'Waterworld': Descartes' Vortical Celestial Mechanics

—A Gambit in the Natural Philosophical Contest of the Early Seventeenth Century

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1. Introduction—Uncommon Vortices

Nearly fifty years ago, Thomas Kuhn, in his best selling and often reprinted, The Copernican Revolution, said this of Descartes' vortex universe: the 'vision was inspired'; the 'scope tremendous'; but 'the amount of critical thinking devoted to any of its parts was negligibly small'.[1] Typically more pointedly and poetically, Gaston Bachelard had in 1938 condemned Descartes' plenist universe, including the vortex mechanics, as the 'metaphysics of the sponge', an exemplary 'pre-scientific' monstrosity, in other words, the sub-scientific progeny of cancerous metaphor and baroque ego projection.[2] Other more mundane brush offs could also be cited.

Of course, Descartes' vortices do not posses for us the straight, presentist scientificity of Newtonian mechanics, but they have an internal density and complex genealogy—in Descartes life work, and later, as Aiton has shown.[3] They are deserving of study if we are to understand the structure and dynamics of natural knowing in the early modern period. We can display how the vortices were intellectually constructed, and why. This I intend to do, concentrating on Descartes' Le Monde, The World or a Treatise of Light, his first systematic statement of the mechanical philosophy, finished in 1633 but unpublished in his lifetime. In saying this I in no way wish to imply that I introduced Bachelard and Kuhn above as mere straw men. These two historian/philosophers of science initially most influenced my understanding of the dynamics of seventeenth and eighteenth century natural philosophy. I have argued elsewhere that Kuhn and Bachelard indeed misunderstood the nature of that natural philosophy and the contestations over it—taking it as the necessary but pre-scientific backcloth to the temporally splayed crystallisation of a heterogeneous set of new 'real' sciences. However, as I have also claimed, that is less important than the fact that their speculations prompted more positive modelling [35] by historians of early modern natural philosophy, its nature, dynamics and trajectory.[4]

This paper is a modest essay in that very problematic. It focuses on a small but essential corner of Descartes' natural philosophical project. It will attempt to show the natural philosophical seriousness of Descartes' vortex universe as an intellectually constructed object and as a strategic gambit. I shall try to place Descartes' earliest celestial mechanics in relation to his manoeuvring in the natural philosophical contestation of his time.[5] This will involve exposing some its minute design and biographical trajectory, thereby also relating it to similar aspirations and strategies of contemporary actors. They, including Descartes, were attempting to displace Aristotelianism, install some version of Copernicanism, and create alternative hegemonic natural philosophical syntheses. For many, Descartes included, such projects battened upon the achievements and promise of what Scholastics termed the mixed mathematical sciences, but which some of our struggling innovators occasionally termed 'physico-mathematical' disciplines, in particular hydrostatics, optics and mechanics. We shall need to say more below about such natural philosophical play upon the subordinate mathematical disciplines.

The argument of the paper will unfold as follows: Section 4 contains the fulcrum of the argument, an extended intellectual reconstruction of the inner toils of the vortex mechanics of Le Monde. Sections 5, 6, and 7 step back in time to trace three key moments in the genealogy of the vortex mechanics in the early work of Descartes, starting in 1619, and focusing, perhaps surprisingly, on his activities in hydrostatics and physical and geometrical optics, and his relations, spanning a decade, with his mentor in corpuscular-mechanism, Isaac Beeckman. The genealogy helps make sense of the already exposed anatomy of the vortex mechanics. Section 8 returns to 1633 and canvasses one small example of the coherence and power of the vortex mechanics. Finally, in Section 9 the vortex mechanics is inserted into the context of the natural philosophical contest of Descartes' generation, with particular reference to Beeckman and Kepler. It also unveils the motivation for use of the odd term 'Waterworld' in the title, an outcome prepared by the genealogical and anatomical dimensions of the argument. But before any of this occurs, there are two items of preparation. Section 2 will explicate the key notions of mixed mathematical science and 'physico-mathematics', whilst Section 3 will introduce the 'dynamics' of Descartes, the doctrine of causation, dealing with motions and tendencies to motion, through which he intended to 'run' the machinery of his vortex world. The evolution of this dynamics will be glimpsed throughout the genealogical Sections 5, 6 and 7, as well, being part of the larger story of how the vortex mechanics became conceptually possible and strategically necessary. [36]

