appendix to the *Art of Weighing"* in*, Principal Works* Vol 1, 507-9; and "*The Practice of Weighing*, 'To the Reader'", *ibid*. Vol. 1,. 297.

[47] For example, the young Galileo had tried, unsuccessfully, to use the Mechanical Questions to found an anti-Aristotelian physics. Stephen Gaukroger, "The Foundational Role of Hydrostatics and Statics in Descartes' Natural Philosophy.", and Gaukroger and Schuster, "The Hydrostatic Paradox."

[48] Just as Descartes ignored the 'declaratory' Scholastic rules about the subordination of mixed mathematics, he ignored Stevin's strictures on the mutual irrelevance of natural philosophy to mixed and practical mathematics.

[49] Indeed the dynamic of research and concept formation unleashed here played out well beyond the optical work of the 1620s and extended directly to the important and little understood details of his vortex celestial mechanics in *Le Monde*, see Schuster, "'Waterworld': Descartes' Vortical Celestial Mechanics"

1627—The Laws of Light and the Laws of Nature [50]

The physico-mathematical hydrostatics of 1619 marked the first partial articulation of the central tenets of Descartes' dynamics. Their path of development between 1619 and the completion of Le Monde in 1633 led not through hydrostatics, but via the most important and fruitful physico-mathematical research Descartes ever attempted—his work in geometrical and physical optics in the 1620s. This involved his discovery of the law of refraction of light around 1627, followed immediately by his exploration of possible mechanical rationales or explanations for the law. The latter attempts in turn were intimately connected with the process by which he crystallised his concepts of dynamics directly out of a 'physico-mathematical' 'reading' of his geometrical optical results.

In 1626/7 Descartes, collaborating with Claude Mydorge, constructed a law of refraction, by working within traditional geometrical optics in the limited mixed mathematical sense and without any corpuscular-mechanical theorising. Descartes and Mydorge, like Harriot earlier, used the traditional image locating rule in order to map the image locations of point sources taken on the lower circumference of a half submerged disk refractometer. **[Fig. 6]**

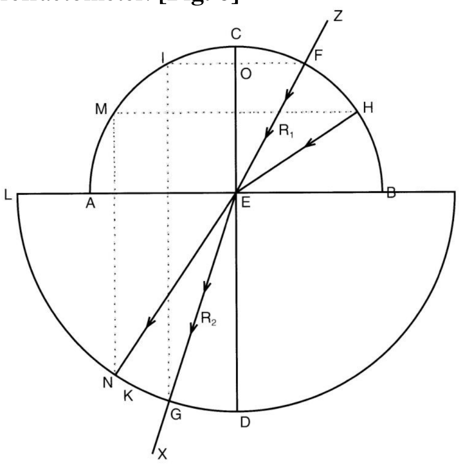

Fig. 6. Harriot's Key Diagram

Even using Witelo's fudged data, one gets a smaller semi-circle as the locus of image points, yielding a law of cosecants. In order to create a refraction predictor, Mydorge flipped the inner semi circle up above the interface as the locus of point sources for the incident light. **[Fig. 7]**

[50] Material in this section derives from Schuster, "Descartes *Opticien:* Descartes' Manufacture of the Law of Refraction and Construction of its Physical and Methodological Rationales 1618-1628".

