What Was Early Seventeenth Century 'Physico-Mathematics'? Or, Did the 'usual suspects' aim to replace natural philosophy with mathematics, or to reform natural philosophy from within?

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What Was Early Seventeenth Century 'Physico-Mathematics'? Or, Did the 'usual suspects' aim to replace natural philosophy with mathematics, or to reform natural philosophy from within?

I. "There are very few physico-mathematicians!" …thus in late 1618 Isaac Beeckman referred to some physico–mathematical exercises Descartes prepared in consultation with him. Beeckman wrote:

> "[Descartes] says he has never met anyone other than me who pursues enquiry the way I do, combining Physics and Mathematics in an exact way; and I have never spoken with anyone other than him who does the same."

Beeckman and Descartes were partly right. Whatever physico–mathematics was, it engaged only a sub-set of the mathematically ***and*** natural philosophically literate.

Stephen Gaukroger and I have written about Descartes and Beeckman's attempt to "physico–mathematicise" Stevin's hydrostatics. I've also presented the trajectories of Descartes in optics and vortex mechanics as physico–mathematical in tenor.

Nevertheless, the posturing of Beeckman and Descartes was also misleading: Of course others were operating in similar ways. Beeckman and Descartes marked merely one position in a wider domain of physico–mathematics. It contained meanings and implications reaching beyond the ken of any given practitioner, but which are recoverable by us through formulation of *an historiographical category of physico–mathematics*. Modelling such categories, applying and revising them, being for me the essence of serious historical work.

A model of physico–mathematics encompasses these two facts: there were differing contemporary interpretations of physico–mathematics; and, not everybody behaving physico-mathematically used that term. An historiographical category of physico-mathematics, through which we explain and narrate events, necessarily entangles us in early 17th century tensions in the overarching field of natural philosophising. But—and I stress this—we're not dealing with the death of natural philosophy and its displacement by mathematicians, despite what some might today think.

II. First we need to revisit the traditional mixed mathematical sciences. The term belonged to Aristotelianism, referring to a group of disciplines intermediate between natural philosophy and mathematics and subordinate to them. A natural philosophical account of something was an explanation in terms of matter and cause…for Aristotle, mathematics couldn't do that. The mixed mathematical sciences, such as optics, mechanics, astronomy or music theory, used mathematics not in an explanatory way, but instrumentally to represent physical things and processes mathematically. So in geometrical optics, one used geometry, representing light as light rays—this might be useful but didn't get at the underlying natural philosophical questions: "the physical nature of light" and "the causes of optical phenomena".

The question of the relation between the subordinate mixed mathematics, on the one hand, and the 'superior', explanatory discipline of natural philosophy, on the other hand, became extremely vexed in the generations around 1600. This is where physico–mathematics really enters the picture: Physico–mathematics was not a hard wired movement, but a diffuse set of agendas sitting loosely inside a strategic frame concerning how the mixed mathematical disciplines should relate to 'natural philosophising' (Note, my terminology, invoking a 'doing' and 'contesting' by natural philosophers in a disciplinary domain.)

Let me exemplify physico–mathematics by summoning two very different natural philosophers, Descartes and Kepler, who both happened to be radical and ambitious physico–mathematicians, and who both aimed thus to transform geometrical optics. Kepler and Descartes each sought closer articulation between optical innovation and natural philosophical explanation. Their respective natural philosophical theories of matter and cause were taken more intimately to control technical details in geometrical optics, and in turn, technical details in geometrical optics exerted pressure on their respective natural philosophical claims about matter and cause. Kepler practised geometrical optics under a neo-Platonic natural philosophy and conception of light, obtaining brilliant results in the theory of the *camera obscura*, theory of vision, and, to some degree, the theory of refraction and the telescope. Descartes, emulating Kepler's technical optical achievements, but competing with his neo-Platonic natural philosophical program, practised geometrical optics under a mechanical conception of light. He achieved a simple law of refraction and a general theory of lenses. Conversely, his optical successes shaped essentials of his natural philosophy, notably what Stephen Gaukroger and I term the principles of his corpuscular dynamics.