2. Mixed Mathematical Sciences and Physico-Mathematics

As noted in the Introduction, Descartes' vortex mechanics emerged within a natural philosophical agenda which, in very general terms, he shared with other key anti-Aristotelian natural philosophical innovators of his generation: to displace Aristotelianism, install some version of Copernicanism, and create alternative hegemonic natural philosophical syntheses. For these innovators, Descartes included, such projects were premised on the exploitation of the achievements and promise of the mixed mathematical sciences, in particular hydrostatics, optics and mechanics. Because the competition to develop the mixed mathematical sciences and exploit them in the contest for natural philosophical dominance plays such an important role in Descartes' case, as well as that of others, we need to consider briefly what was happening and what was at stake in this domain in the generation of Descartes.

The term 'mixed mathematics' belonged to Aristotelianism. It referred to a group of disciplines intermediate between natural philosophy and mathematics. A natural philosophical account of something was an explanation in terms of matter and cause, and for Aristotle, mathematics could not do that. This meant that the mixed mathematical sciences, such as optics, mechanics, astronomy or music theory, used mathematics not in an explanatory way, but merely to represent physical things and processes mathematically. So in geometrical optics, one used geometry, representing light as light rays—this might be useful but did not get at the underlying natural philosophical questions: "the physical nature of light" and "the causes of optical phenomena".

The question of the relation between mixed mathematics, on the one hand, and the 'superior', explanatory, discipline of natural philosophy, on the other hand, became extremely vexed in the generations around 1600. Strict Aristotelians did not grant any natural philosophical relevance to the findings of the mixed mathematical sciences; more avant garde Aristotelians such as some Jesuits, wanted to start extracting some natural philosophical juice out of the ripe fruit of mixed mathematical research discoveries.

Descartes and his mentor Isaac Beeckman, and others as well, used an alternative, more provocative term, 'physico-mathematics', which was gaining some prominence at the time. It signalled a more radical approach to the natural philosophical legitimacy of the mixed mathematical fields. As we shall see, Descartes and Beeckman went even further: they didn't mean mathematical-physical disciplines subordinate to natural philosophy, especially Aristotle's natural philosophy, but a new realm of corpuscular-mechanical natural philosophy, in which the old mixed mathematical fields are explained in corpuscular-mechanical terms and therefore are not subordinate to, but are proper domains of, the new natural philosophy. They were meant to become areas in which could occur true natural philosophical explanation in terms of matter (corpuscles) and cause (the motion, impact and arrangement of corpuscles). Conversely, it meant for Descartes and Beeckman that novel findings in mixed mathematical sciences directly bespoke new insights into the realm of corpuscular-mechanical explanation. All this may seem to us just so much late Scholastic intellectual quibbling, waiting be brushed away with the advent of quite modern mechanics and celestial mechanics just a bit later in the Scientific Revolution. This would be to [37] miss the point: these matters were explosive and challenging issues for contemporaries, and these were the struggles though which some of them, Descartes and Kepler especially, paved the way for those very turns in the Scientific Revolution that the Whig and the populariser are so happy to applaud out of context.[6]

3. Cartesian dynamics—the causal register of corpuscular-mechanism

In the period of the Scientific Revolution 'natural philosophy' as a generic term denoted that common field of endeavour within which particular schools and varieties of systems contended: not only various species of neo-Scholastic Aristotelianism in the universities, but also natural philosophies of neo-Platonist, Stoic and qualitative atomist bent, to which in the generation of Descartes, Beeckman, Mersenne and Gassendi, we can of course add the genus 'corpuscular-mechanist'. Now, in broad terms the scope of 'natural philosophising' involved the identification of what causes material bodies to behave in particular ways. This was understood to be the case whether, as in Aristotelianism, natural processes were explained primarily on the basis of causes identified with the nature or essence of the matter in question, or, as in neo-Platonic natural philosophies, brute matter was worked upon from the outside by various types of non-material causal agents. Theorising about matter and an associated 'causal register' was traditionally taken as constitutive of natural philosophy. Whatever disputes there might have been amongst Platonists, Aristotelians, Stoics, and atomists, there was consensus on what kind of theory provided the ultimate explanation of macroscopic physical phenomena, namely a theory of matter and causation.