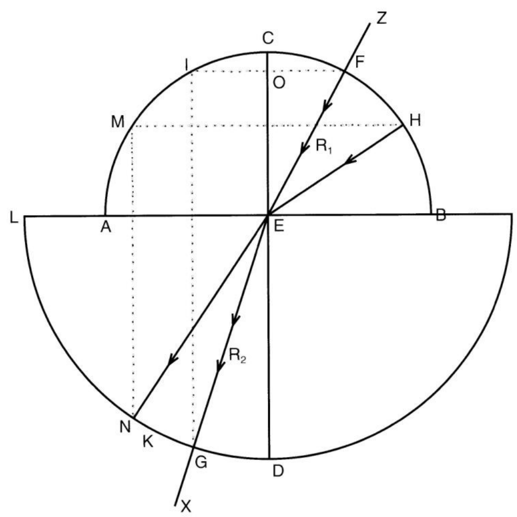

Fig. 7. Mydorge's Refraction Prediction Device

On this representation of the new law Descartes then worked his favoured style of physico-mathematical magic: Looking for a physics of light to explain the law, he transcribed into dynamical terms the geometrical parameters embodied in this diagram. **[Fig. 7]** The resulting dynamical principles concerning the mechanical nature of light were: [1] that the parallel component of the force of a light ray is unaffected by refraction, whilst [2] the quantity of the force of the ray is increased or decreased in a fixed proportion. These then suggested the form of the two central tenets of his mature dynamics. After all, what could be more revealing of the underlying principles of the punctiform dynamics of corpuscles than the basic laws of light—itself an instantaneously transmitted mechanical impulse? Descartes, physico-mathematician, was exploiting geometrical representations of telling phenomena in which no motion took place at all—in hydrostatics, and in refraction of light. In these 'statical' exemplars Descartes found crisp messages about the underlying dynamics of the corpuscular world. Descartes was bidding to transform mixed mathematical optics into a physico-mathematical discipline, and to extract from it conceptual resources for his mechanical philosophy.[51]

[51] There were competing varieties of physico-mathematics. In addition to Descartes' program and the Jesuit mathematicians' attempts to promote mixed mathematics as 'separate but more or less equal' to natural philosophising; there were [1] attempts to bring mechanics, particularly a dynamical approach to the simple machines into natural philosophy; [2] Kepler's profound neo-Platonising of mixed mathematics and redirecting the thus physicalised disciplines back into natural philosophy; [3] Beeckman's linking of an emergent corpuscular mechanism to dynamical interpretations of the simple machines [Gaukroger and Schuster, "The Hydrostatic Paradox" pp.555-7]; finally [4] Galileo's rather more piecemeal physico-mathematical excursions, including his construction of a *sui generis* new kinematical science of motion.

According to the young Descartes' physico-mathematical strategy, any rigorous result in the mixed mathematical sciences bespeaks the discovery of a deep physical truth which can be reduced to corpuscular-mechanical terms.[52] Results in the mixed mathematical sciences are thus reduced and explained, and, by extension, the further realms of practice are subsumed and controlled. These optical researches marked the high point of his work as a physico-mathematician transforming the 'old' mixed mathematical sciences and co-opting the results into a mechanistic natural philosophy: His optical results both confirmed his 1619 agenda of developing a corpuscular ontology and a causal discourse, or dynamics, involving concepts of force and directional 'determinations', and they shaped his conception of light as an instantaneously transmitted mechanical tendency to motion, as well as the precise principles of his dynamics.[53]

These examples are significant in the trajectory of Descartes, but in the larger process of the Scientific Revolution, they are small events. However, they do show how our reformed notions of 'source', 'target' and 'mode of causation' can illuminate specific episodes within the general theme of "what did practical and mixed mathematics have to do with the Scientific Revolution?" Let us take this approach further into the explanatory challenge of this volume, seeking bigger game through two more case studies.

Case Study 2: Sorting Out the 'Causal Mode' of Sixteenth Century Mechanics

A common story of 'source, mode of causation and target' stars sixteenth century mechanics, a dynamic province of mixed mathematics: Sixteenth century mechanics provided core concepts, or metaphors, or values, for the mechanical philosophy, or, for 'the new science' generally. In this regard scholarly attention has recently focussed on one strand of sixteenth century mechanics, the pseudo-Aristotelian *Mechanical Questions*. For example, Helen Hattab, a leading scholar of Renaissance mechanics and philosophy, articulating the work of Rose, Drake and Laird, has documented how some sixteenth century commentators on the *Mechanical Questions* tried to collapse the distinction between physical explanations of natural phenomena and geometrical explanations of machines, thus inviting

---

[52] Gaukroger and Schuster, "The Hydrostatic Paradox", pp.568-70; and Schuster, "Descartes, *Opticien*", pp.279-85, 290-95.