The example suggests that geometrical optics, the mixed mathematical science of the Scholastics, began spasmodically to manifest as a more 'physico-mathematical' discipline, concerned with matter and cause, and from which natural philosophical capital might be extracted.

III. Three interim—perhaps surprising—conclusions:

[1] Physico–mathematics was almost entirely a set of gambits inside the field of natural philosophising: The old mixed mathematical fields would no longer be subordinate to—but rather become proper domains of—one's favoured natural philosophy. Conversely, novel findings in the formerly mixed mathematical sciences would now bespeak new insights into whatever natural philosophy a physico–mathematician favoured.

[2] Physico–mathematics wasn't about the mathematisation of natural philosophy. Physico–mathematical gambits envisioned the physicalisation of the mixed mathematical sciences: We're not talking about mathematisation of anything, but the physicalisation of parts of mixed mathematics, whereby some natural philosophers aim to render the mixed mathematical fields more physical, more about matter and cause discourse within one's favoured natural philosophy.

[3] Similarly, this wasn't an invasion of natural philosophy by mathematicians intent upon destroying or displacing it; the relevant players were *"mathematically adept natural philosophers/slash/natural philosophically literate, and aggressive mathematicians"*. Such people constituted one, small, intersectional sub-set of all European mathematicians and natural philosophers. No circulation or displacement of elites took place.

IV. 'Physico-mathematical' initiatives began to appear in the 16th century. For example, there were attempts to bring mechanics, particularly a dynamical approach to the simple machines into natural philosophy. These gambits—whether classificatory arguments, rhetoric about values, or technical moves—can certainly be labelled **by us** as physico–mathematical. The aim was to modify both natural philosophising and mechanics by shifting the valencies of mechanics, making it natural philosophically relevant, by concerning itself with matter and cause.

An even more subtle tendency toward physico–mathematisation occurred below most actors' radar, as a kind of permanent grammatical possibility in natural philosophising. I'm talking about mixed mathematical sciences as subordinate to natural philosophy. What exactly did subordination mean as a type of systemic discursive relation? In my model of natural philosophy, particular natural philosophies are "articulated" in their own particular ways upon their own selection and interpretation of subordinate fields. 'Articulation' is the explaining of some the fundamental principles of the subordinate field, in terms of the matter and cause registers of the particular natural philosophy in question.

This grammar was part of natural philosophising as a generic game. Aristotelianism institutionalised both the generic grammar and the particular version of articulation, which we've been discussing. Competing natural philosophers operated within that same grammar, but made alternative moves.

The grammar of articulation upon subordinate fields set up an 'objective field of possible moves', in which natural philosophers carried out their own specific articulation strategies—depending upon their respective proclivities, aims and skills.

In this sense even Ptolemaic astronomy was articulated to Aristotelian natural philosophy: The elaborate geometrical details of Ptolemaic modeling fell outside any plausible realistic interpretation, hence outside any natural philosophical gloss. However, the fundamental concepts of Ptolemy's astronomy were shaped by Aristotelian natural philosophy: the finite earth-centred cosmos, the distinction between celestial and terrestrial realms, the primacy of uniform circular motion. Therefore, an articulation of a causal and matter theoretical nature existed between Aristotelianism and Ptolemaic astronomy, and so the latter had a tiny, inescapably physico-mathematical dimension.

But this articulation was fragile. In the topography of the natural philosophical field, it became the site, the hot spot, of the realist Copernican challenge. Beyond astronomical details, the Copernican debate was a battle about articulations of varieties of natural philosophy onto Copernicanism, or not. It was about the challenge in the field of natural philosophising posed by realist Copernicanism--because realist Copernicanism only existed in articulations of non-Aristotelian natural philosophies onto Copernicanism, which articulations were physico-mathematical in new, portentous ways.