Descartes was no exception to this and we may characterise his natural philosophy as concerned with the nature and 'mechanical' properties of microscopic corpuscles and a causal discourse, consisting of a theory of motion and impact, explicated in particular through key concepts of the 'force of motion' and 'tendencies to motion'. It is this causal register within Descartes' natural philosophical discourse which scholars increasingly term his 'dynamics'. Descartes' vortex theory (and his celestial optics as well) depended upon this dynamics. If we do not take his dynamics seriously, we cannot take the vortex theory seriously. Later in Sections 5 and 6 we shall examine some aspects of the genealogy of the dynamics between 1619 and the late 1620s, leading to its initial systematisation in Le Monde. But for the moment, before examining the vortex theory, we need to survey the fundamentals of this dynamics. [38]

In Descartes' Le Monde, the behaviour of Descartes' micro-particles is governed by a carefully articulated theory of dynamics. Descartes' dynamics of micro-particles had nothing to do with the mathematical treatment of velocities, accelerations, masses and forces. Rather it was concerned with accounting for the motion, collision and tendency to motion of corpuscles. Descartes held that bodies in motion, or tending to motion, are characterised from moment to moment by the possession of two sorts of dynamical quantity: (1) the absolute quantity of the 'force of motion'—conserved in the universe according to Le Monde's first rule of nature, and (2) the directional modes of that quantity of force, the directional components along which the force or parts of the force act, introduced in Le Monde's third rule of nature.[7] These Descartes termed actions, tendencies, or most often determinations.[8] Such are the central tenets underlying Descartes' dynamics. [39]

As corpuscles undergo instantaneous collisions with each other, their quantities of force of motion and determinations are adjusted according to certain universal laws of nature, rules of collision. Therefore Descartes' analysis focuses on instantaneous tendencies to motion, rather than finite translations in space and time. Indeed, Descartes offers a metaphysical account of translation which dissolves it into a series of inclinations to motion exercised in consecutive instants of time at consecutive points in space. Whilst the rudiments of this dynamics of instantaneously exerted forces and determinations dates back to Descartes' earliest work, as we shall shortly see, it was first systematically articulated in Le Monde.[9]

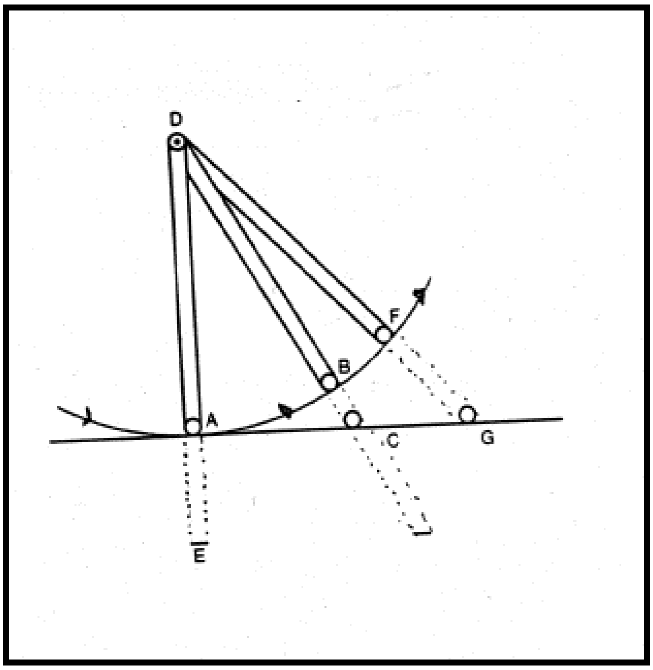

Figure 1. After Descartes, Le Monde, AT XI p.45 and p.85

Descartes' exemplar in Le Monde for applying these concepts to celestial mechanics is the dynamics of a stone rotated in a sling. **[fig.1]** Descartes analyses the dynamical condition of the stone at the precise instant that it passes point A. The [40] instantaneously exerted force of motion of the stone is directed along the tangent AG. If the stone were released and no other hindrances affected its trajectory, it would move along ACG at a uniform speed reflective of the conservation of its quantity of force of motion.[10] However, the sling continuously constrains the privileged, principal determination of the stone and, acting over time, deflects its motion along the circle AF.[11] Descartes considers that the principal determination along AC can be divide into two components : one is a "circular" determination along ABF; the other a centrifugal determination along AE. For present purposes, let us ignore the curious circular tendency. To discuss it would lead us further than we need to go into Descartes' manner of treating circular motion.[12] What Descartes is trying to do is decompose the principal determination into two components: one along AE completely opposed and hindered by the sling--so no actual centrifugal translation can occur--only a tendency to centrifugal motion; the other along the circle, which is as he says, "that part of the tendency along AC which the sling does not hinder".[13] Hence it manifests itself as actual translation. The choice of components of determination is dictated by the configuration of mechanical constraints on the system.