[53] The optical work was indeed the technical high point of his physico-mathematical agenda, but the trajectory into natural philosophical systematics carried Descartes even further, to the 'completion' of this trajectory in the formulation of the vortex mechanics in *Le Monde* as I have argued in Schuster '''Waterworld': Descartes Vortical Celestial Mechanics.

mathematics into discourse concerning physical causation.[54] This is the type of process that should interest us regarding "practical mathematics and the Scientific Revolution". Hattab speculates that these border crossings shaped Descartes' approach to mechanics and mechanical philosophy, "influencing" him to absorb mechanics into physics, and apply mechanics to corpuscles, adding that Descartes derived general mechanical principles from analysing the sling (one of the canonical "mechanical problems" in the text).[55]

Let us submit Hattab's speculative story to an exercise in rectification of explanation. I do this not because of any shortcomings in Hattab's scholarship, which is superb, but rather because her speculation resembles other "source, mode and target" stories common in this area, and we are now well placed to unpack it. First of all, we should recognise that sixteenth century mechanics *per se* exerted no "influence" or "imprinting" upon Descartes. Secondly, as we have seen, Descartes' dynamics was forged in his physico-mathematical hydrostatics and optics. It did not arise via the *Mechanical Questions*, nor was the sling the source of Descartes' dynamical concepts; rather, it illustrated them. Thirdly, in support of Hattab, we can say sixteenth century mechanics was indeed just about the first site where attempts were made, on the level of both declaratory policy and technical practice, to move a mixed mathematical field, closely linked to practical mathematics, into direct contact with natural philosophical issues of matter and cause. Fourthly, Descartes certainly did some appropriating and translating. He picked up and re-emitted the legitimatory rhetoric of sixteenth century mechanics to package detailed, technical work, not deducible from that legitimatory discourse. Those technical resources came from Stevin and geometrical optics. The young Galileo, by contrast, had dipped into Archimedes as well as the *Mechanical Questions* tradition for both sorts of resources.[56]

In sum, as signalled earlier, although technical developments have technical causes (based on appropriation of technical materials in technical contexts), rhetorical transactions are crucial to actors' self understandings and the enrolment of audiences and so must be woven into rectified developmental stories. In explanations of technical and legitimatory developments, influence and imprinting should

---

[54] Hattab, "From Mechanics to Mechanism: The *Quaestiones Mechanicae* and Descartes' Physics"

[55] Hattab, "From Mechanics to Mechanism: The *Quaestiones Mechanicae* and Descartes' Physics", pp.122, 126-7.

[56] None of this is intended to underplay the role of sixteenth century developments in mechanics in the eventual crystallisation of the classical mechanics of Galileo and Newton. Recall our observation that the expression 'mathematisation of 'Science" should be construed as 'physicalisation of the mixed mathematical sciences'. The attempt to 'upgrade' mechanics to natural philosophical status is a key strand in that long process. The construction of classical mechanics involved various strands, in many of which there were 'physico-mathematical' plays by mathematically oriented natural philosophers into the realm of mixed mathematics, for the purpose of physicalising them and enhancing their relevant to natural philosophical issues of matter and cause.

be avoided in favor of some version of a cultural process model, keyed to suitable conceptualisations of the traditions and fields in play.

Case Study 3: Hobnobbing with Practitioners and Machines

During the years he was writing Le Monde and living in the United Provinces, Descartes tried to design a machine to grind parabolic lenses. It differed slightly from the machine described later in the Dioptrique. He attempted to persuade the artisan Jean Ferrier to come join him in the project, and a technical correspondence ensued.[57] What do these transactions say about practical mathematics/natural philosophy relations, and about Descartes' strategies regarding the two fields?

