Descartes Opticien: The Construction of the Law of Refraction and The Manufacture of its Physical Rationales 1618-1629

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1.0. Introduction

In his *Dioptrique* of 1637 Descartes raised numerous difficulties and puzzles. For example, he deduced the laws of reflection and refraction from a model: the motion of some very curious tennis balls. Descartes' contemporaries tended not see any cogency in this model, nor did they grasp the theory of dynamics upon which it is based.[2] Later, questions were raised about how Descartes had obtained the law, if not through his dubious deduction. Had Descartes plagiarised it from Willebrord Snel. If not, where had it come from?[3]

This paper cuts a path of reconstruction through these controversies. First it is shown that the tennis ball model for reflection and refraction links quite coherently to Descartes' impulse theory of light through his dynamics of micro-corpuscles. That dynamics was mooted in his earliest natural philosophical speculations, and first worked out in some detail for Le Monde between 1629 and 1633. Nevertheless, the tennis ball model and its dynamical basis posed a number of problems, seen by Descartes and his critics.. The strengths and the weaknesses of the model will provide clues and tools for the main theme of the paper, a reconstruction of how the law of refraction was discovered. Given that reconstruction, we shall finally be able to explore some of the complicated relations between Descartes geometrical optics and his attempts at mechanistic explanation in the 1620s.

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[2] Pierre de Fermat, *Oeuvres* t. II. pp.108-9, 117-24, 485-9; Paul Mouy, *Le développement de la physique Cartésienne* (Paris, 1934), p.55; Gaston Milhaud, *Descartes savant* (Paris, 1921), p.110

[3] It has long been well established that it is quite unlikely Descartes stole the law from Snel, as some contemporaries maintained. See P.Kramer, 'Descartes und das Brechungsgesetz des Lichtes', *Abhandlungen zur Geschichte der Mathematischer (Natur) Wissenschaften* 4 (1882), 235-78; and D.-J. Korteweg, 'Descartes et les manuscrits de Snellius d'après quelques documents nouveau', *Révue de Métaphysique et de Morale* 4 (1896), 489-501.

2.0. Cartesian Dynamics in Le Monde

This section examines the earliest articulated version of Descartes' dynamics, as offered in Le Monde. This will set the stage for our analysis of the tennis ball proofs in the Dioptrique. These will be fully explicated and consistently reduced to Descartes' actual mechanical theory of light by means of an understanding of this dynamics.

Descartes' dynamics of micro-particles had nothing to do with the mathematical treatment of velocities, accelerations, masses and forces. Rather it was concerned with accounting for the motion, collision and [258] tendency to motion of corpuscles. Descartes held that bodies in motion or even merely tending to motion, can be characterised from moment to moment by the possession of two sorts of dynamical quantity : First, there is the absolute quantity of the 'force of motion'; secondly, there are the directional modes of that quantity of force; the directional components along which the force or parts of the force act. These directional modes of the quantity of force of motion, Descartes termed actions, tendencies, or most often determinations.[4] Whilst the rudiments of this dynamics of instantaneously exerted forces and determinations dates back to Descartes' earliest work, it was first articulated in Le Monde.[5]

Descartes explains natural change mainly by instantaneously occurring corpuscular collisions. At the moment of a corpuscular impact, the God of the Cartesians instantaneously adjusts the quantities of force of motion and the determinations that will characterise the corpuscles concerned in the instant after the impact. God does this by following certain laws and rules of impact he has framed and 'ordinarily' follows. He, God that is, considers the force and determination relations of the two bodies just prior to impact, and upon impact God instantaneously rearranges those forces and determinations in accordance with the rules He has laid down. The laws and rules of impact are Divinely ordained prescriptions, stating what God will do about redistributing the dynamical quantities, given the conditions of the impact. Consider Descartes' the first 'rule of nature' in Le Monde, which reads as follows:

> Each part of matter always continues to exist in the same state as long as other bodies do not constrain it to change that state. If it has a certain size, it will never become smaller, unless other bodies divide it... if a body has stopped in a given place, it will never leave

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[4] The understanding of determination used here develops work of A.I.Sabra, *Theories of Light from Descartes to Newton* (London, ,1967) p.118-121; A. Gabbey, 'Force and Inertia in the Seventeenth Century: Descartes and Newton', in S.Gaukroger, ed. *Descartes: Philosophy, Mathematics and Physics* (Sussex, 1980), pp.230-320; M. Mahoney, *The Mathematical Career of Pierre de Fermat 1601-1665* (Princeton, 1973); S. Gaukroger, *Descartes: An Intellectual Biography* (Chicago 1995); O. Knudsen and K.M Pedersen, 'The Link between "Determination" and Conservation of Motion in Descartes' Dynamics', *Centaurus* 13 (1968), 183-186; T.L. Prendergast, 'Motion, Action and Tendency in Descartes' Physics', *Journal of the History of Philosophy* 13 (1975), 453-62; and Peter McLaughlin in this volume, "Force Determination and Impact'.

[5] These rudiments appear in the so-called hydrostatic manuscript of 1619. See J.A. Schuster, *'Descartes and the Scientific Revolution 1618-34: An Interpretation'* unpublished Ph.D. dissertation, Princeton University, 1977, pp.93-111 ; Gaukroger *op. cit.* (1995).pp.84-9. It should also be noted that *Le Monde* itself contains a reference to the text of the *Dioptrique* attributing the distinction between force of motion and directional force of motion to that earlier text. AT x. 9. cf F. Alquie (ed.) *Descartes Oeuvres philosophiques* , t. 1 p.321 n2.

> that place unless others force it out; and if it has once commenced to move, it will continue along with the same force, until other bodies stop or retard it.[6]

We may take this to assert the conservation of the motion (or rest) of a body in the absence of external constraints. Closer inspection reveals a telling point. Descartes slips into speaking of the "force of motion". This is the quantity which is conserved. This is the force of motion we have been talking about. Descartes uses the term in relation to his Voluntarist understanding of ontology: God must continually support (or re-create) bodies and their attributes from moment to moment. This implies that in the final analysis a body in phenomenal translation, in motion, is really being recreated or continually supported at successive spatial points during successive temporal instants. In addition, and this is the key point, in each of those instants of re- creation, it is characterised by the Divine injection of a certain quantity of "force of motion". We should view the instantaneously conserved "force of motion" as a kind of quantity of efficacy (the phenomenal mirror of the instantaneously injected Divine action). [259]

The third law of motion in Le Monde specifies the direction in which the Divinely conserved quantity of force of motion is to act.[7] The force of motion is directed along the tangent to the path of motion at the point under consideration. We have to be careful here. The third law does not say that merely a direction is conserved. Rather, it asserts that a quantity of force of motion is annexed to a privileged direction. That is, the law specifies a directional quantity of force of motion. It says that in the absence of external constraint, this directional quantity of force of motion would be conserved by God from instant to instant. This directional quantity of force of motion is, of course, that "determination" discussed above.[8] Let us call the directional quantity of force of motion directed along the tangent to the path of motion at a given instant the principal determination of a moving body; following Descartes one can decompose that directional quantity into components, also called determinations. In any given case, mechanical

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[6] AT xi. 38;

[7] AT xi. 43-44: S. Gaukroger, ed. and trans., *Descartes: The World and Other Writings* (Cambridge, 1998): "I shall add as a third rule that, when a body is moving, even if its motion most often takes place along a curved line and, as we said above, it can never make any movement that is not in some way circular, nevertheless each of its parts individually tends always to continue moving along a straight line. And so the action of these parts, that is the inclination they have to move, is different from their motion.[...leur action, c'est à dire l'inclination qu'elles ont à se mouvoir, est different de leur mouvement] " [29] And,

"This rule rests on the same foundation as the other two, and depends solely on God's conserving everything by a continuous action, and consequently on His conserving it not as it may have been some time earlier, but precisely as it is at the very instant He conserves it. So, of all motions, only motion in a straight line is entirely simple and has a nature which may be grasped wholly in an instant. For in order to conceive of such motion it is enough to think that a body is in the process of moving in a certain direction [en action pour se mouvoir ver un certain coté], and that this is the case at each determinable instant during the time it is moving." [29-30]

[8] In the passages cited above Descartes in his discussion of the third law defines 'action' as 'l'inclination à se mouvoir'. He then says that God conserves the body at each instant 'en action pour se mouvoir ver un certain coté'. This would seem to mean that at each instant God conserves both a unique direction of motion and a quantity of 'action' or force of motion. In other words the first law certifies God's instantaneous conservation of the absolute quantity of tendency to motion, the 'force of motion'. The third law specifies that as a matter of fact in conserving 'force of motion' or 'action', God always does this in an associated unique direction. The first law asserts what today one would call the scalar aspect of motion, the third law its necessarily conjoined vector manifestation. Just because he recognises that some rectilinear direction is in fact always annexed to a quantity of force of motion at each instant, Descartes often slips into abbreviating 'directional force of motion' by the terms 'action', 'tendency to motion' or 'inclination to motion', all now seen in context as synonyms for 'determination'.

conditions and the spatial relations of bodies dictate which components of the principal determination come into play. We are going to see that in the demonstrations of the optical laws, the reflecting or refracting surfaces effectively dictate which components of the principal determination of a moving tennis ball come into play in the collision. The only other thing we have to remember is that determination, like force of motion, is a dynamical property predicated of moving bodies (or of bodies tending to motion), from instant to instant. Just as force of motion is injected by God from instant to instant, so is determination, which according to the third law, is only the directional magnitude of that force and the components into which it may be resolved. As God maintains or alters from moment to moment the absolute quantity of force of motion; so he also maintains or alters instantaneously the directional manifestations of that force -- what Descartes calls the determinations.

Let us examine Descartes' chief example of the use of these concepts. **[fig.1]** Consider a stone rotated in a sling. Descartes analyses the dynamical condition of the stone at the precise instant that it passes point A. By the first and third laws of motion, the force of the motion of the stone is directed along the tangent, that is along AG. If the stone were released and no other hindrances affected its trajectory, it would move along ACG at a uniform speed reflective of the conservation of its quantity of force of motion.[9] However, the sling constrains the privileged, principal determination of the stone and deflects its motion along the circle AF.

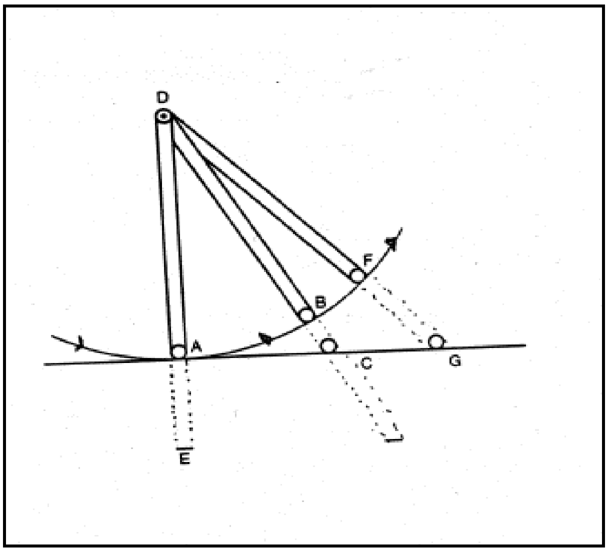

Figure 1. Descartes' Dynamics of the Sling in *Le Monde*

Descartes considers that the principal determination along AC can be divide into two components : one is a "circular" determination along ABF; the other a centrifugal determination along AE. For present purposes, let us ignore the curious circular tendency. To discuss it would lead us further than we need to go into Descartes' manner of treating circular motion.[10] What

[9] *Le Monde*, AT xi. 45-6, 85.

[10] *Le Monde* , AT xi. 85. Descartes argues from the first and third laws of nature that at the instant of time the body is at point A, it tends in and of itself along the tangent AC. The circular tendency along AB is that part of the tangential tendency which is actively opposed by the physical constraint of the sling and hence gives rise to the centrifugal tendency to motion along AE. For the sake of whiggish edification it can be noted that had Descartes dealt with the centrifugal constraint on the

Descartes is trying to do is decompose the principal determination into two components: one along AE completely opposed and hindered by the sling—so no actual centrifugal translation can occur—[260] only a tendency to centrifugal motion; the other along the circle, which is as he says, "that part of the tendency along AC which the sling does not hinder".[11] Hence it manifests itself as actual translation. The choice of components of determination is dictated by the particular configuration of mechanical constraints on the system.

Leaving aside Descartes' theory of elements and his cosmology, his basic theory of light is that light is a tendency to motion, an impulse, propagated instantaneously through continuous optical media. So, light is or has a determination--a directional quantity of force of motion. Note that light, as a tendency to motion, can have a greater or lesser quantity of force--we can have weak light impulses or strong ones--but the speed of propagation in any case is instantaneous. This distinction between the force of light and its instantaneous speed of propagation is about to become very important, having been neglected for three and a half centuries.

3.0 Making Sense of the Proofs of the Laws of Reflection and Refraction in the Dioptrique

We may now turn to the laws of reflection and refraction as they are demonstrated using the tennis ball model in the Dioptrique of 1637. First the case of reflection **[fig.2].** Descartes takes a tennis ball struck by a racket along AB towards surface CBE. We neglect the weight of the tennis ball, its volume, as well as air resistance.[12] The reflecting surface is considered to be perfectly flat and perfectly hard: upon impact it does not absorb any of the force of motion of the ball. The [261] tennis ball is now virtually a mathematical point in motion; it bears a certain quantity of force of motion, divisible into directional components, or determinations.[13] The demonstration of the law of reflection is carried out as a geometrical locus problem: Descartes places two conditions upon the dynamical characterisation of the ball : First, the total quantity of its force of motion is conserved before and after impact--no force can be lost to the surface. Second, the component of the force of motion parallel to the surface is unaffected by the impact. Descartes expresses these conditions geometrically, and uses them to determine the quantity and direction of the force of motion of the ball after impact with the surface.[14]

For the first condition, the conservation of the quantity of force of motion, we draw a circle of radius AB about B. Assume that prior to impact the ball took time t to travel along AB. Having lost no force of motion to the surface, the ball will, in an equal time t after impact, be located somewhere on the circle. The second condition is that the parallel determination, the component of force of motion along the surface, is unaffected by the collision. In time t before

ball offered by the sling, instead of the circular tendency (which violates the first law in any case) he might have moved closer to Newton's subsequent analysis of circular motion.

[11] *Le Monde*, AT xi. 85.

[12] AT vi. 94.

[13] On this interpretation of "determination" in the *Dioptrique* see Sabra, *op. cit.* pp. 118-21.

[14] AT vi. 95-6.

impact, while the ball traversed [262] AB, Descartes says that the parallel determination "caused" the ball to traverse the horizontal distance between AC and HB. In an equal period of time t after impact, the unchanged parallel determination will "cause" the ball to move an equal distance toward the right.[15] We represent this by drawing FED so that the distance between FED and HB equals that between HB and AC. At time t after impact the tennis ball must lie somewhere on this line FED and it must also lie on the circle; that is it must be at F or D. The surface is impenetrable, so at time t after impact the ball must be at F. Geometrical considerations immediately show that the angle of incidence is equal to the angle of reflection.[16] This proof never takes into consideration the behaviour of the component of force of motion perpendicular to the surface, the normal determination as we shall term it.[17]

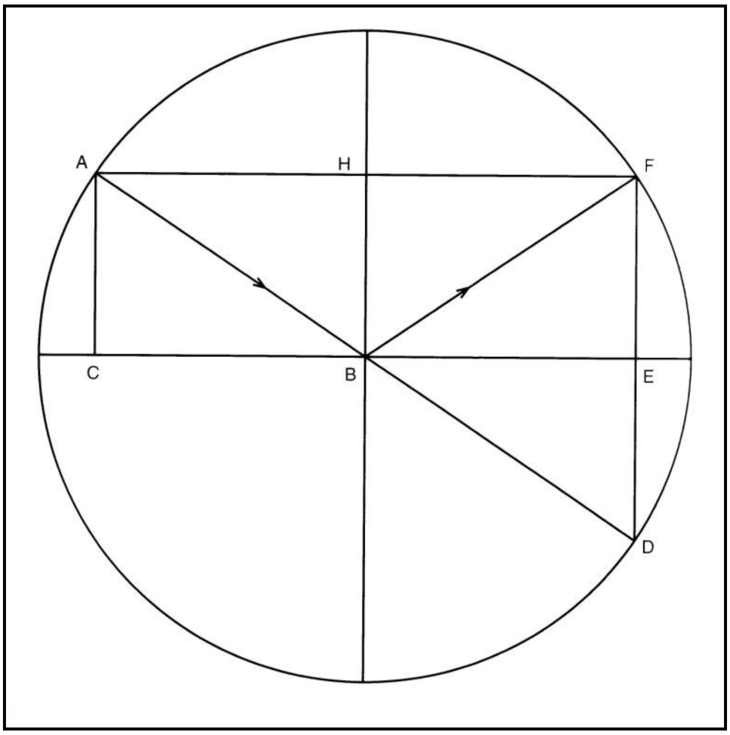

Figure 2. Descartes' Figure for Reflection of Light (Tennis Ball) in *Dioptrique*

I now propose to do something Descartes refused to do in the Dioptrique, even though it is perfectly feasible and follows easily in his overall natural philosophical perspective. I shall translate the tennis ball proof into the terms of Descartes' theory of light, using his dynamics. This is not difficult to do, because the tennis ball has already been stripped of all properties except location, force of motion and its determinations. It is already virtually a mechanical impulse, and that is all a ray of light is in Descartes' theory. So we can assert the same things about the tennis ball at the instant of impact as we would assert about a ray of light at the instant

15 AT vi. 95.

16 AT vi. 96.

17 Cf. Sabra, *op. cit.* pp.85, 110; Mahoney, *op.cit.* pp.379-80; and Richard Westfall, *Force in Newton's Physics* (New York, 1971) pp.65-6, were amongst the first scholars to appreciate this point. Previous students of Descartes' optics, such as Mach, Ronchi, Scott and Boyer, did not, as cited by Sabra, op. cit. p.110.

it meets a perfectly hard reflecting surface.[18] Consider in figure 2 a light ray, AB, a line of tendency to motion, or determination, impacting the surface CBE at B. The surface is perfectly hard, therefore the magnitude or intensity of the impulse is conserved. The parallel component of the impulse is unaffected by the collision.