4. Locking and Extruding—Principles of the vortex celestial mechanics

We turn now to a synthetic recounting of the vortex mechanics of Le Monde, the genealogy of which will be examined in following sections. It is important to note precisely what my interpretive strategy is and what is not, as well as how that strategy subserves the aims of this essay, rather than some aims that might erroneously be attributed to my efforts here.

To be brutally frank, Descartes does not communicate well to the reader in the sections of Le Monde dedicated to the theory of vortices. The text, of course, is incomplete, unpolished and remained unpublished in his lifetime. Indeed, as we shall note in a couple of instances below, Descartes hardly helped his cause by his adoption of a commonsensical, *honnête homme* style. His appeal to commonly experienced analogies and observations—without explicating their limitations or precise modes of articulating to his underlying concepts and theories—tends to [41] swamp and confuse his message. But, and this is the key point, Descartes arguably did possess a coherent and well thought out theory of vortices, of which the surviving text of Le Monde is a rather poor representation. It is, however, a representation that can lead the hermeneut to that underlying theory, provided three conditions of reading and analysis are fulfilled: [1] One must attend constructively to the likely trajectory of Descartes' work and struggle in natural philosophy and the mixed mathematical sciences in the decade or so leading up to the composition of Le Monde; [2] One must probe behind the breezy style of presentation and appeal to easy if somewhat misleading analogies in Le Monde, and interpret the text charitably in the search for deep and coherent theorising, consistent with and evolving out of the material studied in [1]; Finally [3] one must be willing to use the much more systematically and coherently developed explication of vortex mechanics in the Principles of Philosophy as an heuristic guide to what Descartes might possibly have been entertaining in Le Monde (without falling into a vulgar retrospective Whiggism). In many ways, therefore, this reading of Le Monde for a strong and complex underlying theorisation rebounds upon our sense of the text itself, perhaps lending it the coloration of a more private, even solipsistic, document, a bit akin to those sets of working notes and drafts that scaffold our own public utterances without ever seeing the light of day.

Now, the aim of such a reading is not to conclude that Le Monde 'really' teaches such a coherent theory of vortices which later seventeenth century readers, and modern historians of science have, through some cognitive shortcoming, 'failed' to see. Nor is the aim to blame Descartes for failing to express what he had so systematically conceptualised. Such points are irrelevant in regard to my aims here. I aim, rather, to try to capture, via such a reconstruction, what arguably was the state of theorising that Descartes had reached about vortices at the end of almost fifteen years of work in physico-mathematics—a theory that lurks below the surface of Le Monde, but is recoverable from it. We shall see that Descartes' underlying theory was subtle and complex, reflecting upon and exploiting a sequence of technical achievements in physico-mathematics as well as his own lived experience as an increasingly mature, and competitive, player in the struggle to forge a new natural philosophy embodying Copernican realism. In the latter sense, Descartes' underlying conceptualisation was one instance of a more widely pursued problematic, worth studying as part of a mapping of other natural philosophical initiatives and aspirations of similar kind and intended scope—as in the work of Beeckman, Kepler, Gassendi, and Mersenne.

To all this one further and more pragmatic condition has to be added, as far as the account of the vortex mechanics offered here is concerned. My presentation will be synthetic and declarative, there being no space here to offer the more analytical and textual critical account of how Descartes' theory has been teased out of the text; and exactly how textual juxtapositions and interpretations, as well as judicious appeals to the Principles, can be used to clarify his analogies, reorganise his diffuse and confusing order of presentation, and explicate certain half articulated points and [42] claims.[14] I shall, however, at various points indicate in footnotes the degree and type of interpretative work/reconstruction involved in presenting particular concepts and representations. Amongst the concepts and representations I shall use: [1] Some arguably derive quite literally from the text of Le Monde. [2] Some arguably express Descartes' theoretical intentions in ways he did not quite accomplish in the text. [3] Some systematise or clarify concepts confusedly presented in Le Monde (but often better expressed later in the Principia) in a charitable attempt to elicit a coherent theory. [4] Some are novel, my own interpretive inventions, advanced again in a charitable attempt to elicit a coherent theory from Descartes' text. Arguably, they could have been constructed by Descartes himself or a contemporary, but were not to my knowledge. [5] Some representations and concepts correct misleading implications of some of Descartes' analogies in the interest of charitably supporting our vision of his underlying theory, and separating off misleading but understandable implications that have been or could be read into his surface analogies.[15] With all these caveats, let us now begin the explication. [43]