First, Descartes was, in his fashion, making a play inside the field of practical mathematics. He did indeed want to make and 'show' lenses that would embody his law of refraction, and control an improved telescope. Such behaviour is indistinguishable from that of an elite mathematical practitioner. But, secondly, he was manoeuvring within the culture of natural philosophy: The lens grinding machine was also a physical/mechanical instantiation of the law of refraction[58] — not just a bid for fame and profit**.** Indeed, it was a natural philosophical signifier, denoting a concrete and specially valued achievement. His lens machine, guided by natural philosophical principles, surpassed anything that could have been produced by crafty trial and error, and as Ramus and Bacon would have acknowledged, it was both illustrative of the truth and maximally useful.[59]

It should also go without saying that Descartes had not killed off natural philosophy in the interest of modern experimental "method" or "science". Ferrier,

[57] John Schuster, "Descartes and the Scientific Revolution: 1618-34: An Interpretation."pp.580-1; William Shea, *The Magic of Numbers and Motion. The Scientific Career of René Descartes*, pp. 191-201. These transactions are not to be confused with the work Ferrier actually undertook with Descartes and Mydorge regarding refraction earlier in the 1620s. (Schuster, "Descartes, *Opticien*", p.272; Shea, *The Magic of Numbers and Motion*, pp.150-2.)

[58] I am pleased to point out that the late Michael S. Mahoney first made this point to me many years ago in informal discussion.

[59] Rossi, *Philosophy, Technology and Arts in Early Modern Europe,* masterfully established this general perspective. My points here relate to the putative signification of the lens grinding machine as such. Neil Ribe interestingly widens this perspective, by demonstrating that for Descartes the ultimate aim of optical knowledge, practically embodied in telescopes and microscopes, is the improvement of (inherently limited) unaided human vision, in aid of the improvement of genuine knowledge to the purpose of generalised human mastery of nature. Ribe reminds us that at the conclusion of the *Diotprique* Descartes called for a new kind of artisan, from amongst the ranks of the "more curious and skilful persons of our age..." Ribe, "Cartesian Optics and the Mastery of Nature", p. 61.

who had worked with Descartes and Mydorge in the 1620s, came again into potential play regarding the new machine in the early 1630s after the construction of the central concepts of Descartes' dynamics, and as Le Monde was being written. Wanting to hobnob with Ferrier was not driving Descartes' natural philosophical agenda, inscriptions or strategies at all. Inside natural philosophy the instrument was FOR natural philosophical agendas and actions. Descartes had not stopped being a natural philosopher and become a new kind of 'scientist' because he played with instruments and instrument makers. He played with instruments and instrument makers because this fitted his evolving agenda as a natural philosophical contender.[60] The general historiographical lesson here follows from our cultural process model: Suppose we ask, 'What were instruments and their makers FOR inside natural philosophy?' The answer is, they were FOR natural philosophising, FOR natural philosophers' agendas and actions. If we forget that, essence and origin stories will loom up, clouding our historiographical imaginations.

Finally, there is another lesson here for handling claims about the "influence" of the rhetoric and values of mathematical partitioners, because we are dealing with concrete "cultural process" transactions in a specific case. We can temper any claim that Descartes was "influenced" by mathematical practitioners by seeing how the values and rhetoric he appropriated geared into his process of work on a specific natural philosophical project. Hence we can calibrate what can and cannot be attributed to such a vague "influence" as the rhetoric, values or ideology of the mathematical practitioners. So, first of all, it is entirely possible Descartes appropriated practitioners' rhetoric and that this was used to express to others—and to himself—what he was doing and why. But, Descartes was doing more than practicing rhetoric. He was also "doing" optics, and "doing" natural philosophy in specific technical ways. Those "doings" are not deducible from the practitioners' rhetoric, caused or influenced by it. Descartes appropriated the rhetoric to wrap his results in cultural understandings, attractive and persuasive to his audience, and importantly to himself as well, for thematising his own roles and strategies. After all, we have seen how important to him had been his personal twist on the contemporary identity category of *physico-mathematicus.*

Conclusion: Opportunities and Pitfalls

When thinking about 'practical mathematics and the scientific revolution', we encounter a proliferation of uncritical stories of multiple sources and targets, linked by unsatisfactory causal categories, such as influence or imprinting. The answer to 'multiple sources for each given target' and 'multiple targets for each given

---

[60] In short Descartes wished to position himself as the leading philosopher of nature, by means of strategically crucial articulation with, and appropriation of, the rhetoric, as well as the findings, practices and artefacts of practical mathematics.

source' is not imposition of one story, or retreat to local studies only. We can rectify the terms of explanation by modelling both natural philosophy and practical mathematics as contested fields in process over time. In this way the strengths of a reformed externalism and the new historiography of practical mathematics can be realised, and their pitfalls avoided. Events within the field of natural philosophising did not involve members being forced or shaped from the outside by practical mathematics. Rather, players within natural philosophy appropriated, translated and utilised resources from without, with the resulting complex pattern of claims and outcomes—intended and unintended—being played out in the field of natural philosophy over time.