The proof is again a locus problem. After impact, what are the orientation and magnitude of the force of the light impulse? The same two conditions apply. (1) unchanging total quantity of force of the ray; (2) conservation of the parallel component of the force of the ray. Represent (1) by a circle about A. Represent (2) by appropriately spacing FED parallel to HB and AC. Combining our conditions gives BF as the representation of the unchanged magnitude of the force of the ray and its new orientation. We have taken the diagram for the tennis ball model and re-interpreted it as a diagram about forces and determinations. This is obvious, provided (1) you attend to the very instant of impact; and (2) you take the circle and lines to represent the quantity and determination of the force of motion of the ball, as they are instantaneously rearranged in the impact. Descartes' vocabulary of 'forces', 'tendencies' and determinations is already reading the diagram that way, and later correspondence supports this. In this reading, the conceptual distance between the tennis ball model and the impulse theory of light virtually disappears.

Let us now turn to the tennis ball model for the refraction of light. **[fig 3]** [263] Again consider a tennis ball struck along AB toward surface CBE. In this case the surface is a vanishingly thin cloth. The weight, shape and bulk of the ball are again neglected. It is taken to move without air resistance in empty geometrical space on either side of the cloth. In breaking through the cloth, the ball loses a certain fraction of its total quantity of force of motion, say one half. This fractional loss is independent of the angle of approach.[19] Again two conditions are applied to the motion of the ball. First, the new quantity of force of motion (one half the initial amount) is conserved during motion below the sheet. Secondly, the parallel component of the force of motion, the parallel determination, is unaffected by the encounter with the cloth. Descartes takes the breaking through the cloth as an analogue to a surface collision, in which the parallel component is unaffected. We draw a circle about point B. Assume the ball took time t to traverse AB prior to impact. After impact it has lost one half of its force of motion, and hence one half of its speed. It therefore must take 2t to traverse a distance equal to AB. It arrives somewhere on the circle after 2t.[20]

Now, prior to impact the parallel determination "caused" the body to [264] move towards the right between lines AC and HBG.[21] But, after impact, the ball is taking 2t to move to the circumference of the circle, so its unchanged parallel determination has twice as much time in which to act to "cause" the ball to move toward the right. Therefore set FEI parallel to HBG

[18] This crucial point was first noted by Mahoney, *op. cit.* pp.378-9 in the course of his path breaking reinterpretation of Descartes' optical proofs in terms of relations amongst quantities and directional quantities of forces.

[19] AT vi. 97.

[20] AT vi. 97-8.

[21] AT vi. 97.

and AC, but make the distance between FEI and HBG twice as great as that between HBG and AC. At time 2t after impact the ball will be on the circle and on line FEI; that is, at point I, their intersection point below the cloth. The sine of the angle of incidence, AH, is to the sine of the angle of refraction, IK, as one is to two; that is as the force in lower medium is to the force in upper medium--which ratio is constant for all angles of incidence.[22]

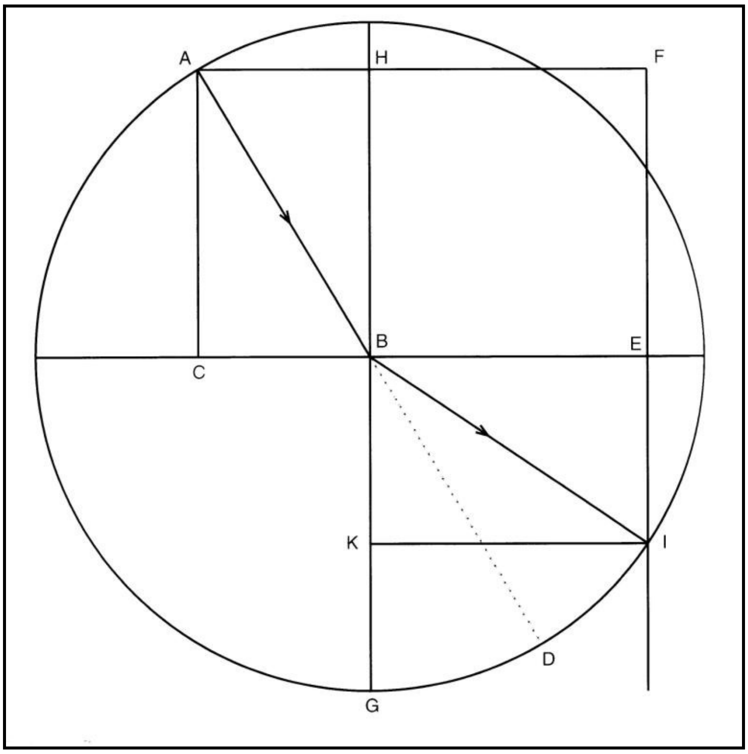

Figure 3. Descartes' Figure for Refraction of Light (Tennis Ball) in *Dioptrique*

Now, as we did in the case of reflection, let us sketch a proof of the law of refraction in the case of a light ray and Descartes' dynamics. **[fig 4]** This will prove most instructive and consequential for our inquiry into how Descartes first constructed the law and how he subsequently came to design his dynamical rationale of the law.[23] Consider a ray incident upon refracting surface CBE. Let length AB represent the magnitude of the force of the light impulse. The orientation and length of AB represent the principal determination of the ray; that is, condition one. The force of the ray is diminished by half in crossing the surface, so we must draw a semi-circle below the surface about B with a radius equal to one half of AB; that is condition two. We also know that the parallel determination of the force of the ray is unchanged in crossing the surface. The distance between AC and HBG represents that parallel determination. Therefore, we must set out line FEI parallel to the two former lines and with the distance between FEI and HBG equal to that between HBG and AC. Again the intersection of the lower semi-circle and line FEI gives the new orientation and magnitude of the force of the ray of light, BI and the law of sines follows.

[22]AT vi. 97-8. Descartes later supplies arguments concerning the mechanical structure of optical media to explain why light bends toward the normal when passing into a denser medium. AT vi. 103.

[23] Mahoney, *op. cit.* p.379, was the first to suggest how the tennis ball model could be referred back to an imputed Cartesian dynamics in order to explicate Descartes' proof.

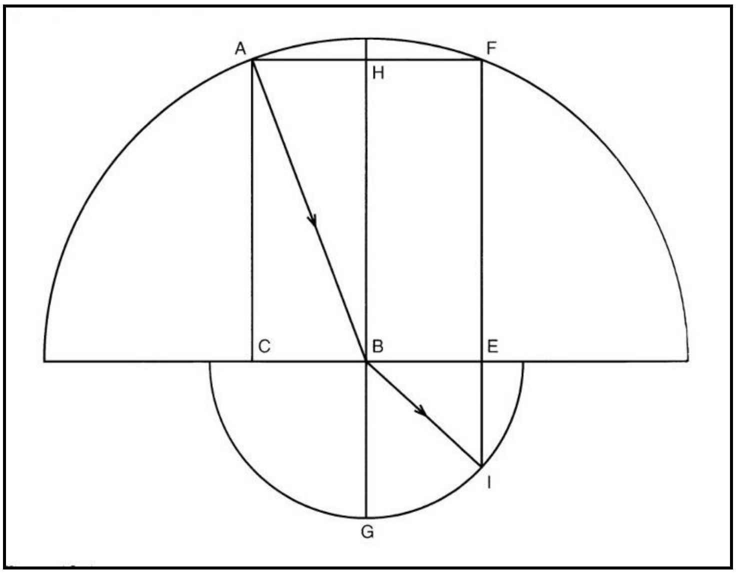

Figure 4. Refraction of Light Using Descartes' Dynamics and Real Theory of Light

The case of the light ray **[fig 4]** requires manipulation of two unequal semi-circles. These directly represent the ratio of the force of light in the two media. In the tennis ball case **[fig.3]** we went from ratio of forces to ratio of speeds and hence differential times to cross <u>equal</u> circles. But, in both cases at bottom we are attributing the same type of force and determination relations to the ball, and to the light ray, at the instant of impact.[24]

It is sometimes said Descartes fell into a contradiction, because in his theory of light, light rays move instantaneously through any medium, whilst in the tennis ball model we must deal with a ratio of finite speeds. We **can** now see that Descartes had no problem: one must distinguish the <u>speed</u> of propagation of a light ray, which is instantaneous, from the <u>magnitude</u> of its force of propagation, which can take any finite positive value. The <u>speed</u> of Descartes' tennis ball corresponds not to the speed of propagation of light but to the intensity of the force of its propagation. [265]

<u>4.0. Descartes' Dynamical Premises: Demonstrative Efficacy and Empirical Weakness</u>

Our analysis thus far goes some way toward vindicating the plausibility and coherence of Descartes' attempted demonstrations. Having decided in 1633 not to publish his first system of natural philosophy, <u>Le Monde</u>, Descartes offered the public in 1637 the <u>Discours de la méthode</u>

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24 It is noteworthy that Descartes himself thought about his tennis ball model proof in precisely the manner we have just used to render it in terms of his dynamics and apply it to light rays. He later wrote to Mydorge for Fermat to explain the manipulation of the speeds (forces of motion) and determinations in the tennis ball proof:

> The [principal] determination is forced to change in various ways, in accordance with the requirement that it accommodate itself to the speed [force of motion]. And the force of my demonstration consists in the fact that I infer what the [principal refracted] determination must be, on the basis that it cannot be otherwise than I explain in order to correspond to the speed, or rather the force which comes into play at B.

Here Descartes views his proof in dynamical terms, as a deduction of the new refracted principal determination induced at the instant of impact with the surface, rather than in kinematical terms, as a deduction of the position of the tennis ball at a certain time after impact with the surface. To Mydorge for Fermat, 1 March 1638, AT ii. 20.

and its three supporting Essais. The Dioptrique therefore appeared without the full backing of Descartes' principles of dynamics and real theory of light. Yet, we have now seen that the proofs were set up in such a way that their dependence upon the dynamics and pertinence to the real theory of light lurked between the lines, and hence could have been brought into the open in case of the eventual revelation of the full system. We have simply tried to read the proofs across a prior knowledge of the relevant contents of Le Monde. The dynamics of light which we can read out of Le Monde makes good sense of the core aspects of the optical proofs. Using Descartes dynamical principles we can relate the tennis ball model back to the real theory of light, and hence vindicate Descartes of the traditional charge that the variable speed of the tennis ball bears no analogy within the real theory of light. We have also seen that recent interpreters are correct to interpret "determination" as a coherent mechanical concept, denoting the directional magnitude of the force of motion. There are, however, definite limitations to this procedure of interpretive vindication, for even in our interpretation many problems surround Descartes' [266] presentation, and the analysis of these problems is going to provide some signposts both for the reconstruction of Descartes' route to the law of refraction and about its manner of "demonstration".

The difficulties with Descartes' theory of refraction arise from the very core of his presentation, from the two principal dynamical premises used in deducing the law of refraction. One may formulate his premises as follows:

(1) For any two optical media, the quantity of the force of light in the upper 'incident' medium bears to the quantity of the force of light in the lower 'refracting' medium a constant ratio, characteristic of the two media and independent of the path of propagation, or

$$\frac{|Fi|}{|Fr|} = \text{const}$$

where $|Fi|$ is the quantity of force of light in the upper medium and $|Fr|$ the quantity of the force of light in the lower medium.

(2) The component of the determination of the force of light parallel to the refracting surface is unaffected by the refraction of the ray, or

$$|Fi| \ \sin i = |Fr| \ \sin r$$

Combining (1) and (2), we obtain, following Descartes[25]

$$\frac{\sin i}{\sin r} = \frac{|Fr|}{|Fi|} = \frac{1}{\text{const}}$$

We have seen that these premises can be grounded in Descartes' dynamics; that they mesh with his real theory of light as an instantaneously transmitted mechanical impulse; and that they allow

25 This derivation merely reworks Sabra's well known analysis of Descartes' demonstration. [Sabra op. cit. pp.97-100, 105-6, 116.] The only difference is that here we deal with quantities of forces and their directional components (determinations), rather than with quantities of speed and their directional components, as Sabra did. The reason is that we have insisted upon the centrality of the former concepts for Descartes and we have argued that Descartes could reduce phenomenal speeds to instantaneously exerted quantities of force of motion, so that speeds and tendencies to motion could be treated under the same conceptual and geometrical framework. We shall return to Sabra's analysis below in Section 6.1.

a plausible deduction of the law of refraction in an idealised case in which a vanishingly thin sheet, separating two void spaces, refracts an incident tennis ball which for all practical purposes has been reduced to a point localisation of an instantaneously exerted quantity and directional magnitude of force. But although the premises work well in this limited and idealised context, as soon as one considers more complex and less idealised cases, they begin to reveal certain problems of empirical plausibility and logical consistency.

To put the matter in a nutshell, when one considers real space-filling media, Descartes' first dynamical assumption--path independent ratio of the force of light--seems to entail that optical media are isotropic, whilst the second dynamical assumption--conservation of the parallel determination--seems to entail that they are not. We are about to see that Descartes was aware of some of the difficulties consequent upon so construing the premises, and that he tried both to finesse and ignore them whilst holding firm to the premises themselves. His determined investment in [267] premises which permit derivation of the law of refraction, yet which are so empirically questionable in themselves can provide us with clues about how the law was originally discovered and when and why the premises were devised. We shall first look at these difficulties in an abstract and slightly 'Whiggish' fashion, and then show how they manifested themselves in Descartes' articulation of his theory of refraction.[26]

At first sight Descartes' assumption (1) would seem to entail that optical media are isotropic, for the force ratio depends only upon the nature of the media and is independent of the incident and refracted paths of the tennis ball or light ray. The most superficial examination of assumption (2), however, shows that this must be an oversimplification. Assumption (2) maintains the conservation of the parallel component of the principal determination before and after refraction, and hence it entails that in refraction all dynamical changes affecting the ball or the ray in fact come about through variation in the normal component of the incident principal determination. Of course Descartes' proofs assign no quantitative or geometrically constructive role to the comportment of the normal component: the locus problems are solved using only the absolute quantities of force and the parallel components of the determination (laid off by lines normal to the refracting surface). Clearly, then, assumption (2) entails that Descartes' implied sense of 'isotropic' must differ from ours. His 'isotropic' media effect changes in the normal components of the determination of the incident ray which are complicated functions of the angle of incidence, while they leave the parallel component untouched.

Assumption (2), which raises difficulties for the isotropic character of optical media suggested by assumption (1), also generates some empirical implausibilities when considered on its own. While one can perhaps intuitively grasp how a vanishingly thin sheet might affect only the normal component of the incident determination, is this really plausible in the case of real space filling media? In such media collision with the surface may well affect only the normal determination; but, what about the ball's or ray's subsequent penetration of a finite thickness of the medium? Would not the ball or ray now encounter altered conditions of motion (or of tendency to motion) in the direction parallel to the surface? If (1) really entails that media are

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26 We take it that in the spirit of Bachelard's epistemological and historiographical conception of *récurrence*, such analytical whiggism is not at all a thing to be avoided. Cf. S. Gaukroger, 'Bachelard and the Problem of Epistemological Analysis', *Studies in the History and Philosophy of Science* 7 (1976), 189-244, at pp.229-34..

isotropic in some sense, then the parallel component must be affected in precisely the same way the normal component was. So, depending upon how one views Descartes" implied notion of isotropic media, his assumptions are either contradictory or simply wildly implausible in an empirical sense: either (1) entails our notion of isotropic media while (2) denies it; or (1) entails path dependent variations in the normal component which are then most implausibly denied to the parallel component by (2) in the case of space filling media.

Returning to the Dioptrique, one finds that Descartes began to [268] encounter difficulties reflective of these deeper problems as soon as he moved beyond the case of the thin sheet separating two void spaces. When he turns to space filling media, Descartes harks back to figure 3 in which he now takes CBE to be the upper surface of a volume of water. He argues that if the tennis ball loses, as before, one-third of its force of motion in encountering the surface, then the derivation of the new refracted principal determination will also follow as before and the ball will be refracted toward I..