V. The heightened natural philosophical contestation of the early 17th century intensified the proliferation, and competition of physico-mathematical gambits. Here are eight examples:

[1] As Peter Dear established, some leading Jesuit mathematicians pursued what I'd term a 'conservative' physico–mathematical program. They wanted the mixed mathematical fields to enjoy a status 'separate but more or less equal' to natural philosophising'. This liberated the mixed sciences from Aristotelian constraints but handicapped their ability to enrich natural philosophical discourse of matter and cause. Compare this strategy to that of the proponents of natural philosophically relevant mechanics. The latter would put a very marginal mixed mathematical field into the core of natural philosophy, not preciously separate the two.

[2] Kepler's profound neo-Platonising of mixed mathematics and redirecting the thus physicalised disciplines back into natural philosophy, including creating a new physico-mathematical field, celestial physics;

[3] Descartes' physico-mathematics: Descartes made radical attempts to ground a corpuscular-mechanism and establish the principles of its causal register (laws controlling force and determination of motion). He did this through physico-mathematical inquires in hydrostatics and optics.

As Stephen Gaukroger and I have argued, his hydrostatical work implied a radically non–Aristotelian vision of the relation of the mixed mathematical sciences to this emergent form of corpuscular-mechanical natural philosophising. Descartes' aimed to shift hydrostatics from mixed mathematics unambiguously into the realm of natural philosophy. Using a corpuscularian matter theory, learnt from Beeckman, he tried to redescribe what causes the pressure exerted by a fluid on the floor of the vessel containing it.

Indeed, Descartes' approach was ultra radical: The version of hydrostatics from which he started was that of Stevin: mathematically rigorous, and rigorously statical in the Archimedean style, and hence unpromising as the key to finding dynamical concepts for a corpuscular–mechanism. (A more popular approach, followed by the young Galileo, and by Beeckman, sought physico-mathematical capital in the dynamical approach to statical problems found in the pseudo-Aristotelian Mechanica.)

Descartes envisioned the mixed mathematical fields would be explained in corpuscular-mechanical terms and therefore not be subordinate to, but domains of, his new natural philosophy. Conversely, for Descartes novel findings in mixed mathematical sciences directly bespeak insights into the realm of corpuscular-mechanical explanation. Indeed in Descartes (and also in Kepler) we have a physico-mathematical strategy of trying directly to "see or intuit" the natural philosophical' causes, in and through representations of significant results in geometrical optics. Ofer Gal, Sven Dupré and I have a project about this. Elsewhere I've argued that this approach marked Descartes' optical work in the 1620s, and that it later shows up, somewhat sublimated, in his vortex celestial mechanics.

[4] As mentioned, Beeckman had a slightly different sub-species of physico-mathematics.

[5] Galileo's rather piecemeal physico-mathematical excursions, including his construction of a sui generis new science of motion. Like other radicals Galileo made natural philosophical capital out of mixed mathematics. But, he didn't pursue a *systematic natural philosophy*; rather, he tried to establish a realist Copernican cosmology and a strong anti-Aristotelian stance. Still, like Kepler and Descartes, Galileo was breaking the declaratory Scholastic rules about subordination of mixed mathematics, in pursuit of what amounted to gambits in the field of natural philosophising.

[6] Marin Mersenne. Like the Jesuits, he was an outlier, because he had no coherent natural philosophical program, no agenda about matter and cause. His odd case, however, proves the rule that physico-mathematics was mainly in and of the game of natural philosophy. More on this in a moment.

[7] For Gilbert, a remarkable case, there's no time, except to say that he was trading off practical mathematical materials rather than mixed mathematical ones, articulating some bits of practical mathematics to his own novel natural philosophy, as Jim Bennet has pregnantly observed.[1]

[1] Jim Bennet, writing of how in Gilbert, "navigational magnetism" a practical mathematical node of instrumentation, theory and practice, "impinged on natural philosophy through the need to characterise and codify declination and inclination in their terrestrial distribution."

[8] As mentioned: The hottest of physico-mathematical hot spots was in the Copernican realist articulation struggle.

So that makes for the early 17th century at least eight types of physico-mathematics, regardless of whether actors invoked the term, all but two types involving moves inside the realm of natural philosophising.