Finally, the approach taken in this paper may allow us to resolve the problem vulgarly expressed as 'how did science become mathematical'. The issue was not the 'mathematisation of science' but rather the 'physicalisation of the traditional mixed mathematical sciences' by radical natural philosophical challengers to neo–Scholastic hegemony, challengers who were, amongst other things, willing to appropriate and translate rhetorical and technical resources from the tradition of practical mathematics. We examined Descartes' strategies and gambits in this regard; but, Descartes was only one player in a competitive early to mid seventeenth century natural philosophical environment, where appropriation and natural philosophical deployment of mixed and practical mathematics—under the category 'physico-mathematics'—was cutting edge practice for some contenders. To conclude, therefore, 'the story of practical (and mixed) mathematics and the Scientific Revolution' is really the sum of largely yet to be written critical narratives of these activities.

References:

Anstey, Peter and John Schuster, "Introduction." In *The Science of Nature in the Seventeenth Century: Changing Patterns of Early Modern Natural Philosophy*. Edited by Peter Anstey and John Schuster, pp.1-7. Dordrecht: Springer, 2005.

Barnes, Barry. *T.S.Kuhn and Social Science*. London: MacMillan, 1982.

Beeckman, Isaac. *Journal tenu par Isaac Beeckman de 1604 à 1634*. 4 vols. Edited by C. de Waard. The Hague: Nijhoff, 1939-53

Bennett, Jim. "The Mechanics' Philosophy and the Mechanical Philosophy." *History of Science* 24 (1986): 1-28.

------ "The Challenge of Practical Mathematics." in *Science, culture and popular belief in Renaissance Europe* . Edited by Stephen Pumfrey, Paolo L. Rossi and Maurice Slawinski , pp. 176-190. Manchester: Manchester University Press, 1991.

------ "Practical Geometry and Operative Knowledge" in *Configurations* 6 (2) (1998): 195-222.

Bloor, David. *Knowledge and Social Imagery*. London: Routledge and Keagan Paul, 1976.

Borkenau, Franz. *Der Ubergang vom feudalen zum burgerlichen Weltbild. Studien zur Geschichte der Manufakturperiode.* Paris: 1934.

Carteron, Henri. *La Notion de force dans la système d'Aristote.* Paris: Vrin, 1923.

Cormack, Lesley. 'Geography.' in *Encyclopedia of the Scientific Revolution: from Copernicus to Newton*. Edited by Wilbur Applebaum, pp.261-264. New York: Garland Publishing, 2000.

Cunningham, Andrew. "Getting the game right: some plain words on the identity and invention of science." *Studies in History and Philosophy of Science*, 19 (1988): 365-89.

------"How the *Principia* Got its Name; or, Taking Natural Philosophy Seriously." *History of Science* 24 (1991):377-92.

Cunningham, Andrew, and Perry Williams. "De-centring the 'Big Picture': *The Origins of Modern Science* and the Modern Origins of Science." *British Journal for the History of Science* 26 (1993): 407-32.

Dear, Peter. "The Church and the New Philosophy." In *Science, Culture and Popular Belief in Renaissance Europe*. Edited by Stephen Pumfrey, Paolo. L. Rossi and Maurice. Slawinski, pp.119-39. Manchester: Manchester University Press, 1991.

------ *Discipline and Experience: The mathematical way in the Scientific Revolution*. Chicago: University of Chicago Press, 1995.

------ "Religion, Science and Natural Philosophy: Thoughts on Cunningham's Thesis." *Studies in History and Philosophy of Science* 32 (2001): 377-86.