> ...first of all, it is certain that the surface of the water must deflect it toward there in the same way as did the cloth, seeing that it takes from the ball the same amount of its force, and that it is opposed to it in the same direction.[27]

So, as one expects, refraction is still held to be an interface phenomenon, the new principal determination being set at the instant the ball encounters the surface, by the alteration of the quantity of force of motion, conjoined with the conservation of the incident parallel determination. It makes no difference, Descartes next argues, that the ball, after refraction, passes through a real, dense volume filling medium, for the medium is isotropic in the sense that it offers the same resistance to the passage of the ball, regardless of the angle of path 'set' by the refraction at the interface.

> Then, as for the rest of the body of water that fills all the space between B and I, although it may resist the ball more or less than did the air that we assumed to be there before, this is not to say that because of this it must deflect it more or less: for it can open in order to permit it passage, just as easily in one direction as in another, at least if we always assume, as we do, that neither the heaviness or lightness of this ball, nor its bulk, nor its shape, nor any other such foreign causes changes its course.[28]

Descartes apparently expects readers to accept that by appealing to the isotropic character of the medium, he can thus separate the setting of the refracted determination at the moment of encountering the interface from any mechanical effect the ball might undergo in passing through a finite thickness of the medium.

Descartes' strategy here seems to be to preserve at all costs the locus construction in figure 3, centering on the circle AHF and the lines AC, HB and FE, the representations of his two central assumptions. He fails to explain why the parallel component should be conserved during the passage of a finite thickness of the medium, and simply tries to persuade us that since media are isotropic in the Cartesian sense, whatever determination is set at the interface will be preserved

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27 AT vi. 98.

28 AT vi. 98-9.

within the medium. It was quite feasible for a contemporary reader to question Descartes' implied [269] concept of isotropic media as both ad hoc and empirically implausible. In 1640 Père Bourdin explicitly questioned why the ball, in entering the water, is not retarded in moving from left to right just as it is retarded in moving from high to low. Descartes' less than edifying response was that he had already dealt with this problem in the Dioptrique when he considered refraction through a thin sheet (sic):

> ...in order to show that it does not occur in the depth of the water, but only on its surface; and... that it is necessary to consider only the determination of the ball [ver quel côte se determine la bale] upon entering the water, because afterwards, whatever resistance the water exerts upon it will not change its determination.[29]

This adds virtually nothing to the argument in the Dioptrique, and it in no way justifies Descartes' premises or answers Bourdin's penetrating query. For what is at issue is how can it possibly be, given Descartes' premises, that refraction does in fact only occur at the interface. Descartes' answer amounts to the claim that since in fact refraction occurs only at the interface, his premises explaining refraction must surely be adequate to that fact, as indeed they are, if only one conceptually separates consideration of the causes of refraction at the interface from the effect upon the ray of the isotropic character of any finite thickness of the medium. Hence, we are forced to the following conclusion: The cash value of these manoeuverings can only have been the staunch defence of the premises as such, and of the construction and demonstration which they ground.[30]

The difficulties posed by the two premises emerge more subtly when Descartes deals in the Dioptrique with the case of refraction toward the normal. In the tennis ball model the racket is taken to strike the ball again at the moment of incidence, thus increasing its speed, or quantity of force of motion, in a given ratio to the incident speed.[31] Commentators have often noted the sheer ad hocness of this strategy, as well as the even more damaging point that in the real theory of light there is virtually no analogue for this providentially adjusted stroke of the racket. But it is less the ad hocness of the argument which interests us here than the deeper conceptual embarrassments of which it is merely a symptom. Note that according to Descartes' theory the second stroke of the racket must act in the normal direction, for there can be no alteration in the parallel component of the determination. This means that depending upon the angle of incidence, the racket acts in the normal direction to increase the normal component in such a manner that, as a consequence, the overall absolute quantity of force of motion is increased in just the prescribed ratio. Descartes could hardly have failed to realise this, since it is an immediate consequence of the explicitly stated portion of his theory. However, he astutely avoided a clear indication that the racket must act in the normal [270] direction (much less that its normal action is a function of the angle of incidence).

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[29] to Mersenne, for Bourdin, 3 December 1640 AT iii. 250.

[30] Descartes is tacitly appealing on the empirical level to an indubitable fact: when dealing with a pair of homogenous media, refraction is an interface phenomenon. His dynamical premises are consistent with this fact, but they cannot be consistently articulated so as to allow the deduction of this fact, and this fact only.

[31] AT vi. 99-100.

> But let us make yet another assumption here, and consider that the ball, having been first of all impelled from A toward B, is impelled again, once it is at point B, by the racket CBE which augments the force of its movement by for instance one-third, so that afterwards it can make as much headway in two moments as it previously made in three. This will have the same effect as if the ball were to meet, at point B, a body of such a nature that it could pass through the surface CBE one-third again more easily than through the air.[32]

Descartes' form of words is designed so as not to reveal to the reader the deeper consequences of the theory. His concern was well justified, because, of course, these consequences attach as well to the previous case of refraction away from the normal: Although the metaphor of penetrating a thin sheet tends to hide the relevant dynamical considerations, it remains the case that the <u>loss</u> of force of motion in a fixed ratio to the incident force of motion can only be accomplished on Descartes' premises through a path dependent decrease in the normal component of the incident determination. Descartes was and remained unwilling to bring these consequences into the open, for they threatened the plausibility of his central assumptions and their presumed ties to his larger views on dynamics and the real theory of light. By what Cartesian mechanical means, after all, is such a path dependent variation in normal component to be effected, in the case of the decrease <u>or</u> increase of the incident force of motion? And if such a path dependent variation in normal component must occur, why then, to resume the earlier critique, does this not also occur in the parallel direction in the case of penetration of a finite thickness of the 'isotropic' refracting medium?

In sum, Descartes' two dynamical premises permitted a plausible deduction of the law of refraction, but they generated what seemed to some of his readers, and arguably to Descartes himself, to be crippling difficulties. His theory deals poorly with volume filling media, with refractions toward the normal, and more generally with the question of how it happens that the alteration in the normal determination is variable, depending upon the angle of incidence. Indeed virtually the only strength of Descartes' central assumptions resides in their pleasing ability to rationalise the geometrical steps in his construction of the path of a refracted ray or ball. Descartes was willing to try to ride out likely accusations that the premises are empirically implausible, dynamically ad hoc, and in some interpretations, logically inconsistent, because the premises provided elegant and more or less convincing rationalisations for the geometrical moves in his demonstration. All this suggests that Descartes did not obtain his premises through a deep inquiry into the conceptual and empirical requirements of a mechanical [271] theory of the propagation and refraction of light. It seems more plausible to associate the premises closely with the very geometry of the diagrams in which Descartes depicts and constructs the paths of refracted rays. The issue then turns on whether the premises are post-facto glosses of geometrical constructions arrived at in some other way; or whether the diagrams themselves were invented to illustrate previously held mechanical principles concerning the behaviour of light. In the following sections it will be suggested that the former hypothesis is the more likely. In particular it will be argued [1] that Descartes probably discovered the law of refraction independently of any mechanical assumptions, although he held a number of unsystematised and

32 *ibid*.

abortive ideas about the mechanics of light as early as 1620; and [2] that it was the geometrical diagrams expressing his newly found law which suggested to him the precise form and content of his two dynamical premises and their mode of relation in explaining refraction.

5.0. Descartes' Route to the Law of Refraction 1619-1627

In this section we turn to the discovery of the law of refraction. As indicated above, our unearthing of the dynamical framework of the optical proofs will ultimately aid our detective work.

5.1 The Mydorge Letter of 1626/27

Thomas Harriot discovered the law in exact form around 1598 and Willebrord Snel, who died in 1626, discovered it sometime after 1620.[33] Descartes, working with Claude Mydorge, discovered it in 1626/27. The chief document supporting this conclusion is a letter from Mydorge to Mersenne.[34] It is well known to students of seventeenth century optics, but I suggest that it has not yet been properly understood. That depends upon its dating, and the dating depends upon its content.

Mydorge's first claim is that if he is "Given the inclination and refraction of any one ray at the surface of any refracting medium" he can "find the refraction of any other ray incident on the same surface."[35] This is Mydorge's procedure: **[fig 5]** Ray ZE is refracted at surface AEB, along EX. Draw a semi-circle above AEB cutting the ray at F. Draw FI parallel to the surface. From I, where FI intersects the semi-circle, drop IG perpendicular to the surface until it cuts the refracted ray at G. Then with radius EG draw another semi circle about E, this time below the surface. This figure now permits the construction of the refracted path of any other incident ray, say HE. Draw HM parallel to the surface cutting the upper semi-circle at M. Drop MN normal to the surface until it meets the lower semi-circle. Connect E and N, then EN is the refracted ray.[36]

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33 J.Lohne, 'Zur Geschichte des Brechungsgesetzes', *Sudhoffs Archiv* 47 (1963), 152-72; J. Lohne, 'Thomas Harriot (1560-1621) The Tycho Brahe of Optics', *Centaurus* 6 (1959), 113-21; J.A.Vollgraff, 'Pierre de la Ramée (1515-1572) et Willebrord Snel van Royen (1580-1626)', *Janus* 18 (1913), 595-625; J.A.Vollgraff, 'Snellius Notes on the Reflection and Refraction of Rays', *Osiris* 1 (1936), 718-25; C. deWaard, 'Le manuscrit perdu de Snellius sur la refraction', *Janus* 39-40 (1935-6), 51-73.

34 Mersenne, *Correspondence* (ed. C. deWaard) (Paris 1945ff). I. 404-415.

35 *loc cit.* p.404.

36 *loc cit.* p.405.

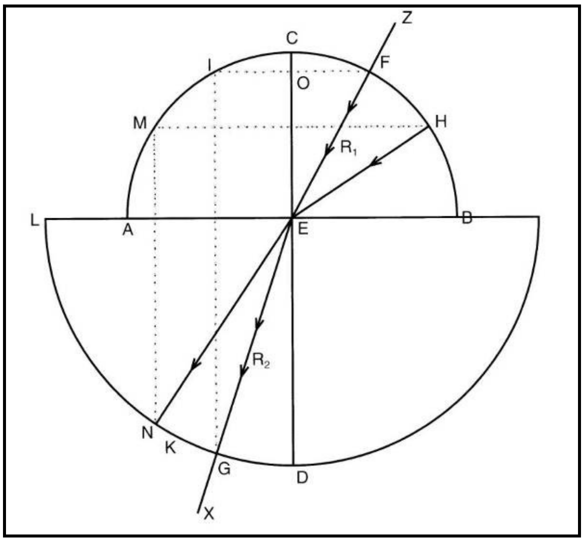

Figure 5. Mydorge's Refraction Prediction Device

Mydorge observes that the law is given here as a law of cosecants. That is, taking the first ray [272]

$$\frac{\text{cosec } i}{\text{cosec } r} = \frac{R1/OF}{R2/OI}$$

since OF = OI, the cosecants are as radius of upper semi-circle is to radius of lower semi-circle.[37]

Let us call this the 'cosecants' or 'unequal radii' form of the law of refraction, compared to Descartes' Dioptrique form, which we shall call the 'sine' form or 'equal radius' form. We have seen this diagram before--it is identical to our figure 4 for refraction using Descartes' theory of light as an instantaneous impulse. Mydorge uses two conditions to calculate the refracted ray. They are the same conditions that Descartes uses in his theory of light. The difference is that Mydorge states them only as rules of geometrical construction, while Descartes also gives them a dynamical rationale. The two conditions of course are:

(1) the constant ratio of the radii of the upper and lower semi-circles for all angles of incidence. This, in Descartes' theory, becomes the path independent constant ratio of force of light in the two media.

(2) The equality of lines FO, OI, the parallel component of the line representing the ray. This later becomes the conservation of the parallel determination of the ray.

Note that Mydorge's figure gives a clearer picture of Descartes' two assumptions than does Descartes' one circle diagram (figure 3) in the [273] Dioptrique. Why is this so? And why did Descartes invoke tennis balls in actual translation? Before we can find out, we must date the material in the letter.

[37] *loc. cit.* p.406.

5.2 Lens Theory and the Date of the Material in Mydorge's Letter

Descartes' earliest recorded statement of the sine law of refraction dates from a report to Isaac Beeckman in October 1628.[38] Descartes consistently identified 1626/27 as the crucial period for his optical studies.[39] He collaborated with Mydorge in that period, and Mydorge credited Descartes with the discovery of the law.[40] De Waard dated this letter from 1626, but that was merely a conjecture based on this collateral evidence.[41] Costabel, Shea and others date the letter from 1631 at the earliest.[42] But, evidence in the letter concerning the presentation of the law and the development of lens theory, strongly suggests this material is from 1626/7, and is contemporary with the initial construction of the law and first articulation of lens theory.

After presenting the cosecant form of the law, Mydorge outlines a theory of lenses clearly antecedent to the theory of lenses offered in the Dioptrique. The key difference is that Mydorge does not initially use the sine law in constructing lens theory. Rather, starting with the cosecant form of the law, he only strikes a sine formulation in the course of his opening analysis of the anaclastic problem: it is a simple matter of adding a few lines.[43] He does not seem to know the sine form before that constructive manoeuvre. Then he deploys the sine form in the following synthetic demonstrations.[44]

Moreover, Descartes own synthetic lens theory demonstrations in the Dioptrique differ from those of Mydorge in another historically revealing way. Mydorge had set up the sines of the angles of incidence and refraction by reference to a semi-circle on one side of the interface.[45] In the Dioptrique, as we have seen, Descartes directly relates the sines to their respective rays.[46] Isaac Beeckman seems to have been the author of Descartes' more 'natural' representation of the

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38 AT x. pp.336ff; also Beeckman *Journal* (ed. C. deWaard) fol. 333v ff.

39 Descartes repeatedly mentioned that during this period he recruited Mydorge and the master artisan Ferrier in an attempt to confirm the law and construct a plano-hyperbolic lens. Eg. Descartes to Golius, 2 February 1632, AT i. p.239; Descartes to C. Huygens, December 1635, AT i. pp.335-6.

40 In addition to the material cited in previous note, see Descartes to Ferrier, 8 October 1629, AT i. 32; 13 November 1629, AT i. 53ff; Ferrier to Descartes, 26 October 1629, AT i.38ff. In the mid 1620s Mydorge annotated Leurechon's *Récréations mathématiques*, a popular work dealing with mathematical tricks and fancies of a natural magical character. Leurechon's work was first published anonymously in 1624 and reprinted several times thereafter with additional notes, including those by Mydorge. I have consulted (Jacques Ozanam) *Les Récréations Mathématiques...Premierement revu par D. Henrion depuis par M. Mydorge* (Rouen, 1669). Mydorge notes concerning the nature of refraction "Ce noble sujet de refractions dont la nature n'est point esté cogneue n'y aux anciens, n'y aux modernes Philosophes et Mathematiciens iusque à present, doit maintenant l'honneur de sa découverte à un brave Gentilhomme de nos amis, autant admirable en scavoir et subilité d'esprit." p.157.

41 DeWaard admits that the copy he examined dated from 1631 at the earliest, Mersenne, *Corr.* I. p.404:

42 W. Shea, *The Magic of Motion and Numbers: The Scientific Career of René Descartes* (Canton, MA.,1991), p.243 n.38.

43 Mersenne, *Corr.* I 411-413.

44 Mersenne, *Corr.* pp.408-11. Schuster, *op. cit.* (1977), pp.321-7 documents the textual and mathematical claims made in this paragraph.

45 Mersenne, *Corr.* I 408-9.

46 cf. Figure 3 above.

sines. In October 1628 Descartes asked Beeckman to prove the refractive properties Descartes claimed for the hyperbola. Beeckman's proof is geometrically identical to Descartes' figure in the Dioptrique and was 'approved' by Descartes.[47] At the same time Descartes showed Beeckman an elegant proof for the ellipse case.[48] However, he did not use that proof in the Dioptrique, probably because the sines of incidence and refraction are not related to their respective rays in the obvious way Beeckman achieved for the plano-hyperbolic case.[49]

I conclude that in the Dioptrique Descartes used Beeckman's more 'natural' representation of the sines in both cases, ellipse and hyperbola, thus rejecting his own elegant ellipse proof and Mydorge's early 'one sided' representation of the sines. The Mydorge letter therefore contains [274] Mydorge and Descartes' earliest lens theory, and arguably their first form of the law, the cosecant form. The material in the letter, if not the artifact itself, pre-dates October 1628, certainly predates composition of the Dioptrique and very plausibly is as early as 1626/7--but not earlier as we shall soon see. So this dating points to the cosecant form of the law as the first form Mydorge and Descartes possessed. And this, it transpires, is the key to reconstructing how they obtained it, because the other independent discovers first obtained it in the same unequal radius form.

5.3 Traditional Geometrical Optics and the Discovery of the Cosecant Form of the Law

To reconstruct how Descartes found the law, let us first follow Johannes Lohne's important analysis of how Thomas Harriot discovered the law, because, as we shall see, Mydorge's letter provides evidence for an identical path of discovery.

One obvious phenomenological expression of the behaviour of refracted rays is the displacement of images of objects viewed under refracting media. Traditional geometrical optics had a rule for constructing the image locations of such sources. Lohne supposed that Harriot attempted to discover a general relation between the incident and refracted rays using the image rule; and that the cosecant form of the law resulted from this strategy of research.