People often take the celestial mechanics on its most superficial level, as if it was just a historical holding action waiting for Newton: **[Fig. 2]**

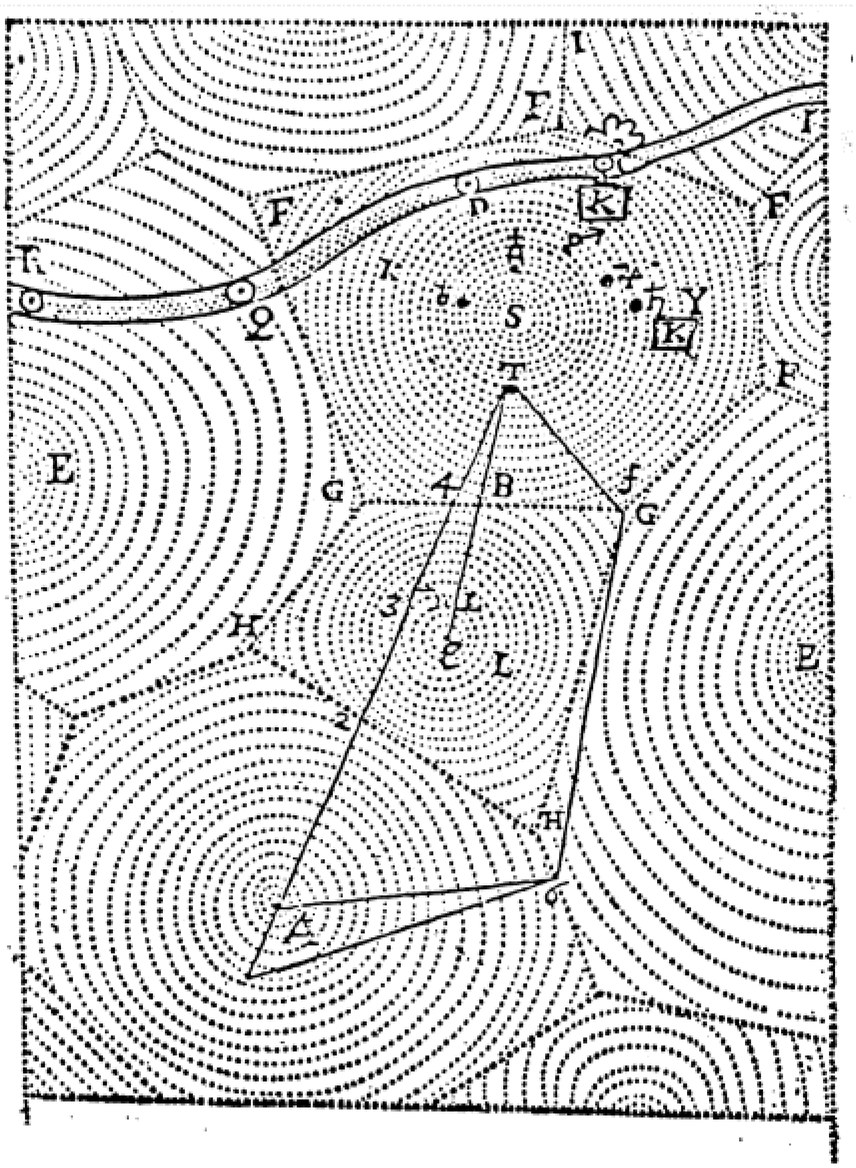

Figure 2. Descartes, Le Monde, AT XI p.54

Descartes imagined a whirlpools or vortices of second element, rotating around their respective central [44] stars to sweep along their planets like boats in a strong current.[16] In fact the swishing along of the planets in the vortex was the least of his concerns. He thought that the mere existence of a whirlpool of second element accounted for the orbital movement. What interested him was why the planets maintain relatively stable celestial distances and different distances; and why comets do what he imagined them to do, that is, continually oscillate between vortices, spiralling in toward the central star of one vortex, up to a specific, theoretically given radial distance, and then spiralling out again into a neighboring vortex, up to a similar theoretically given minimum radial distance from its central star, and so on.[17]