Descartes, René. *Oeuvres de Descartes*. Edited by Charles Adam and Paul Tannery, 11 vols. Paris: J. Vrin, 1996

------ *Descartes, The World and Other Writings*. Edited and translated by Stephen Gaukroger. Cambridge: Cambridge University Press, 1998.

de Solla Price, Derek J., "Automata and the Origins of Mechanism and the Mechanistic Philosophy." *Technology and Culture* 5 (1964):9-42.

Douglas, Mary. *Natural Symbol*s. New York: Penguin 1970.

Duhem, Pierre *Les origines de la statique*, 2 vols. Paris: A. Herman, 1905-6.

Gaukroger, Stephen "The Foundational Role of Hydrostatics and Statics in Descartes' Natural Philosophy.", In *Descartes' Natural Philosophy*. Edited by Stephen Gaukroger, John Schuster and John Sutton, pp.60-80. Routledge: London, 2000.

------ *Descartes: An Intellectual Biography*, Oxford: Oxford University Press, 1995.

Gaukroger Stephen W. and John A. Schuster, "The hydrostatic paradox and the origins of Cartesian dynamics.", *Studies in History and Philosophy of Science* 33 (2002):535–572.

Golinski, Jan. *Making Natural Knowledge: Constructivism and the History of Science.* Cambridge: CUP, 1998.

Harrison, Peter. "The Influence of Cartesian Cosmology in England." In *Descartes' Natural Philosophy*. Edited by Stephen Gaukroger, John Schuster and John Sutton, pp.168-92. Routledge: London, 2000.

------ "Voluntarism and Early Modern science", *History of Science*, 40 (2002): 63-89.

------ "Physico-Theology and the Mixed Sciences: The Role of Theology in Early Modern Natural Philosophy." In *The Science of Nature in the Seventeenth Century: Changing Patterns of Early Modern Natural Philosophy*. Edited by Peter Anstey and John Schuster, pp.165-183. Dordrecht: Springer, 2005.

Hattab, Helen. "From Mechanics to Mechanism: The *Quaestiones Mechanicae* and Descartes' Physics." In *The Science of Nature in the Seventeenth Century: Changing Patterns of Early Modern Natural Philosophy*. Edited by Peter Anstey and John Schuster, pp.99-129. Dordrecht: Springer, 2005.

Henry, John. *The Scientific Revolution and the Origins of Modern Science*. London: Macmillan,1997.

Hessen, Boris. "The Social and Economic Roots of Newton's "Principia" in *Science at the Crossroads, Papers Presented to the International Congress of the History of Science and Technology Held in London from June 29th to July 3rd, 1931, by the Delegates of the USSR*, pp. 149-212. London: Kniga (England) Ltd, 1931.

Hill, Katherine [Katherine Neal] "`Juglers or Schollers?': Negotiating the Role of a Mathematical Practitioner." *British Journal for the History of Science* 31 (1998): 253-74.

Johnston, Stephen. "Mathematical practitioners and instruments in Elizabethan England.", *Annals of Science* 48 (1991): 319-44.

------ "The identity of the mathematical practitioner in 16th century England." In *Der `mathematicus': Zur Entwicklung und Bedeutung einer neun Berufsgruppe in der Zeit Gerhard Mercators.*. Edited by Irmgard Hantsche, pp.93-120. Bochum, 1996.

Laird, W. R. “The Scope of Renaissance Mechanics.” *Osiris* 2 (1986): 43-68.

Livingstone, David. “Geography” in *The Companion to the History of Modern Science*. Edited by Robert. Olby, Geoffrey. Cantor, John. Christie and M.J.S Hodge, pp. 743-760. London: Routledge, 1990.

Mahoney, Michael S. “The Beginnings of Algebraic Thought in the Seventeenth Century.” In *Descartes: Philosophy, Mathematics and Physics*. Edited by Stephen Gaukroger, pp.141-155. Brighton, Sussex: Harvester Press, 1980.

Martens, Rhonda. *Kepler’s Philosophy and the New Philosophy*. Princeton: Princeton University Press, 2000.

Mayr, Otto. *Authority, Liberty and Automatic Machinery in Early Modern Europe*. Baltimore: Johns Hopkins Press, 1986.