The traditional image placement rule ran as follows: **[fig 6]** [275] AB is a refracting interface; CD the normal to AB at O, the point of incidence. E is a point source emitting ray EO, refracted at O to the eye at F. Experience teaches that E will not appear at E. Where does it seem to appear? The rule says that it will appear at I, which is the intersection point between the refracted ray FO drawn back into the first medium, and EG, which is the normal to the surface from E.[50]

47 AT x. 341-2; Beeckman, *Journal* fol. 338r.

48 *ibid.*

49 Schuster, *op.cit.* (1977),pp. 326-7 documents the textual and mathematical claims made in this paragraph.

50 This principle appears in Alhazen, Pecham, Witello, Roger Bacon and Maurolico; cf Robert Smith, *A Compleat System of Optics* (Cambridge, 1738) para 212, cited in C. Turbayne, 'Grosseteste and an Ancient Optical Principle', *Isis* 50 (1959), 467-472, at p.467.

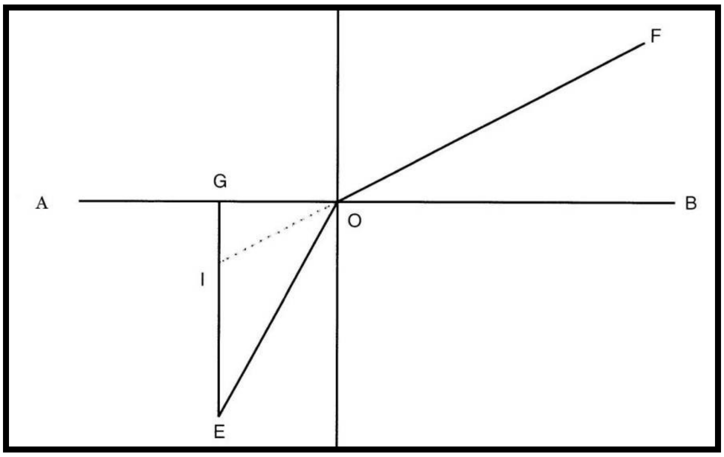

Figure 6 The Traditional Image Locating Rule

Harriot used this rule in conjunction with observations made with a disk refractometer half immersed in water. Taking source points at 10 degree intervals around the lower circumference of the disk, he observed the corresponding angles of refraction. He then constructed the image places for the source points, by applying the image rule. With the source points located around the circumference of the disk, he found the calculated image places lie roughly on a smaller, concentric circle. If you suspect the plot is really a circle, a little trigonometric analysis gives you the cosecant form of the law. Harriot's key diagram **[fig 7]** is indistinguishable from Mydorge's diagram.[51]

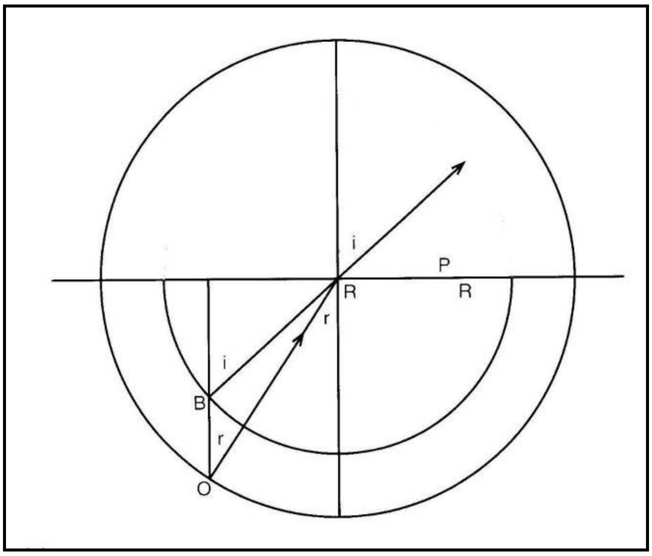

Figure 7. Harriot's Key Diagram

It is important to note that Mydorge and Descartes need not even have made <u>any</u> such observations. They could have used Witelo's rather cooked data for water/air and glass/air interfaces. I have followed calculations originally provided by Bossha and found that this data is

51 Lohne, *op. cit.* 1959, pp.116-7; Lohne, *op. cit.* 1963 p.160. Gerd Buchdahl provides a particularly clear statement of the methodological role played by the image principle in Harriot's discovery of the law in his 'Methodological Aspects of Kepler's Theory of Refraction', *Studies in the History and Philosophy of Science* 3 (1972), 265-98 at p.284. Willebrord Snel's initial construction of the law of refraction also followed the type of path indicated by the Lohne analysis. See Vollgraff, *op. cit.* (1913) (1936); deWaard, *op. cit.* (1935-6) and Schuster, *op. cit.* (1977) pp.313-5..

good enough to give a strong suggestion of a semi-circular plot when used in Harriot's manner.[52] Trigonometricians of the power of Snel, Mydorge and Descartes need only have suspected the circular plot to seize upon it and explore it further. Mydorge's diagram arguable has the form it does because he and Descartes proceeded in the same way as [276] Harriot [and Snel], leading to the same cosecant form of the law. Mydorge probably took a diagram like Harriot's and then flipped the smaller semi-circle up above the surface to create the path predicting device in his letter.

In sum, strong evidence exists that the law was constructed by traditional optical means, using data and concepts familiar to skilled students of geometrical optics. This account involves nothing about the dynamics of light or of tennis balls. What then is the relation between the cosecant form of the law and Descartes' two dynamical assumptions? Did Descartes perhaps have the two assumptions prior to 1626/7? And, if he did, is it still possible that despite our reconstruction, he arrived at the law by deducing the cosecant form from the assumptions? Whilst that is, of course, logically possible, it is not supported by the existing evidence, as we shall learn in detail in the following section.

6.0. The Dynamical Premises for the Deduction of the Sine Law of Refraction: Their Pre-History and History 1618-1629

In this section we shall see that by the early 1620s Descartes did possess some intriguing views about the dynamics of light, but that these conceptions could not have directed him to the law. Indeed they constituted an obstacle to his ever finding it. This will prompt the firm conclusion that the two dynamical premises were initially seen in, and modelled upon, the Mydorge diagram, when Descartes saw that the geometry of that diagram clarified and modified his earlier, inefficacious, dynamical notions. We can unpack all of this by examining the main alternative conjecture as to Descartes' route to the law of refraction, that owing to A.I.Sabra. The evidence that dismisses his explanation further buttresses my version of the story.

6.1 Sabra's Conjectural Discovery Path

Sabra holds that Descartes could have discovered the sine law in the very way he deduces it in the Dioptrique. Suppose Descartes possessed the two key assumptions used in this proof; he could then have discovered the law by deduction.[53] We have already foreshadowed Sabra's argument in Section 4.0.[54] The first assumption is that the ratio of the force of light in two media is a constant for all angles of incidence.

$$\frac{|Fi|}{|Fr|} = \text{const.}$$

[52] J. Bossa, 'Annexe note', *Archives Neerlandaises des Sciences Exactes et Naturelles*, ser II t. XIII (1908) xii-xiv. Cf Schuster, *op. cit.* (1977) p.311.

[53] Sabra, *op. cit.* pp.97-100, 105-6,116.

[54] cf Note 25 above .

The second assumption is that the component of the force of light parallel to the refracting surface is unchanged by refraction.

$$|Fi| \sin i = |Fr| \sin r \quad [277]$$

Combining (1) and (2) we get the sine law.

$$\frac{\sin i}{\sin r} = \frac{|Fr|}{|Fi|} = \frac{1}{\text{const}}$$

The essential question is, did Descartes have the two assumptions before the Mydorge letter. Sabra seizes on an early fragment of Descartes, dating from 1620, which, he argues, implies possession of both assumptions. This means Descartes could have deduced the law any time from about 1620. The fragment reads in part:

> Because light can only be produced in matter, where there is more matter there it is more easily generated; therefore, it more easily penetrates a denser medium than a rarer one. Whence, it happens that refraction occurs in the rarer medium from the perpendicular, in the denser medium toward the perpendicular.[55]

For Sabra the first sentence is assumption 1: the force of light is as the density of the media--independently of path. Sabra then notes the sentence: "whence refraction occurs toward the normal in the denser medium, and away from the normal in the rarer medium". He asks, how can Descartes say that unless he also has the second assumption? And, of course, given the two assumptions, Descartes could have deduced the law of refraction.[56] Sabra is thinking of a diagram very much like the Mydorge diagram, which, of course, neatly represents these assumptions.

Nevertheless, we hold that Sabra is mistaken: Descartes' first sentence does not contain or entail assumption (1). Rather Descartes is assuming that the normal component of the force of light is increased in a denser medium. In other words in 1620 he holds **:**

$$\frac{Fi^{\perp}}{Fr^{\perp}} = \text{const} \quad \text{rather than} \quad \frac{|Fi|}{|Fr|} = \text{const.}$$

To establish this we have to consider first what Sabra ignores--Descartes' style of natural philosophising and doing optics **in** 1620.

6.2 Descartes: "Physico-Mathematicus"

In November 1618 Descartes met Isaac Beeckman and fell in with his dream of a natural philosophy that would be both corpuscular-mechanical and properly (rather than metaphorically) "mathematical". They termed this project 'physico-mathematics'. Although Descartes and Beeckman produced no convincing examples of this discipline, in one or two special cases it is clear that they thought they possessed real instances of it.

55 AT x. 242-3

56 Sabra, *op.cit.* pp.106, 111.

An example of this is one of Descartes' fragments from the period, the so-called 'the hydrostatics manuscript'.[57] Beeckman had asked Descartes to explain in 'physico-mathematical' terms some of Simon Stevin's hydrostatics. [278] Stevin had demonstrated what is essentially a special case of the hydrostatic paradox by employing a rigorously Archimedean style of mathematical argument. He applied <u>reductio ad absurdum</u> arguments showing that conditions of static equilibrium obtain between specified macroscopic volumes and weights of water (and of notional solids of equal specific gravity). Descartes' attempted 'physico-mathematical' gloss of Stevin is a <u>bricolage</u> of ad hoc assertions about, and geometrical representations of, the corpuscular structure of fluids and the characteristic motions and tendencies to motion of their constituent particles. Descartes took this to be a promising piece of 'physico-mathematics', what with its geometrical representation of the tendencies to motion of the constituent corpuscles.

<u>6.3 The 1620 Optical Fragment: A Physico-Mathematical Reading of Kepler</u>

Now let's return to the optical fragment of 1620: Sabra used only part of it. Examination of all of it shows that Descartes was studying Kepler's <u>Ad Vitellionem paralipomena</u> [1604] and that his fragment is a 'physico-mathematical 'reading' of a set of texts and figures in Kepler's work. Descartes was reading Kepler the way he had read Stevin: seeking grist for the physico-mathematical mill, he attempted to elicit some physical theory from a set of compelling geometrical diagrams and texts for refraction presented by Kepler.

The most important claims in Descartes' fragment are (1) that the 'penetration' of light varies positively with the density of the medium; and (2) that consequently light is refracted toward the normal in the denser medium, and away from the normal in the rarer one. It is essential to realise that in the traditional optical literature there is no precedent for this sort of sketch physical theory of refraction. Earlier major authorities on optics, such as Alhazan, Witelo, Roger Bacon and Peckham, as well as contemporary ones such as Snel, had maintained in one fashion or another that media resist the passage of light in proportion to their densities, and that the path of motion normal to the refracting surface is the easiest or one of least resistance. From these premises opticians contrived to conclude that a ray obliquely entering a denser medium, and hence meeting increased resistance at the interface, must be refracted in a path lying closer to the easiest, normal path; and that a ray obliquely entering a rarer medium, and hence meeting decreased resistance at the interface, must be refracted into a path lying farther from the easiest, normal path.[58] Various explications were offered in attempting to link these conclusions to the premises. What one might term Kepler's 'official' qualitative theory of refraction, published in Chapter 1 of <u>Ad Vitellionem,</u> differed considerably from that of the Medieval and Renaissance perspectivists; but even he retained the stress on the denser medium weakening the incident light.[59]

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57 Schuster, *op. cit.* (1977),pp.93-111, Gaukroger, *op. cit.* (1995), pp. 84-9. J.A. Schuster, 'Descartes' *Mathesis Universalis* 1619-1628', in S. Gaukroger, ed., *Descartes: Philosophy, Mathematics and Physics* (Sussex, 1980),, pp.41-96, at 48-9.

58 D.Lindberg, 'The Cause of Refraction in Medieval Optics', *British Journal for the History of Science*, 4 (1968), 23-38; on Snel's adherence to this type of conceptualisation see Vollgraff, *op. cit.* (1913) pp.622-3.

59 Kepler held that light is an immaterial emanation propagated spherically in an instant from each point of a luminous object. Refraction, he maintained, is a surface phenomenon, occurring at the interface between media. The movement of the expanding surface of light is affected by the surface of the refracting medium, because, according to Kepler, like affects like,

It is quite obvious that Descartes' sketch theory of refraction rejects the [279] central elements of the Medieval and official Keplerian theories of refraction. For example, refraction toward the normal in denser media in no way depends upon a weakening or obstructing of the incident light; quite the contrary, refraction toward the normal is said to depend directly upon the greater 'penetration' or 'generation' of light in denser media. A fortiori, there is no role for a compensating bending toward the easier, normal path, as in the Medieval theories, nor does Descartes envision that a weakened parallel component causes the bending toward the normal, as in Kepler's official theory. So Descartes certainly did not obtain his 1620 theory of refraction by reworking those of his predecessors. The conceptual resources upon which he was drawing are likely to have resided, if at all, in less obvious corners of the traditional optical literature. As suggested above, there is strong evidence that Descartes was reflecting upon certain parts of Kepler's work on refraction in Ad Vitellionem. This line of investigation was initially prompted by the concluding portion of the 1620 fragment, not cited earlier, which discusses image places in the context of Kepler's new theory of vision. Examining the portions of Ad Vitellionem which deal with refraction, whilst bearing in mind Descartes' 'physico-mathematical' interests, brought to light two sets of passages which do seem to have provided the starting point for his curious 1620 theory of refraction.

The first and most important passage occurs in Chapter IV of Ad Vitellionem, where Kepler attempts to discover a simple law of refraction by means of an analysis of its putative physical causes. Kepler asserts that there are two fundamental physical factors which any adequate theory of refraction must take into account: the inclination of the incident rays, and the densities of the media. (These points are consistent with his 'official' theory of refraction, described above.) He offers a geometrical construction representing these factors **[fig.8].** Take AG incident upon a basin of water. The density of water is said to be twice that of air, so Kepler lowers the bottom of the basin DE to LK so that the new basin contains 'as much matter in the rarer form of air as the old basin contained in the doubly dense form of water'. Kepler then extends AG to I and drops a normal from I to LK. Connecting M and G gives the refracted ray GM. Its construction involves the obliquity of incidence and densities of media.[60] Although Kepler then goes on to reject this construction on empirical grounds,[61] the question is, did this text speak of René Descartes, the 'physico-mathematician' and budding optician, and what did it say?

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hence surface can only affect surface, and the surface of the refracting medium 'partakes' in the density of the medium. He analysed the effect of the refracting surface upon the incident light by decomposing its motion into components normal and parallel to the surface. The surface of a denser medium weakens the parallel component of the motion of the incident light, bending the light toward the normal; a rarer refracting medium facilitates or gives way more easily to the parallel component of the motion of the incident light, deflecting it away from the normal. (The normal component of the motion of light is also affected at the surface by the density of the refracting medium, weakening or facilitating its passage, but not contributing to the change of direction). *Ad Vitellionem Paralipomena,* Chap.1 Prop. 12, 13, 14, 20, in J. Kepler, *Gesammelte Werke*, ed. M Caspar (Munich, 1938ff), vol.II pp. 21-3, 26-7. I have termed this Kepler's official theory of refraction, because it is not his only articulated discussion of the causes of refraction (and their geometrical representation) offered in *Ad Vitellionem*, as we are now about to see.

60 *Ad Vitellionem Paralipomena,* in J. Kepler, *Gesammelte Werke*, ed. M Caspar (Munich, 1938ff), vol.II pp.81-5.