VI. Let's reflect on what it means to say natural philosophical players obeyed, or bent 'rules'. Most physico-mathematicians were indeed rebels, but not against the culture of natural philosophising. Rather, they tried to alter the rules under which the game should be played.

I'd say (in the spirit of interpretive sociology) that Aristotle's rules about natural philosophy and the mixed mathematical sciences were 'declaratory' rules; formally invoked and usually obeyed; but constantly open to renegotiation. The letter of Aristotle's law was slightly violated in geometrical astronomy, as we saw. The Jesuit mathematicians tried to step around the rules, but more radical natural philosophers, like Kepler and Descartes, de facto altered them, by forging new physico-mathematical meanings and practices. Again, I stress: *most of the jockeying over the physicalisation of mixed mathematical fields occurred inside the field of natural philosophising, not outside it.*

VII. Therefore, let's return to those termed 'outliers, the Jesuit mathematicians and Mersenne. Their work was piecemeal in relation to the main lines of development of physico-mathematical disciplines within natural philosophy. However, the outliers can fool us, because they misleadingly foreshadow what occurred later in the physicalisation of mixed mathematics.

Let me explain: Earlier we saw some incipient physico-mathematical moves in optics, as Kepler and Descartes extracted natural philosophical capital from optical work. Correspondingly, at each turn a slightly more dense, slightly more independent domain of physico–mathematical optics could be glimpsed. This process deepened later in the century, in the physico-mathematical optical work of Hobbes, Huygens, Robert Hooke, Newton and others. Physico–mathematical optics floated more freely than ever before from the demands of any given natural philosophical system. Geometrical optics, the Aristotelian mixed mathematical science, was evolving into a much more obviously physico-mathematical discipline, crystallising within, yet drifting apart from, natural philosophising.

Just as a more modern looking discipline of physical optics was tending to detach itself from the field of natural philosophising, so this process was occurring across other domains. The field of natural philosophy was entering a process of disintegration, its long term successor products being a suite of more narrow, technical and autonomous scientific disciplines. Optics and other mixed mathematical sciences emerged from their Aristotelian cocoons, and became physico-mathematical disciplines, increasingly independent of any particular natural philosophical system and relatively independent of the domain of natural philosophising as such. Two new physico–mathematical domains also arose with this same centrifugal élan toward separation from natural philosophy: celestial mechanics and classical mechanics per se.

Well, our early 17th century physico-mathematical outliers to natural philosophising may look like part of these later outcomes; but, in fact, they were wandering up dead ends of piecemeal results off the main highway of unfolding secular trends within natural philosophising and its attendant sciences. The closest analogy in contemporary terms to these outliers came from master practical mathematicians, such as Stevin, who played upon mixed mathematical fields from outside the realm of natural philosophising, aiming not to make natural philosophical capital, but to expand and systematise the realm of practical mathematics.

VIII. First, what has been called the '*mathematisation* of science' actually looks like attempts to *physicalise* the mixed mathematical sciences and invent some new ones.

Second, we should take seriously the existence of a complex, contested field of natural philosophising. The issue then becomes: "What were the gambits and strategies of mathematically literate natural philosophers or natural philosophically ambitious mathematicians (who were the same people!)?"

Third, natural philosophy was not killed, or replaced by mathematics and mathematicians. Some mathematics always existed in the field of natural philosophy, even in the declaratory purview of Scholasticism. Rather, hegemonic Aristotelianism set up the rules, initial opportunities and limits, which then were stretched and rewoven. And what eventuated were physicalisations of the mixed mathematical disciplines and in time some new ones—celestial physics; a mathematico-experimental mechanics.

In the end, of course, natural philosophy did die, not through the displacement of elites; or, on a certain day in London in the 1660s, so that 'Modern Experimental Science' could burst out. No, it died, eventually, through the kind of dynamics we've hinted at here—slowly and unintentionally, on a secular time scale, through a dynamics we can discern and narratives we can construct. Thinking through the category of physico-mathematics opens one window onto the dynamics of the field of natural philosophising: which in turn is the key to unpacking that process we term the Scientific Revolution.

Some references alluded to in the script:

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