Neal, Katherine. “The Rhetoric of Utility: Avoiding Occult Associations for Mathematics Through Profitability and Pleasure.” *History of Science* 37 (1999): 151- 178.

Needham, Joseph. *The Great Titration: Science and Society East and West*. London: Allen & Unwin, 1969.

Olsler, Margaret. “Mixing metaphors: Science and Religion or Natural Philosophy and Theology in Early Modern Science.” *History of Science* 36 (1998):91-113.

Ribe, Neal. “Cartesian Optics and the Mastery of Nature.” *Isis* 88 (1997): 42-61.

Rose, Paul Lawrence and Stillman Drake. “The Pseudo-Aristotelian *Questions of Mechanics* in Renaissance Culture”, *Studies in the Renaissance*18 (1971) 65-104.

Rossi, Paolo. *Philosophy, Technology and Arts in Early Modern Europe.* New York: Harper and Row, 1970.

Sahlins, Marshall. "Goodbye to *Tristes Tropes*: Ethnography in the Context of Modern World History.", *Journal of Modern History* 65 (1993): 1-25.

Schuster, John A. “Descartes’ *Mathesis Universalis*, 1619-28.” In *Descartes: Philosophy, Mathematics and Physics*. Edited by Stephen Gaukroger, pp.41-96. Brighton, Sussex: Harvester Press, 1980.

------ “Descartes and the Scientific Revolution: 1618-34: An Interpretation.” Ph.D. diss., Princeton University, 1977.

------. “The Scientific Revolution.” In *The Companion to the History of Modern Science*. Edited by Robert. Olby, Geoffrey. Cantor, John. Christie and M.J.S. Hodge, pp. 217-42. London: Routledge, 1990.

------ "Descartes *Agonistes* New Tales of Cartesian Mechanism." *Perspectives on Science* 3 (1995): 99-145.

------ “L’Aristotelismo e le sue Alternative.” In *La Rivoluzione Scientifica.* Edited by Daniel Garber, pp. 337-357. Rome: Instituto della Enciclopedia Italiana, 2002.

------ “Internalist and Externalist Historiographies of the Scientific Revolution.”, In *Encyclopedia of the Scientific Revolution*. Edited by Wilbur Applebaum, pp.334-6. New York: Garland Publishing, 2000.

------ “‘Waterworld’: Descartes’ Vortical Celestial Mechanics: A Gambit in the Natural Philosophical Contest of the Early Seventeenth Century.” In *The Science of Nature in the Seventeenth Century: Changing Patterns of Early Modern Natural Philosophy*. Edited by Peter Anstey and John Schuster, pp.35-79. Dordrecht: Springer, 2005.

Schuster, John and Graeme Watchirs. “Natural Philosophy, Experiment and Discourse in the Eighteenth Century: Beyond the Kuhn/Bachelard Problematic.” In *Experimental Inquiries: Historical, Philosophical and Social Studies of Experiment*. Edited by Homer. E. LeGrand, pp. 1-48. Dordrecht: Reidel, 1990.

Schuster, John and Alan B. H. Taylor. “Blind Trust: The Gentlemanly Origins of Experimental Science.” *Social Studies of Science* 27 (1997): 503-536.

Schuster, John and Alan B. H. Taylor. ‘Seized by the Spirit of Modern Science.’ *MetaScience*, 9 (1996): 9-26.

Shapin, Steven. "Discipline and Bounding: The History and Sociology of Science As Seen Through the Externalism-Internalism Debate." *History of Science* 30 (1992): 333-369.

Shea, William R. *The Magic of Numbers and Motion. The Scientific Career of René Descartes.* Canton, Ma.: Science History Publications, 1991.

Skinner, Quentin. “Meaning and Understanding in the History of Ideas.” *History and Theory* 8 (1969):3-53

Stevin Simon. *The Principal Works of Simon Stevin*, 5 vols, Edited by E. Cronie et al., Amsterdam: Swets & Zeitlinger, 1955-66

Zilsel. Edgar. “The Sociological Roots of Science.” *American Journal of Sociology* 47 (1942): 544-62

______. “The Genesis of the Concept of Physical Law.” *Philosophical Review* 51 (1942):245-79