61 Kepler, *loc. cit. p.86.*

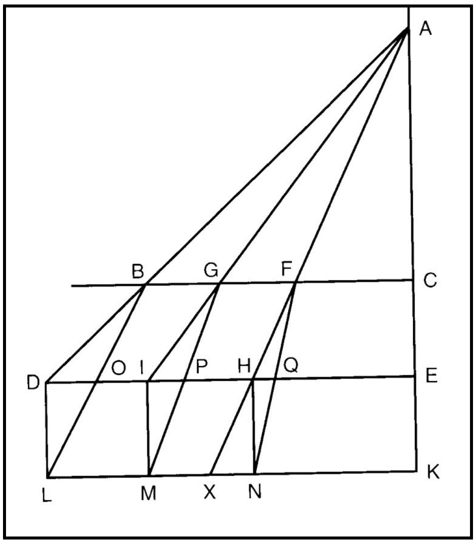

Figure 8

The first thing to notice is that Descartes' fragment and Kepler's text resemble one another in precisely those respects in which they are anomalous with regard to the traditional theories of refraction. Kepler's construction, like the Medieval theories and his own official theory, stresses the role of the greater density in bending rays towards the normal. But, in his figure Kepler directly represents the greater density (by lowering bottom DE) and he then utilises that representation in an unmediated fashion to construct the refraction of the ray toward the normal. It is [280] strongly implied that greater density is a direct cause of bending toward the normal. Kepler does not argue, as had the Medieval perspectivists, from greater density of the medium, to more resistance to the passage of light, and thence to a compensating bending toward the 'easier' normal path. Nor does he argue, as he had in his official theory, from greater density of the medium, to weakening of the parallel component of the motion of the light, and thence to bending toward the normal. Descartes' fragment is peculiar in precisely this same respect. There is no mention of a weakening of the light or of any of its components, nor of a compensating bending toward the normal. Instead, greater density is connected with greater 'generation'/'penetration', which apparently directly causes refraction toward the normal. Descartes' fragment would therefore appear to be based in some way upon Kepler's text and construction.

It is not difficult to see why Descartes, the aspiring 'physico-mathematician', would have been attracted to the non-traditional approach manifested in Kepler's text. Kepler was trying to penetrate beyond the mere phenomenon of refraction and to identify its physical causes; he wanted to represent geometrically the action of these causes and build the representations into a method of generating, by geometrical construction, [281] the paths of refracted rays. Descartes, who had already attempted to identify and geometrically represent the true causes of the paradoxical statical behaviours of fluids, 'superficially' examined by Stevin, probably saw Kepler's construction as a promising step toward the physico-geometrisation of the problem of explaining refraction.[62]

This may explain Descartes' source and his motivation, but it does not yet elucidate the precise wording of his fragment. Here one has to be careful in teasing out the relationship between Descartes' fragment and Kepler's passage; for the fragment is not a simple verbal transcription of Kepler's construction technique (and verbal gloss), but rather an elaboration and explication of them. As we have seen, the two texts share the same anomalous posture vis-à-vis traditional theories of refraction. But within that broad similarity there resides an important residual difference. Kepler's construction technique does not focus upon, or work with, the parallel and normal components of the motion of the incident light or light ray. He directly represents the causally efficacious greater density of the lower medium and postulates a construction technique which uses that representation of density, and the obliquity of the incident ray, to manufacture a ray path bent toward the normal: the greater the obliquity of incidence and the farther the bottom DE has been lowered, the greater the resultant refraction toward the normal. In contrast, Descartes' fragment introduces the concept of 'generation'/'penetration' of light which varies with density. It is the increased or decreased 'penetration' (itself the product of greater or lesser density) which causes refraction toward or away from the normal. Descartes, unlike Kepler, wishes to characterise the properties of the light or light ray itself and to insert the characterisation between the talk of 'density' and of 'refraction'.

Why should Descartes have been led to view the Kepler diagram in these terms; why mention 'penetration'/'generation' at all; why not just say that greater or lesser density causes refraction toward or way from the normal? The answer would seem to be that Descartes, in interpreting Kepler's passage, was reintroducing quite customary questions about the comportment of the parallel and normal components of the motion of the incident light, or of the ray that represents it. Kepler, in other contexts in which he deals with refraction (and reflection), typically considers the comportment of these components, even though he does not always deduce changes in direction of light by (re-)composing altered components of its motion.[63] Descartes' contention that the 'penetration' of light varies with the density of the medium makes sense as a reading of Kepler's text, provided one takes Descartes to be thinking in terms of the comportment of the parallel and normal components of the motion of the incident light or of the incident ray. When approached in this way, Kepler's diagram and construction technique would be taken as saying that the denser medium has the effect of increasing one or both of these components, hence causing refraction toward the normal. [282] Only a little reflection is required to see that this in turn boils down to the claim that the normal component of the motion of the incident light increases upon entering a denser medium, while the parallel component can remain constant, increase in appropriate proportion, or even decrease.

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62 Schuster, *op. cit.* (1977), pp.336-9. Cf. also the problem solving techniques attributed to the young Descartes by Sepper in this volume, 'The Problem of Figuration: Or How the Young Descartes Figured Things Out'.

63 For example, Kepler's official theory of refraction [Note 59 above] dealt with the parallel and normal components of the motion of the light, asserting that both are weakened at the interface, whilst attributing the refraction to the alteration in the parallel component alone. In the traditional optical literature it was, of course, also thoroughly commonplace to attend to the comportment of the normal and parallel components of the motion of light when discussing its refraction and reflection.

The literal text of Descartes' optical fragment is therefore to be explained as follows. Descartes was pursuing the central idea of Kepler's passage, the direct causal role of greater or lesser density in bending light to or from the normal. But Descartes translated that physico-mathematical insight into the customary mode of discourse about the parallel and normal components of the motion of light or of light rays, and so produced his proposition about 'penetration' varying with density. Hence, when Descartes writes of the 'penetration'/'generation' of light being directly related to the density of the medium, he is envisioning the behaviour of the normal components of incident light rays. The magnitude of these components (the 'penetration') varies with the density of the medium. Increase in the normal component (with conservation or appropriate alteration in the parallel component) will bend the refracted ray toward the normal; decrease in the normal component (with conservation or appropriate alteration in the parallel component) will bend the ray away from the normal.[64] This then also explains the entailment between the first and second sentences of the fragment, claimed by Descartes and discerned by Sabra: Descartes can say that greater or lesser 'penetration' causes refraction toward or away from the normal, because he identifies greater/lesser 'penetration' with increase/decrease in the normal component, which can be represented in ray diagrams and used in the construction of refractions toward/away from the normal.

This reading of Descartes' fragment can be confirmed by looking at a second set of passages in Ad Vitellionem which conditioned his thinking about the 'physico-mathematics' of refraction. Descartes' fragment, quoted above, continues,

> Moreover the greatest refraction of all should be in the densest medium of all...[65]

In his analysis of the fragment, Sabra did not cite or discuss this remark; yet, it is of vital importance in understanding Descartes as a 'physico-mathematical' reader-interpreter of Ad Vitellionem.

As it happens, in Ad Vitellionem Kepler twice considers the notion of 'the most dense medium possible', pointing out on both occasions that any ray entering such a medium will be refracted into the normal direction. **[fig.9]**.

> In the most dense medium of all refractions are performed toward the perpendiculars themselves, and are equal in respect of (all) inclinations.[66]

And, [283]

> ...if you should ponder what ought to occur in the most dense medium (or medium of infinite density), you would comprehend from the analogy of other media that, if there could be such a medium, it is necessary that all rays falling from one point onto the

64 Again, our interpretation should be compared to Sepper's thesis in this volume about how the young Descartes solved problems via strategies of figural representation ('The Problem of Figuration: Or How the Young Descartes Figured Things Out'). Here Descartes uses the routine representation of the components of rays to represent and articulate Kepler's interesting physical hypothesis.

65 AT x. 242-3.

66 Kepler, *Ad Vitellionem*, *Gesammelte Werke* II p.107.

surface would be fully refracted, that is, after refraction they would coincide with the perpendicular itself.[67]

The context of these remarks is Kepler's official theory of refraction. The infinite density of the refracting medium destroys the parallel component of the motion of the light, leaving it only its normal component.

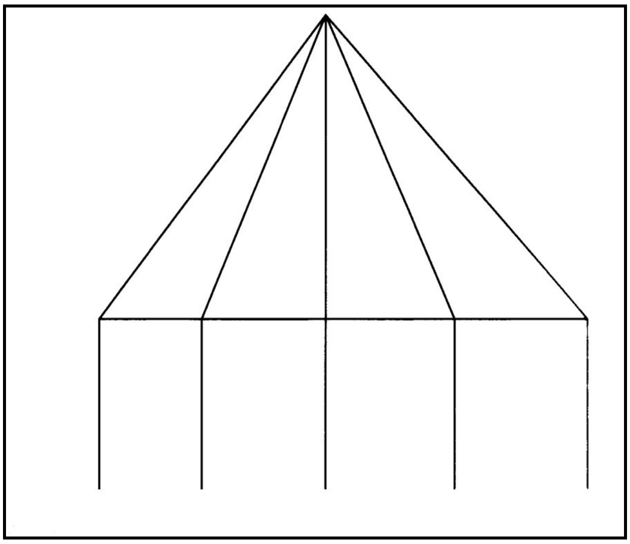

Figure 9

When Descartes echoes these passages in his fragment the context is not Kepler's official theory of refraction, but rather the first two sentences of his own 1620 text, as we have learned to read them. Clearly, Descartes intended to present the case of the 'most dense medium' as a limiting case of the general proposition that 'penetration varies with density and causes refraction to or from the normal'. That is, when a ray enters the most dense medium possible, the normal component is infinitely (or as Descartes probably would have had it, indefinitely) increased and the ray bent into the normal, regardless of whether the parallel component suffers a finite increase, decrease or merely stays the same. If Descartes drew his limiting case from Kepler, this lends extra weight to the claim that the first two sentences of the fragment constitute a 'physico-mathematical' reading of the other passage in Ad Vitellionem.[68] In sum, Descartes connected two lines of speculation present in Ad Vitellionem but not explicitly linked by Kepler: (1) The geometrical representation of the claim that 'the greater [284] the density, the greater the refraction toward the normal'. And, (2) The claim that infinitely dense media would refract all incident rays into the normal. It was Descartes, not Kepler, who first related (2) to (1), using (2) to illustrate the limiting case of his own explicated version of (1) which related change in density to change in 'penetration' (normal component) to change in direction.

So, in 1620, Descartes embraced an assumption which would have hindered his ever deducing the sine law. He held that in two media the normal components of the force of light are in a

[67] *loc. cit.* pp.89-90.

[68] Issues about Descartes' ontology of light in the fragment--whether Aristotelian, Keplerian or mechanistic--are addressed below in Section 7.2 as part of the explication of the development of his mechanistic rational of the law of refraction in the 1620s.

constant ratio. Had he then assumed that the parallel components are constant, he would have gotten a law of tangents.[69] How then did Descartes ever devise his two assumptions--and in particular why did he ever decide that the constant force ratio applies to media, in a path independent manner? All the evidence examined thus far suggests that a likely answer is this: Descartes only formulated his two dynamical assumptions <u>after</u> he had constructed the law in cosecant form, using traditional means--issuing in the Mydorge diagram. The Mydorge diagram--the cosecant form--gives you the two assumptions <u>if you are looking to read them out of the diagram</u>. And in 1626 Descartes, physico-mathematician, was very interested to read out of his ray diagram some mechanical theory explaining that diagram. In short, he did to the Mydorge diagram exactly what he earlier did to diagrams in Stevin and Kepler: he took a geometrical picture of a macroscopic phenomenon and read out of it the underlying dynamical causes at the corpuscular level. Viewed through physico-mathematical spectacles, the Mydorge diagram was the locus where the two dynamical assumptions were forged and coordinated.

This reconstruction thus helps us understand why, after 1627, Descartes moved to a dynamical rationale for the law; and why that rationale took the form it did. Having been thwarted in his early attempt to arrive at the law of refraction by physico-mathematical analysis of its purported physical causes, Descartes would have seized upon the newly discovered and arguably correct cosecant form of the law of refraction. He decoded the Mydorge diagram as a message concerning the causes of refraction. This account also helps us deal with the problem of why Descartes embraced such problematical dynamical premises for explaining refraction: Why, as we noted earlier, he used dynamical premises which simultaneously entail that optical media are and are not isotropic. The most likely answer is that having formulated the premises by inspecting the geometry of the already discovered cosecant form of the law of refraction, he accepted and defended these premises because of their supreme value in grounding a deductive physical rationale for the law.[70] [285]

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69 Had Descartes assumed that the parallel component varies either directly or inversely with the density, he would have again deduced 'tangent laws' with slightly differing indices of refraction. There seems no way to proceed directly from the assumptions of 1620 to the sine law of refraction, unless one is prepared to introduce Newtonian complications about the variation in components as functions of the angle of incidence, a way of conceiving the problem foreign to Descartes in 1620, 1626, as well as 1637.

70 The discerning reader will note a difficulty in this reconstruction. It has been argued that Descartes and Mydorge (as well as Snel) used the traditional image finding rule in their path of research leading to the law of refraction. But, unlike Harriot, the three later discoverers presumably were well aware of Kepler's new theory of vision which cast grave doubt on the use of the traditional rule. Descartes, after all, was working on a mechanistic version of Kepler's theory of vision around the same time he and Mydorge discovered the law and his 1620 optical fragment indicates familiarity with Kepler's new work on vision. This fascinating issue cannot be addressed in full here. Suffice it to say that the problem is more Descartes' than our own. That is, there is evidence that Descartes suppressed discussion of his actual path of discovery for several reasons, one of which was the embarrassing point that his work depended upon an optical principle he could no longer accept. For example, his odd methodological story about how the law might be discovered, offered in rule 8 of the *Regulae ad directionem ingenii*, seems intended to occlude this fact, and to mythologise several of his other theoretical quandaries, under a cloak of persuasive, but necessarily vacuous 'method talk'. The matter is briefly touched upon in J.A Schuster, 'Whatever Should We Do With Cartesian Method: Reclaiming Descartes for the History of Science' in S. Voss (ed.), *Essays on the Philosophy and Science of René Descartes* (Oxford, 1992), pp. 195-223, and below in Section 9.1. I am working on a study of Descartes' methodological tales of optics in the *Regulae* and *Dioptrique* , and how other reconstructions of his discovery path compare to that offered in the present paper, eg. Kramer, *op. cit.;* Milhaud, *op. cit.* pp.108ff; Milhaud, 'Descartes et la loi des sinus', *Revue general des Sciences* 18 (1907), 223-228; Sabra, *op. cit.* ; and Mark Smith, 'Descartes's Theory of Light and Refraction: A Discourse on Method', *Transactions of the American Philosophical Society* 77 pt 3 (1987), pp.1-92.

7.0. The Mechanical Theory of Light 1620-1628

7.1 Expository Strategy and Working Distinctions

Thus far we have reconstructed how Descartes developed his two dynamical premises while bracketing the question of what he actually took to be the nature of light. The reconstruction presupposed only that Descartes had thought of himself as a 'physico-mathematician', and that he had been committed in some sense to a mechanisation of optics. Beyond that, the discussion was intentionally non-committal about details: Descartes was said to have realised that the parallel component of the force or motion of the incident light is conserved before and after refraction, and that the quantity of the force or motion of the light varies with the density of the medium and is path independent. Problems of exposition necessitated this strategy. For example, the place of the 1620 optical fragment in the development of the law of refraction can be assessed without having to linger over the ontological problems it poses and which we shall discuss below. Similarly, we shall see that evidence relating to Descartes' mechanistic theory of light in the period 1626-28 can only be decoded on the basis of a prima facie account of how and when the law of refraction was discovered. So, in this section we examine Descartes' commitment to a mechanistic theory of light between 1620 and 1628 with the goal of confirming and deepening the findings of Section 6.0

When investigating Descartes' commitments to mechanism and to a mechanistic optics certain working categories need to be kept in mind. It is useful to distinguish between (1) fundamental ontological convictions in general, and (2) theories about the nature of light in particular. Furthermore, when considering (1) or (2), one needs to distinguish between (a) relatively articulated or systematised commitments or theories, and (b) relatively unarticulated commitments or theories. Combining these possibilities one obtains a set of four broad analytic categories

(1a) A systematic corpuscular-mechanical ontology: such as is found in Descartes' two systematic treatises on the philosophy of nature, Le Monde (1629-33) and Principia philosophiae (1644). This involves an elaboration of the corpuscular-mechanical structure of matter, leading on to a theory of 'elements', a theory of the 'cosmological' structuring of matter, and an explicit doctrine concerning the laws of motion, collision and tendency to motion, or what we have termed Cartesian dynamics.

(1b) An unarticulated corpuscular-mechanical ontology: such as is found in Beeckman's Journal, or in Descartes' work prior to his commencement of Le Monde. This involves a general belief in corpuscular-mechanism and piecemeal appeals to it in formulating particular explanations, without a sustained attempt to organise or mediate between these particular applications. Certain consistencies might [286] run through these applications and to that extent one might speak of an 'element theory', 'cosmology' or 'dynamics' implied in them; but, in general, the more that the theme of systematisation emerges and claims to control the applications, the more articulated and systematised the ontology can be judged to be.

(2a) An articulated corpuscular-mechanical theory of light, such as is found in the explanations of light in Le Monde or Principia philosophiae. In the broadest sense this would therefore involve the attempt to explain the true nature of light as part of the sort of system envisioned in (1a), in which the theory of light is articulated to the matter theory, cosmological setting and controlling principles of motion and dynamics.[71]

(2b) An unarticulated mechanical theory of light: such as we shall find in Descartes' optical work in 1626-28. This would involve a loose commitment to the mechanistic nature of light, based on piecemeal and unsystematised appeals to mechanistic causes, and to 'mechanistic principles' which have not taken the form of a systematised dynamics. This can involve a background belief in the corpuscular-mechanical character of matter and light.

One needs also to note that two broad options were open to Descartes in constructing a theory of light, whether under (2a) or (2b). Light could be taken to consist in the translation of pieces of matter or in mechanical impulses or tendencies to motion transmitted through media. Finally, under both (2a) and (2b) a theory of light could be elucidated or applied by means of explicit mechanical analogies. So, by the early 1630s Descartes had to hand his tennis ball model, which, as we have seen, was really offered under the tacit aegis of his (2a). Similarly we shall see that in the late 1620s he employed a balance beam model for the refraction of light which is meant to clarify the version of (2b) which he then held.

7.2. Reprise--The Optical Fragment of 1620

Descartes' optical fragment of 1620 makes no direct reference to a corpuscular-mechanical ontology. Indeed it appears to take a quasi-Aristotelian view of the nature of light, with Descartes writing of the 'generation' of light, although if taken literally this would imply light to be a substance rather the actualisation of a potential property of the medium, as Aristotle held. The generally Keplerian context of the fragment, which we established in Section 6.0, might suggest an underlying ontology of light as immaterial emanation. Yet, Descartes' apparent concern with quantifying the variation of 'penetration' (normal component) with density might also bespeak an unarticulated theory of light as mechanical impulse or tendency to motion. For example, in the hydrostatics manuscript of 1619, Descartes had already explained gross weight as [287] the product of summed corpuscular tendencies to (downward) motion, and he had analysed the 'weight-producing' normal components of those tendencies.[72]

However, teasing deep ontological commitments out of the optical fragment of 1620 may be slightly beside the point. Descartes seems less interested in ontology in general, or with the theory of light in particular, than with explaining refraction by relating density to 'generation/penetration' (magnitude of normal component), and expressing the relation geometrically. 'Physico-mathematics', as Descartes understood it, sought to combine corpuscular-mechanical ontology with genuine mathematisation. In so far as Descartes sought to explain refraction by mathematicising the density-penetration relation, he has comporting himself as a physico-mathematicus. The question of how (or even whether) the corpuscular-mechanical

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[71] One can also imagine slightly lesser degrees of articulation, involving, for example, merely a corpuscular-mechanical explanation of optical sources and media and lacking cosmological articulation, and possibly lacking a highly articulated theory of dynamics.

[72] cf. Note 5 above.

ontology applied was pushed to the periphery, as was any unequivocal commitment about the nature of light.

One possible explanation for Descartes' reticence about these issues may perhaps reside in his comparing Kepler's approach to refraction with Beeckman's corpuscular speculations about the phenomenon. To explain refraction Beeckman explicitly employed his corpuscular-mechanical ontology and a theory of light as the translation of light corpuscles. The macroscopic refraction of light results from a complex series of collisions between light corpuscles and the constituent particles of the refracting medium.[73] The explanation was qualitative and discursive, incapable of mathematical treatment, and, if we may judge by Descartes' eager appropriation of Kepler's texts, was thought unlikely to lead to the discovery of the law governing refraction. Encountering Kepler's physico-mathematical approach to refraction, Descartes may well have faced a choice: either to pursue Beeckmanian qualitative corpuscular-mechanical speculations about light and refraction, or, to follow Kepler's attempt to identify and mathematicise the causes of refraction as a step toward the discovery of the law. In the latter case a corpuscular-mechanical explanation need not have been rejected in principle, but merely deferred until such time as the law of refraction might be discovered (and indeed this is the pattern our analysis has suggested thus far.)

Descartes' physico-mathematical encounter with Kepler's optics, recorded in the 1620 fragment, therefore probably affected his views about ontology in two ways. First, it discredited, for the time being, detailed corpuscular-mechanical stories about light, media, sources and the micro-mechanics of refraction, because these eluded and obstructed attempts at mathematisation. Second, at the level of even unarticulated theories of light, it exerted pressure away from explicit kinematic models and toward models involving no passage of any material entity. Beeckman's kinematic fantasies were avoided, but there were permitted models of light as mechanical impulse or as tendency to motion, or indeed as Keplerian immaterial substance, or even as Aristotelian actualisation of a [288] potential property of the medium.[74] In this sense the 1620 fragment displays the boundaries within which Descartes' optical theorising would move during the next eight years. We shall now see that by 1626 he was firmly convinced of an unarticulated theory of light as instantaneously transmitted mechanical impulse or tendency to motion. Only in 1629/30, when he had begun to compose Le Monde, did Descartes attempt to devise an articulated corpuscular-mechanical theory of light (2a) within his emerging system of mechanical natural philosophy (1a). Likewise, it was apparently at this same time that he designed the tennis ball model for use in the Dioptrique. This was his only foray into the "corpuscular"-kinematic modelling of refraction, and its use is quite circumscribed. On the one hand the tennis ball model is, after all, only a model for the corpuscular-mechanical theory of light as tendency to motion, and, on the other hand, the model itself is essentially premised on the principles of his dynamics of instantaneously exerted forces and determinations, as we have seen.[75]

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73 eg. Beeckman, *Journal* III pp.27-28.

74 For Descartes' similar reaction to Beeckman's celestial mechanical speculations see Schuster, *op. cit.* (1977), pp.567-79.

75 On the larger functions and uses of the tennis ball model and Descartes' difficulties with it, see below Section 8.0.

7.3. Light as an Instantaneously Transmitted Mechanical Impulse 1626-28

Whatever the ambiguities of the 1620 fragment on the issue of the nature of light, one can be reasonably certain that by 1626 Descartes had opted for an unarticulated theory of light as mechanical impulse or tendency to motion transmitted instantaneously through corpuscular media, though the microstructures of those media were not as yet a matter of concern, for the very reasons we have just canvassed. The main evidence on this point comes from parts of Descartes' Regulae ad directionem ingenii which he wrote in Paris between 1626 and 1628, after the discovery of the law of refraction, discussed in this section, as well as from discussions he held with Beeckman in 1628, discussed in the next section.

Limitations of space prevent a full discussion of the relevant portions of the Regulae, but the key point for our concerns here is that a theory of light as an instantaneously transmitted mechanical impulse plays a central role in and between the lines of the latter portion of the text, written in Paris between 1626 and 1628. I have argued elsewhere that the Regulae really consist in three main textual strata, written at different times between 1619 and 1628 with rather different aims in view.[76] The first stratum, consisting in a portion of rule 4, is the remnant of a treatise which Descartes planned to compose in mid 1619 on the subject of 'universal mathematics'. Descartes conceived of this 'discipline' in mid 1619, viewing it as some sort of synthesis of his physico-mathematical project and his more purely mathematical researches, in particular his recent work in the generalisation of analytical procedures applied to classes of geometrical and algebraic problems. Universal mathematics was supposed to embody general analytical methods applicable to all genuinely mathematical fields, whether pure or physico-mathematical. It was more [289] an enthusiastic post-adolescent dream of than a practical reality. Descartes overestimated the generality and power of his analytical findings, and, as has been seen, his physico-mathematics was itself a loose assemblage of piecemeal and often quite tendentious initiatives. These difficulties most likely did not become clear to Descartes at the time, for by November 1619 his horizons widened even farther. The half-baked project of universal mathematics was superseded by and encysted within the main lines of his method, the dream of a general analytical machinery suitable for all rational disciplines, mathematical or not. I have shown that Descartes' constructed his doctrine of method by analogically extending concepts embedded in his none too efficacious discourse about universal mathematics. This was done in the winter of 1619-20, the results being recorded in the second stratum in the Regulae, rules 1 to 3, part of 4, and 5 to 11, excluding some material in rule 8.[77]

With his apparently effective method in hand, Descartes arrived back in Paris in 1625 and there was led into the composition of the third and final stratum of the Regulae. As I have established elsewhere, under the influence of Marin Mersenne, Descartes' Regulae project now took the form

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[76] Schuster, *op. cit.* (1980).

[77] I have also shown that the structure of Descartes' method discourse necessarily prevented it being able to achieve what it proclaims itself able to achieve, whilst, at the same time, that structure necessarily produced, for Descartes and other believers, a connected set of illusions, literary effects about the method's unity, applicability and efficacy. Cf above Note 70 and Section 9.1 below, as well as J.A.Schuster, 'Cartesian Method as Mythic Speech: A Diachronic and Structural Analysis', in J.A. Schuster and R.R. Yeo, eds., *The Politics and Rhetoric of Scientific Method: Historical Studies* (Dordrecht, 1986), pp.33-95. and Schuster, *op. cit.* (1992).

of returning to the universal mathematics of 1619, which he would now attempt to construct in detail, by expanding and extending his 1619/20 text on method, that is, roughly rules 1 to 11 of the Regulae. Universal mathematics would appear to grow out of the doctrine of method. The shift from the second to third stratum in the Regulae can be located inside the present text of rule 8 and leads immediately to the methodological tale of how the anaclastic problem can be solved on the basis of discovery of the law of refraction. This reinforces a dating for the third stratum of the Regulae after 1626/7 and prior to 1629. In order to underwrite universal mathematics Descartes, in Rules 12 to 14, sketched the outline of a mechanistic theory of nervous function and perception.[78] The key point in the present context is that a mechanistic theory of light as instantaneously transmitted impulse underlay this enterprise, along with a 'mechanisation' of Kepler's new theory of vision.[79]

7.4. Light as Mechanical Impulse and the Explanation of the Law of Refraction 1626-28--The Balance Beam Model

The theory of light as an instantaneously transmitted mechanical impulse, unarticulated as it was in 1626-28, would still have been sufficient to provide the conceptual framework for Descartes' 'physico-mathematical' reading of the Mydorge diagram, as discussed in Section 6.0. Descartes, physico-mathematicus, operating with an unarticulated theory of light as mechanical impulse, could have read the Mydorge diagram as bespeaking the true physical premises necessary for the demonstration of the law of refraction, premises which corrected and reformed the ideas about density and penetration (normal component) evident in the 1620 [290] fragment: (1) A light impulse, or ray, has a force, strength, or perhaps (retaining the language of 1620) a 'penetration', which varies when the impulse passes from one medium to another. For a given pair of media the ratio of these forces or 'penetrations' is constant and independent of the angle of incidence; (2) The force or 'penetration' of an impulse or ray may also be considered directionally, in the usual terms of components parallel and normal to the refracting surface. The force or penetration of the ray or impulse acting parallel to the surface must be unaffected by the refraction. This, of course, is a 'rational reconstruction' of how Descartes might have interpreted the Mydorge

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78 Schuster, *op. cit.* (1980) pp.59-64. Descartes writes in Rule 12 that the external senses 'perceive in virtue of passivity alone, just in the way that wax receives an impression /figuram/ from a seal.' He intends no mere analogy: just as the wax is physically impressed with the image of the seal, 'the exterior figure of the sentient body is really modified by the object'. All of our sensations, whether of light, colour, odour, savour, sound or touch, are ultimately caused by the mechanical disturbance of the external sense organs. From the sense organs the impressed 'figures' are transmitted to the common sense via the nerves. This occurs 'instantaneously' by the passing of a pattern of mechanical disturbance. 'No real entity travels from one organ to the other', just as the motions of the tip of a pen are instantaneously communicated to its other end, for 'who could suppose that the parts of the human body have less interconnection than those of the pen'. Patterns are so registered in the common sense can then be imprinted in the imagination, there to be stored in memory from the future 'attention' of the vis cognoscens, or to be immediately attended to in sense perception. AT x. 412-4.

79 Schuster, *op. cit.* (1980), pp.61-2. Although Descartes focuses upon the mechanical causation of sensation and perception, it is clear that a mechanical theory of light underpins the entire discussion. Whatever the essential nature of external objects may be, Descartes implies, they act upon the perceiving subject in a mechanical manner. In the case of visual perception, therefore, light (or the optical media through which it acts) mechanically impresses the 'figures'. Presumably light is an instantaneously transmitted mechanical impulse: Descartes' mention of instantaneous mechanical nervous action, and his analogy of it to the instantaneous transmission of motion from one end of a pen to the other, suggest that light is considered to act in the same fashion. Note also that although the pen analogy is applied to nervous action, it is similar to the analogy of the blind man's staff, used later in the *Dioptrique* to illustrate the instantaneous mechanical transmission of light.

diagram, using a theory of light as mechanical impulse in the interests of designing a 'physico-mathematical' explanation of the new law. This rational reconstruction fills up the interpretive and evidential void left at the common terminus of several lines of textual and contextual reconstruction. There is, however, a very remarkable piece of evidence, dating from 1628, which we are now finally in a position to examine, and which shows Descartes striving to elucidate how the theory of light as mechanical impulse could be used in the demonstration of the law of refraction. Although it does not record Descartes' initial 'physico-mathematical' reading of the Mydorge diagram, it is arguably a product of research and reflection which followed very closely upon that event.

In the fall of 1628 Descartes paid a short visit to the Low Countries prior to his settling there permanently early the next year. On 4 October he met with his old friend Isaac Beeckman for the first time since early 1619. He sketched for Beeckman some of his discoveries of the previous nine years, including the work on lens theory (cf Section 5.2). This was prefaced by a statement of the (sine) law of refraction which Beeckman recorded in a short memorandum, illustrated by **figure 10**, in which for rays aeg and cef, (ab/hg) = (cd/if). There immediately follows Beeckman's description of an analogy through which Descartes sought to explain the law to him,

> [Descartes] considers water to be under st and the rays to be aeg, cef. They seem to undergo the same /change/ as the arms of an equal arm balance, on the ends of which are fixed weights, of which that in water is lighter and raises the arm.[80]

This passage certainly is cryptic; even so patient a Cartesian scholar as Gaston Milhaud was moved to dismiss the analogy as 'bizarre'.[81] But, Descartes' conception can be reconstructed, provided one is willing to grant that Beeckman, in an understandable way, garbled or mistook part of the sense of Descartes' exposition.

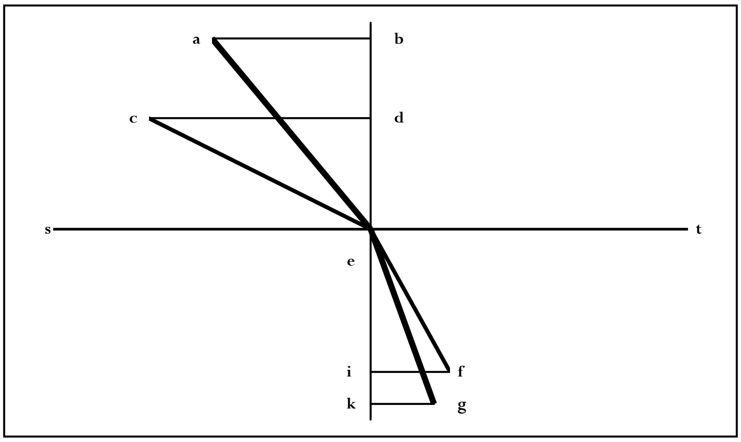

Figure 10: Beeckman's 1628 illustration of discussion of the sine law of refraction

Let us take Descartes to be suggesting that the behaviour of the incident and refracted rays of light is analogous to the behaviour of an equal arm balance, the arms of which must be bent, or refracted, at the fulcrum [291] to maintain equilibrium under varying conditions of loading. The

[80] AT x 336; Beeckman, *Journal* fol. 333v.

[81] Milhaud, *op. cit.* p.110.

constant ratio of the force of light in a given pair of media is likened to the constant ratio of the 'effective' weights of identical bodies immersed in a pair of fluids differing in specific gravity. In **figures 11 and 12** we have a balance who equal arms can be pivoted about the fulcrum and fixed at the settings required to maintain equilibrium under differing conditions of 'effective' weight. The arms are loaded with two identical bodies of specific gravity SGb. The specific gravity of the upper medium, SGu, and the specific gravity of the lower medium, SGl, are each less than SGb, so the weights 'weigh down' from both ends of the balance. In figure 11, SGu>SGl and in figure 12, SGu<SGl. Then, in figure 11, the effective weight of body in the upper medium, Wt'u bears to the effective weight of the body in the lower medium, Wt'l, the ratio

$$\frac{Wt'u}{Wt'l} = \frac{SGb - SGu}{SGb - SGl} = \text{const.} < 1$$

And, in figure 12 the corresponding ratio is:

$$\frac{Wt'u}{Wt'l} = \frac{SGb - SGu}{SGb - SGl} = \text{const.} > 1$$

In either case at equilibrium,

$$(Wt'u)\ (r \sin i) = (Wt'l)\ (r' \sin r) \qquad [292]$$

[293] where r sin i and r' sin r are the effective lever arms

$$\text{but,} \qquad r = r'$$

therefore,

$$\frac{\sin i}{\sin r} = \frac{Wt'l}{Wt'u} = \text{const.}$$

Thus, if equilibrium is to be maintained, in figure 11 the right arm must be dropped toward the normal; in figure 12 it must be removed away from the normal. For a given pair of media, the 'refraction' of the right arm will always be given by the last equation, a veritable 'law of sines' telling us how to adjust the right arm at the fulcrum, for a given setting of the left arm, in order to maintain the condition of equilibrium.

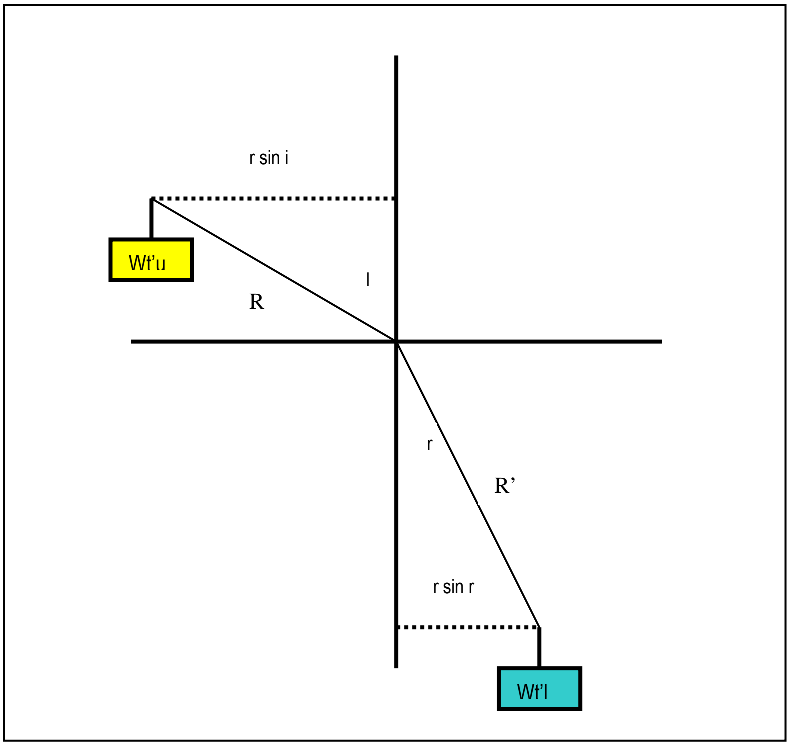

Figure 11 Reconstruction of Beeckman's Bent Arm Balance: Refraction toward the Normal

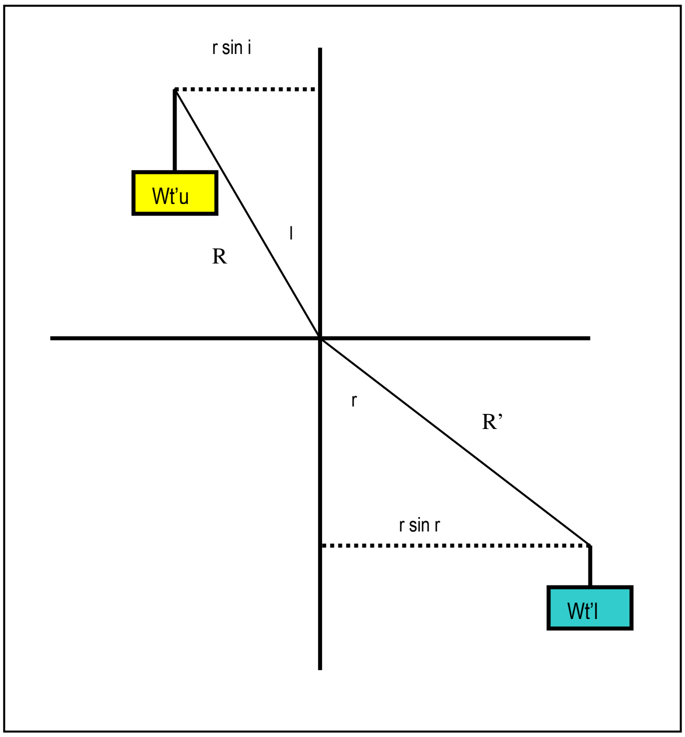

Figure 12 Reconstruction of Beeckman's Bent Arm Balance: Refraction from the Normal

Returning to the entry in Beeckman's Journal, we see that his diagram **[fig.10]** indicates refraction toward the normal in water, but that his discussion specifies that the weight on the right rises due to the buoyancy of the water being greater than that of the air. The inconsistency can be explained by Beeckman having garbled Descartes' explanation. Figure 13 illustrates what Descartes intended in the case of a real balance with weights immersed in air and water.[82] But, as we also know from the Dioptrique, when Descartes switched from tennis balls to light rays, he had to argue that the force of light is greater in water than in air, in order to explain its refraction toward the normal in water. Accordingly, to apply the balance analogy to the case of light, Descartes must have claimed that the lower medium is rarer than the upper one, so that the effective weight of the body in the lower medium (analogous to the force of the refracted light) is greater than the effective weight of the body in the upper medium (analogous to the force of light in the upper medium), hence that Wt'l>Wt'u. This, of course, makes no sense if one still has in mind a real balance with one arm plunged into a real vat of water. To make the balance germane to the behaviour of light passing from air into water, one must abstract from the concrete situation and invoke different media with the appropriate ratio of densities. Beeckman may have gotten confused in the shift from the concrete case of a balance beam with weights in air and water, to the abstract case where the balance illustrates by analogy the force changes light undergoes in different media. In any case, Beeckman must have garbled the sense of his discussion with Descartes, for he cannot have both his figure and his text.

On this reading, Descartes was offering to Beeckman a particularly fine model for his two recently devised dynamical premises, as conceived against the background of the theory of light as instantaneous mechanical impulse [as in the later portions of the Regulae]:

(1) The path independent ratio of the force of light in the two media is modelled by the ratio of 'effective' weights, which depend, of course, on the ratio of the densities of the media.[83] The 'effective' weights, moreover, are beautifully 'path independent'. The weights hang down perpendicularly from the ends of the arms regardless of the direction [294] in which the left arm, the 'arm of incidence' if you will, has been set, and regardless of the direction then assumed by the right arm, the 'arm of refraction', in order to maintain equilibrium.

(2) The conservation of the parallel component of the force of the light is modelled by the condition of equilibrium, which requires the equality of statical moments about the fulcrum.[84]

One should also note that if, as seems likely, Descartes was thinking of his premises against the background of a theory of light as instantaneous impulse or tendency to motion, then the model is particularly apt for two further reasons. Firstly, weight may be interpreted as a tendency to motion (as Descartes did indeed conceive of it as early as 1619 in the hydrostatics manuscript), and hence as a kind of impulse reiterated from moment to moment; and, secondly, weight, like a

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82 It would also illustrate the case of a 'tennis' or cannon ball whose motion is refracted away from the normal in water, as discussed later in the *Dioptrique* (AT vi. 97-8). Beeckman and Descartes might perhaps have discussed this phenomenon in 1628.

83 The only problem with Descartes' analogy of course is that greater force [effective weight] depends upon placement in a rarer medium and vice versa, thus implying a disanalogy between specific gravity and refractive 'density' of an optical medium.

84 This again is a figural modelling of the conditions of the problem, in the manner suggested by Sepper's analysis in this volume, 'The Problem of Figuration: Or How the Young Descartes Figured Things Out'.

tendency to motion or a light impulse, can be conceived to have a certain gross magnitude (measured by weighing), as well as specifiable components of 'directional magnitude'.[85]

8.0 Full Circle: Cartesian Dynamics, Optics and the Tennis Ball Model 1628-1633

Our argument has now travelled almost full circle. It began with an analysis of Descartes' dynamics, which was then used to unpack the tennis ball model and optical proofs in the Dioptrique. The reinterpretation of the Dioptrique was an important, yet secondary goal; the strategic aim was to take some bearings which could orient our reconstruction of Descartes' route to the sine law and of his struggle to explain it in mechanistic terms. The analysis of the Dioptrique uncovered Descartes' two dynamical premises and the hidden radius form of the law of refraction to which they are best adapted. These findings provided questions and points of reference around which the reconstruction was developed . We can now reverse the process, using the reconstruction of the course of Descartes' optical researches in order to throw new interpretive light on the status of his tennis ball model. We shall discover that far from being central to Descartes' physical optical research, it was really a rather contingent element, explicable by the circumstances and needs which shaped the writing of the text of the Dioptrique, and consequently that it does not reflect the trajectory of Descartes' earlier optical researches and is likely to mislead us about them.

Anyone the least familiar with the Dioptrique and who has followed the argument thus far will no doubt be wondering why Descartes chose to employ the tennis ball model in the first public exposition of his optics. In Section 3.0 we saw that the tennis ball demonstrations of the laws of optics make sense only when supplemented by a knowledge of Descartes' dynamics, which contemporary readers could only have gained from the [295] suppressed Le Monde. We were able to recognise cryptic hints of Descartes' dynamics between the lines of the Dioptrique only after familiarising ourselves with the relevant portions of Le Monde. What is more, we have discovered that kinematic tennis ball type models of light probably played no role in the long gestation of Descartes' physical optics from the 1620 fragment down to the Regulae and bent arm balance beam analogy of the late 1620s. If our reconstructions are accepted, they seem to entail that Descartes committed a miscalculation in the Dioptrique when he suddenly elected to use a kinematic model for light and almost completely neglected to provide it with an adequate and explicit dynamical rationale which could link it to his real theory of light as a mechanical impulse.

The canons of historical interpretation suggest that perhaps there is something wrong with our reconstruction if it entails such an unflattering picture of Descartes' capacities. In this section I want to avoid this conclusion by showing why Descartes himself probably believed that the tennis ball model could do an adequate job in the Dioptrique, despite certain gross limitations of which he was arguably aware. The answer resides in the demands of Descartes' theory of colour which figures prominently later in the Dioptrique and Météores. That requires the real spatial

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85 As Stevin, the stimulus for the hydrostatic manuscript, had taught with his near approach to the parallelogram of forces, mainly applied to the non-vertical components of weight. S.Stevin, *The Principle Works,* Vol. 1 (E.J.Dijksterhuis, ed.) (Amsterdam 1955) pp.183-5.

translation of balls or corpuscles, so that spin/speed ratios can account for colours; yet, you cannot have a ratio of a tendency to spin to a tendency to move. We are about to see that this problem partially explains Descartes' characteristic reticence about colour theory at the level of the real theory and natural philosophical systematics. Using tennis balls at least allowed Descartes to finesse the problem in his 1637 texts. The tennis ball model directly linked to the real theory of light, and it could bear the weight of the colour theory. Unfortunately, his colour theory and real theory of light did not cohere. Descartes, I suggest, knew this and struggled with the tensions it generated.

The first step toward grasping Descartes' rationale for the tennis ball model is to understand its wider range of functions in the Dioptrique and in the optical portions of the Météores. Thus far we have only discussed its use in the demonstration of the optical laws in the second discourse of the Dioptrique. In the Météores Descartes employed the model in a mechanistic explanation of the causes of the sensations of colours. Descartes was particularly interested in the production of spectral colours when a thin beam of light is refracted through a prism. The explanation of this phenomenon then served as the basis for the explanation of the colours of the rainbow and parhelia. These were among the first problems he addressed in 1629, when he began the work which eventually was embodied in Le Monde, the Dioptrique and the Météores.[86] One must appreciate the importance Descartes would have attached to a general solution to the problem of the (apparent) production of colours through the reflection and refraction of light.

According to Descartes, the tennis balls, whose rectilinear translation [296] models the transmission of light, may also have spin imparted to them when they collide with 'reflecting' or 'refracting' surfaces. In certain situations the spin imparted to the balls is 'nearly equal to their motion in a straight line', and no colours result. But, in other situations, what we may term the ratio of 'spin to speed (of translation)' will be increased or decreased relative to the 'normal' ratio. Such non-normal spin to speed ratios are taken to explain the triggering of sensations of colours, red in the former case, 'blue or violet' in the latter.[87]

Descartes lays the basis for this approach early in the Dioptrique in the third of a series of analogies or 'comparisons' through which he proposes to explain and illustrate those properties of light relevant to the understanding of the Dioptrique and Météores, without having to enter upon the details of his 'philosophy' (element theory, dynamics and real theory of light). The first two analogies explain properties of light travelling through uniform optical media.[88] To explain the phenomena which occur when light encounters a second medium, Descartes introduces the

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[86] To Mersenne, 8 October 1629, AT i. 23.

[87] *Météores* , AT vi. 331-32.

[88] First, he uses the analogy of the blind man's staff to illustrate the instantaneous propagation of light without the passage of any material (or immaterial) entity. The analogy clearly derives from the pen analogy used earlier in the *Regulae*. As the blind man receives from the far end of his staff only instantaneously conveyed tendencies or resistances to motion, so light rays are only lines of tendency to motion propagated instantaneously through the contiguous particles of optical media. [AT vi. 84-6] The second analogy deals with the rectilinear propagation of light rays, their propagation in infinitely many directions from a luminous point, and their ability to cross without impeding each other. Descartes' model is a vat filled with half crushed grapes and new wine. The analogy is carried out by manipulating putative lines of tendency-to-descend running from wine particles on the surface of the vat to hypothetically voided points on its bottom, a procedure clearly borrowed from the hydrostatics manuscript of 1619. [AT vi. 86-8].

tennis ball model, to which he then adds the spin/speed articulation. He describes how one may impart spin to a tennis ball by grazing or 'cutting' it obliquely with a racket, and he points out how the same thing can happen when a ball bounces obliquely off uneven surfaces. Analogously, colours are produced when rays encounter uneven reflecting surfaces. And, as smooth regular surfaces do not graze the ball, so smooth regular reflecting surfaces do not endow the reflected light with the property of causing the sensation of colours.[89]

Later, in the Météores, the explanation of the generation of spectral colours through prismatic refraction, which is fundamental to the explanation of the rainbow and parhelia, proceeds on the basis set down at the beginning of the Dioptrique. Dropping all reference to macroscopic tennis balls, Descartes boldly descends to the micro level, to those "petites boules d'une matière fort subtile", whose "action or movement" constitutes the true nature of light, as, he says, was "described" in the Dioptrique.[90] The boules, passing (or tending to pass)[91] through the pores of "terrestrial bodies", can also acquire spin in certain circumstances. When such boules pass obliquely out of the glass prism into the air, their paths are, of course, refracted, and, entering a medium which alters their force of motion, they all acquire a uniform spin in the same direction "equal to" their rectilinear motion. In this case no colours are produced. But if what we might term the 'beam' of boules is narrowed by blocking off with a shade all but a small area of exit on the refracting surface of the prism, then the boules in and near one side of the beam will have their spin/speed ratios increased above their normal amount, whilst those in or near the other side of the beam will have theirs lowered. In the former case the sensation of the colour red will be produced in observers, in the latter case "blue or violet". The alteration of the spin/speed ratios necessarily follows from the fact that the boules at the edges of the beam must graze boules at rest, nestled [297] amid the grosser particles of the shade (and of the air proper). Given their previously acquired uniform speed and sense of spin, the boules at one edge have their spin increased and those at the other edge have theirs decreased, and these respective effects also propagate inward from the edges of the beam to some distance, through the contact and interaction among the boules making up the beam.[92]

From Descartes' perspective the tennis ball model therefore works rather elegantly within the texts of the Dioptrique and Météores: in unarticulated form the model facilitates the deduction of the laws of reflection and refraction; then a simple articulation allows Descartes to explain the production of colours in these same processes. (In addition, the articulated model at least held out the promise of a general explanation of colour phenomena, through the study of the reflection and absorption of light by the varied surfaces of coloured bodies.) However, this elegance is achieved in Descartes' texts at some considerable cost, which is chargeable to his

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[89]Although he will later deal with the production of colours through refraction of light, Descartes introduces the 'spin/speed' articulation of the tennis ball model in the case of reflection [AT vi. 90-1], because it is much more easily grasped in common sense terms, and because, of course, he has not yet even shown how the simple tennis ball model can be applied to the law of reflection and then extended to the law of refraction.

[90] *Météores*, AT vi. 331.

[91] *loc. cit.* p. 332.

[92] *loc. cit.* pp.331-4.

views about the real nature of light, and hence to the coherence of the system of natural philosophy he had just created. Descartes, we shall see, was well aware of this cost.

Unfortunately for Descartes, the model for the production of colours works only on condition that the balls, whether tennis balls or boules of "subtle matter", undergo real rectilinear translation, and not merely a "tendency to motion" or "action". "Grazing" or "cutting" imparts a real spin, and can do so in the systems of interest to Descartes only as the balls pass by the grazing or cutting surfaces.[93] In such cases there can be no question of merely a "tendency to rectilinear motion", which might bear some ratio to a spin, or, even worse, to a "tendency to spin". There simply is no coherent and convincing analogy in the real theory of light for the spin of the tennis ball or boules, or for their mode of acquisition of spin.[94] The articulated tennis ball model therefore cannot be translated into the terms Descartes' real theory of light as an instantaneously propagated mechanical impulse. In this it differs from the unarticulated tennis ball model used in the proofs of the optical laws. There the model and the real theory collapse into one another, provided one attends to the crucial instant of impact with the reflecting or refracting surface, and concentrates upon the instantaneous, rule-bound alteration of the force and/or determination which occurs at that moment.[95]

Nevertheless, this difficulty need not have worried Descartes all that much in so far as he was concerned with the internal coherence and presentation of Dioptrique and Météores. Since the full details of his real theory of light and of his dynamics were not on display, because of his decision to abandon publication of Le Monde consequent upon the condemnation of Galileo, the tennis ball model could be deployed in these texts without appearing to violate the tenets of his real theory. The very absence of the full details allowed Descartes to write in the Météores of the translation of the boules, a violation of his real theory of light, but a [298] neat and consistent sequel to the (superficially) kinematical optical proofs.[96]

In Section 3.0 we in effect cast doubt upon Descartes' conceptual and literary skills when we discovered how little of the real dynamical rationale for the optical proofs is present in the Dioptrique. Now, however, we can perhaps appreciate that Descartes was cleverly adapting to the fact that Le Monde had been suppressed and the Dioptrique and Météores would therefore

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[93]At times Descartes speaks of a part of the speed of translation of a ball being converted into spin. [eg. AT vi. 90] He was no doubt thinking of everyday macroscopic analogies, such as a tennis ball appearing to lose some its incident speed upon acquiring a spin after bouncing obliquely on the ground.

[94] Descartes uses this infelicitous locution at AT vi. 333.

[95] For, as we have established above, at the moment of impact, the tennis ball (reduced to a weightless, frictionless point) behaves exactly the way a light impulse would--indeed dynamically speaking the two are identical--and the superficially kinematical aspects of the model 'momentarily' drop from view.

[96] Looking more deeply into this, one realises that at the level of the published texts the coherence of Descartes' presentation really turned on the dual character of the proofs of the optical laws: On the one hand, the tennis ball optical proofs were based on his dynamics and drew their cogency from the way they modelled instantaneous alterations of force and/or determination. Of course, their true character was only partly inscribed in the text, and for the most part had to be sought between the lines. The dynamical underpinnings were hinted at, and could be mobilised if questions arose, as occurred in the subsequent debates concerning the proofs, for example in the remarks cited above at Note 24. On the other hand, the optical proofs were presented in an overtly, if superficially, kinematical fashion. As such, they motivated and paved the way for the spin/speed articulation which would explain colours.

appear without any extended discussion of dynamics of the real theory of light. What from one perspective seems to have been an error in Descartes' presentation appears from this new perspective as a quite reasonable strategy of argument, adopted after he had decided that could not then publish Le Monde and the system it contained.

This interpretation obviously assumes that Descartes was aware of the difficulty of identifying the spin/speed model with his real theory of light, and that he made his strategic decisions on that basis. Evidence on this score can be gleaned from both of Descartes' treatises of systematic natural philosophy, Le Monde and Principia philosophiae, as well as from the Météores itself. In sum, Descartes never discussed the spin/speed explanation in detail in his systematic treatises: In Le Monde Descartes refuses to mention colour as an essential phenomenon of light and so relieves himself of the onus of having to explain colour in a manner inconsistent with the rest of his discussion.[97] His behaviour, I contend, was quite intentional. Later, in Principia philosophiae he still avoided explicit discussion of the spin/speed explanation. With one exception, all questions about the causes of colour were dealt with by referring the reader to the Dioptrique and Météores.[98]

But to say that Descartes was aware of this problem is not to suggest that it always haunted him with equal vigour. The intensity of the problem would have varied from context to context and from time to time. When, in 1644, Descartes finally published a system of mechanical natural philosophy, the problem would have loomed large and caused his evasions. But earlier, in the mid 1630s, when he was committed to suppressing Le Monde and only publishing the Discours de la méthode and its three Essais, he could well have been satisfied with the heuristic and organisational role played by the tennis ball model within the combined texts of the Dioptrique and the Météores.[99]

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97 When presenting his real theory of light in chapter fourteen of *Le Monde*, he listed twelve properties of light and explained them as arising from tendencies to motion transmitted through the spherical *boules* of his "second element". Colour is not mentioned explicitly as one of these properties; but, it is implicitly contained in the last two properties, described in terms of capacity of the "force" of a light ray to be increased or decreased "by the diverse dispositions or qualities of the matter that receives them". Descartes' 'explanation' of these properties makes no mention of colour and seems intended more to elaborate the explanation of the tenth property, refraction. As for refraction and reflection themselves, Descartes passes up the opportunity of introducing the tennis ball model (or moving *boules*), and simply refers the reader to the *Dioptrique*. [AT x. 97-103].

98 The exception occurs in an obscure corner of the final part of the French version of the treatise [*Princ*. IV 131, AT ixB. 274], where Descartes explains the properties of coloured glass. Leaving aside this limited and late passage, which is Descartes' and/or Picot's afterthought, we see that Descartes steadfastly refused to introduce the spin/speed model into his systematic work. And the likely reason for this is that the model cannot be made to agree with his real theory of light as a tendency to motion. Further evidence of Descartes' awareness of the problem and its intractability may be found in the *Météores*. In the passages discussed above [Note 94 above], Descartes twice writes of the *boules* "tendency" to move and "tendency" to spin. Evidently he was caught between the content and the grammar of his real theory on the one hand, and the mechanical rationale of his spin/speed model on the other. At this point of tension his discourse falters and wavers, despite the fact that here in the published text of 1637 he could (for the foreseeable future) have gotten away with the consistent pretence that light consists in the translation of *boules*.

99 The little we know about the course of composition of the *Dioptrique* tends to confirm this picture of a Descartes reluctantly satisfied, for the time being, with the tennis ball model in the publications of 1637. The *Dioptrique* is first mentioned in a letter to Mersenne of 25 November 1630 [AT i. 179], over a year after the problems of parhelia and the rainbow had first stimulated his work on a system of corpuscular-mechanical natural philosophy. Descartes writes that he wishes to insert into the *Dioptrique* an explanation of "the nature of light and colours", a task which has held him up for six months. This will virtually turn the *Dioptrique* into a "system of physics", an "abridgment of *Le Monde*", and so acquit him of

9.0 Conclusion

In conclusion we briefly note two further developments related to the reconstruction offered thus far. Taken in conjunction with the reconstruction, they open a rather wide perspective on the interrelation of Descartes' agendas in corpuscular mechanism, geometrical optics, physico-mathematics and methodology between 1618 and 1637. The first issue [9.1] deals with Descartes' attempt in 1626-8 to weave a methodological [299] tale of discovery around his experience in optics over the previous several years. Properly deciphered, Descartes' tale bears witness to some of the complexities, quandaries and pitfalls of his optical work, as revealed by our reconstruction. The second issue [9.2] relates to Descartes' exploitation of his dynamical rationalisation of the law of refraction in his attempt to frame general laws of nature in his first systematic natural philosophical treatise, Le Monde.

9.1 Descartes Mythologises his Experience as an Optician: Method and Optics in **Regulae** 8

In rule 8 of the Regulae Descartes describes, in a carefully chosen subjunctive mood, how the law of refraction, the anaclastic curve, and the physical explanation of refraction might all have been discovered by using his method. This part of the rule dates from 1626-28: it obviously post-dates the discovery of the law of refraction, the first elaboration of lens theory and the initial attempts to provide a physical rationalisation of the law.[100]

Descartes' story in Rule 8 of the methodological investigation of the anaclastic and other problems unsurprisingly contains an initial analysis and a concluding, demonstrative synthesis,

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his promise to Mersenne, made in April 1630, to finish the system within three years. He adds that if the reception of the *Dioptrique* shows he can persuade people of the truth, then he will proceed to complete his treatise on metaphysics begun earlier in 1629.

Two main difficulties seem to have been haunting Descartes. First, the explanation of the nature of colour had proven a most difficult proposition. One suspects this was not only due to the intricacies of his articulated tennis ball model, but also because of the dawning realisation that it bore no convincing analogy in the real theory of the "nature of light". Second, Descartes was clearly still undecided about how much material from his emerging system of corpuscular-mechanism should or could appear in the *Dioptrique* . In the letter he toys with the idea of adding a section on the true nature of light and colour, and thus implying that he already possessed some version of the model-based presentation he later published. Again, part of his hesitation and indecision may have related to the difficulty of linking the spin/speed articulation to his real theory of light. In January 1632 he sent to Golius what he termed "the first portion of the *Dioptrique*, dealing with "refractions without touching upon the rest of philosophy". [AT i. 235] This, too, tends to indicate that Descartes still contemplated publishing in the *Dioptrique* more of his dynamics and real theory of light than we find in the publication of 1637. If so, he was probably then still facing the problem of the relevance of the spin/speed articulation to the real theory.

In the end Descartes' problems were solved on a pragmatic basis, motivated by external events. When he learned of the condemnation of Galileo and decided to withhold *Le Monde* from publication, he reorganised his publication program, producing within three years the *Discours* and three *Essais* in the form with which we are now familiar. The reorganisation allowed him to design the *Dioptrique* and the optical portions allotted to the *Météores* around the tennis ball model, without having to face up to the problem of whether the model in its articulated form could represent aspects of the real theory of light. In this respect, perhaps, he came to see the demise of *Le Monde* as something less than a complete disaster, since it allowed him to resolve the problem of presenting and justifying his optical achievements. Again, from this perspective, he may well have viewed the tennis ball model as a qualified success.

[100] It can also be shown that it is the first of the passages added to the *Regulae* in Paris and leads directly to the core of the third stratum of the text. Cf. above Note 76 and Schuster, *op. cit.* (1980), pp.58-9.

and follows the general lines of the method doctrine extractable from the early Regulae.[101] The analysis consists in the discovery of that ordered series of questions upon the solution of which the resolution of the anaclastic problem ultimately depends. If, Descartes begins, one were going to search for the anaclastic curve using the method, the first step would be to see that the solution depends upon first discovering the law of refraction, 'the relation which the angles of refraction bear to the angles of incidence'. At this point, Descartes observes, a mathematician would have to give up the search, for all he can do is assume some relation and work out the consequences. Further analysis shows that the problem of the law of refraction in turn depends upon knowledge of 'physics' as well; for the relation between the angles of refraction and incidence depends in some way upon the manner in which light passes through media. But the answer to that question would be seen to depend on them more general issue of 'what is the action of light', and the answer to that question would be seen to depend in turn upon the answer to the ultimate question in this series, 'what is a natural power?' One would have to determine, by a 'mental intuition' what this 'absolute nature' is. This would be the last step in the analysis and the first in the deductive synthesis. Unfortunately, Descartes does not inform us as to the content of this 'intuition'; but, we can presume that light and all other natural 'powers' are to be explained mechanically, by corpuscular motion, impact or tendency to motion. In any case, having discovered this by 'intuition' one would have to pursue the rest of the synthesis by proceeding back along the chain of questions, deducing the more relative natures from the less relative ones. However, our deduction might stall at some [300] point, for example at the step of trying to deduce the nature of light from the nature of natural powers in general. In such cases one would have to proceed by 'analogy'. The investigator must 'enumerate all the other natural powers, in order that the knowledge of some other of them may help him, at least by analogy...to understand this one.' Again, we are not told anything more here about the analogies, but we are acquainted with one of Descartes' favourites from this period, the bent arm balance he was soon to expound to Beeckman.[102] Allowing for such occasional and unpredictable recourse to analogy, the synthesis would ultimately lead from a theory of natural powers, via a theory of light to a deduction (and explanation) of the law of refraction, and thence to a theory of lenses.

As one would expect, Descartes' methodological tale about how he 'could have done optics' bears no relation to the complex trails of research reconstructed in this paper. Elsewhere I have argued what when such tales of particular researches are woven out of the discursive cloth of a grand doctrine of method (Descartes' or anyone else's) some characteristic effects follow. On the one hand, the 'thick', sui generis conceptual and procedural density of the field of inquiry in question is necessarily suppressed and lost from view. This entails that the method story really cannot accurately describe any actual or even possible course of genuine practice in that field; it necessarily structurally mystifies the dynamics of knowledge production and evaluation in that field. On the other hand, the little methodological story bears structural similarities to other such stories which can be generated within the same method discourse. To the methodologist,

101 AT x 393-5.

102 Perhaps he also had in mind other analogies for the action and refraction of light, for example, a rudimentary and unarticulated kinematic model, a tennis ball model; we simply do not know.

therefore, the story seems to be true, or at least possibly true, and his belief in the unity and efficacy of his method are enhanced by this further 'evidence' of its value.[103]

Given all this, Descartes' story is to be construed as a rationalisation of the complex and sometimes abortive course of his researches; as an attempt to show that since the results could in principle have been produced by using method, they should enjoy certain epistemological and methodological accolades. After all, our reconstruction indicates that Descartes' lived experience of 'being an optician and physico-mathematician' had not been entirely happy or tidy. On the one hand there was the tortuous and none too orderly course of his researches, which had at long last produced some results of note. On the other hand, despite or indeed because of these results, he confronted a confusing array of resources, theories, programs and commitments, the disorderly residues of eight or nine years of endeavour. Among these we can number (a) a law of refraction discovered using the possibly discredited image locating principle; (b) an unarticulated theory of light as mechanical impulse; (c) two dynamical premises read out of (a) in the light of (b); (d) a body of lens theory in the process of refinement and alteration; and, (e) at least one analogy for the deduction of (a) from (c). Upon this chaos of personal history and conceptual baggage the method tale imposes a double order. [301] There is the diachronic order of an ideal course and flow of research, and conflated with, or contained within, that diachronic order is a logical/explanatory order, revealing the deductive relations holding amongst his theories and principles.[104]

This interpretation further allows us to make sense of two otherwise peculiar aspects of Descartes' tale: (1) his appeal to the use of analogies, and (2) his reticence about the nature of light and natural powers in general.

(1) It would appear likely that Descartes introduced an analogy when moving to the step of deducing the nature of light because he simply did not quite know what else to say about the issue. At the time he possessed an unarticulated theory of light as mechanical impulse, two rough hewn premises read from the Mydorge diagram, and the balance analogy. The theory of light was not closely articulated to a system of mechanistic natural philosophy; he simply did not have one. Similarly, the dynamical premises were not yet part of a system of dynamics, forming part of that larger system of natural philosophy. Leaving aside the Mydorge diagram, read 'physico-mathematically', the only thing holding together the theory of light and the premises was the balance analogy: it modelled light as an impulse and it modelled the two premises; and, it could

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103 Schuster, *op. cit.* (1986, 1992).

104 Like a myth viewed in a Lévi-Straussian perspective, the method discourse provides a structure which imposes order on this jumble of biographical and in part contradictory conceptual meaning-tokens, by means of a narrative of particular events and actions which is at bottom yet another instance of his core myth of method. C. Lévi-Strauss, *Structural Anthropology*, trans. C.Jacobson and B.G.Schoepf, (Norwich 1972), pp.216, 224. Alternatively, if one prefers Roland Barthes' view of myth, we might say Descartes' account amounts to a none too convincing rational reconstruction, motivated by a host of personal, philosophical and ideological concerns, and posing as a true story of the discovery. R.Barthes, 'Myth Today' in *Mythologies*, trans. A.Lavers, (St. Albans,1973), pp.109-59.

be used to explain/deduce the law of refraction. In rule 8 Descartes is probably simply echoing this as yet unsystematised and unresolved state of affairs.[105]

(2) A similar sort of explanation applies to the question of why Descartes **was** coy and reticent about the 'nature of natural powers' in general and about the 'nature of light' in particular. We may surmise that Descartes preferred to be non-committal, because he had not yet committed himself to articulated theories on either topic. The beauty of the method tale is that it can accommodate this vagueness and hide it by enfolding it in 'orderliness'. Certainly he had a theory of light, a mechanistic outlook on nature and premises from which to deduce the law; but none of this was settled or elaborated. Since he had to hand a workable analogy for deducing the law and modelling light, it was better in such circumstances to inject into the tale a sub-discourse on the use of analogy, than it was to imply that any of his currently unsettled ideas might have the status of products of 'intellectual intuition' or 'deduction' therefrom.

9.2 The Optical Exemplar for Descartes Laws of Dynamics?

As we have seen, Descartes' mature dynamics distinguishes between the absolute quantity of a force of motion, and its directional manifestations, expressed in laws 1 and law 3 in Le Monde. We may now suggest that these principles derived from a further generalisation of his original reading of the Mydorge diagram. Descartes first read the diagram for some basic principles of physical optics, assumptions about the quantity and directional quantity of the force of light. But what about the laws of nature [302] which he had to construct after 1629, when he began to write Le Monde? How better to base the laws of nature than to use as an exemplar the dynamical principles revealed by successful optical research: Light, after all is just an impulse, so its behaviour clearly reveals the basic dynamics of forces and determinations. Descartes would have had every reason to be confident that his optical exemplar was well chosen and correctly analysed, and so he would have had every reason to think that his dynamics of force and determination could be premised upon his having cracked the code of the physics of refraction.

If the reconstruction offered in this paper carr**ies** some degree of plausibility, it brings into relief complex diachronic and conceptual relations amongst Descartes' early enterprises and demonstrates the centrality of geometrical optics, and optical concerns in general, in the evolution and cross fertilisation of his agendas in physico-mathematics and corpuscular-mechanism. It also suggests, consistent with Sepper's contentions elsewhere in this volume, something about the manner in which Descartes practiced physico-mathematics and 'figured out' solutions to its problems. There was more to the young Descartes' projects than has yet been clarified in the literature, despite the fact that it was from these complex and intertwined endeavours that there emerged his first seminal texts, the Regulae, Le Monde, the Discours and the Essais. [303]

105 His only alternative would have been to begin discoursing about the Mydorge diagram, physico-mathematics, how to read Kepler, as well as admitting to having used the now superseded traditional image location rule etc, a most unmethodical undertaking, if our reconstructions are to be believed.