The overall condition for stability of the vortex is that there be a uniform and continuous increase in the centrifugal tendency of the particles making up the vortex as one goes away from the center.[18] Now, according to Descartes' dynamics, discussed above, centrifugal tendency is proportional to the force of motion in the tangent direction, and force of motion is measured by quantity of matter times speed, or more technically, quantity of matter and the instantaneously exercised principal tendency to motion. Descartes wants to specify how size and speed of the particles of the vortex vary with distance from the centre. He does this twice and we shall need to attend closely to both moments in his exposition.

First Descartes describes the speed/size distribution of the particles making up the vortex in the earliest stages of vortex formation, prior to the production of his three types of stable particle, or elements, and hence prior to the formation of the sun, which, of course, is made up entirely of the highly agitated particles of the 'first element'—a critical moment in the theory as we shall shortly see. So, Descartes tells us that in this first, very early stage, as the vortex settled out of the original chaos, the larger corpuscles were, of course, harder to move, so there was a tendency for the smaller ones to acquire higher speeds more easily. Accordingly, in these early stages, the size of particles decreased and their speed increased from the center out.[19] But the speed of the particles increased proportionately faster, so that force of motion increased continuously. In **Fig 3** we see Descartes' first declared distribution of size and speed of the particles making up the vortex in the period before the formation of the three elements and the emergence of a star in the center of the vortex: Force of motion constantly rises, as does speed, while size decreases proportionately less than speed.[20] [45]

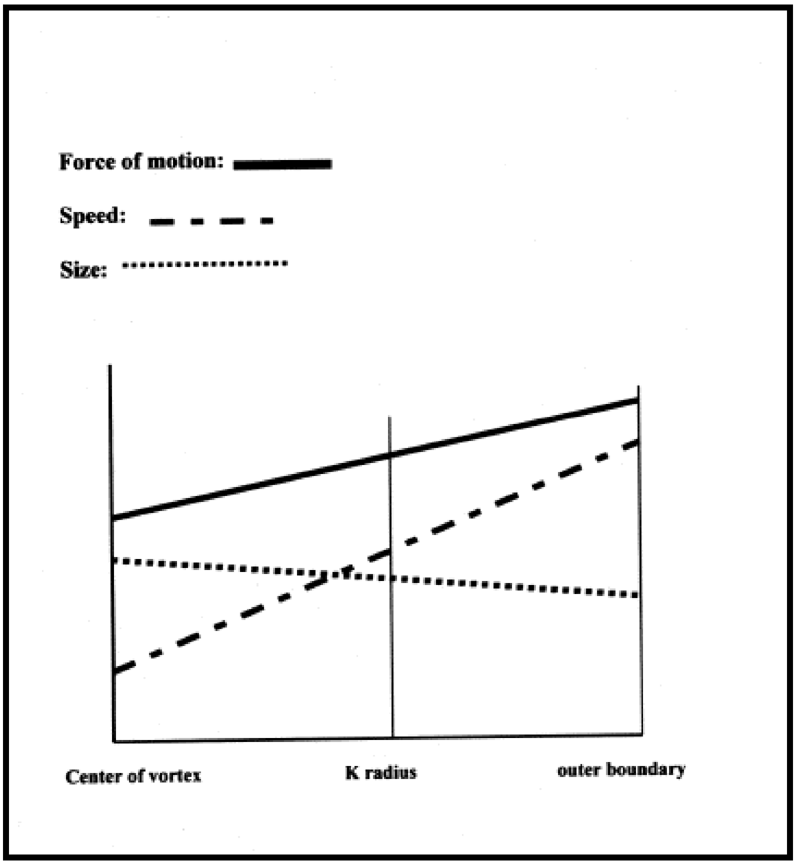

Figure 3. Size, Speed and Force of Motion Distribution of Particles of 2nd Element, Prior to Existence of Central Star

Descartes' second description of the speed/size distribution of the particles making up the vortex applies to the period after the formation of the three elements. [21] Descartes explains that as the vortex rotates in its first stage, the particles [46] collide with one another, breaking off their rough angles and points. These cosmic scrapings form the first matter. Much of the first matter is forced to the center of the vortex while the remainder fills the interstices left between the particles of the vortex. The latter particles, smoothed and polished by this process, become the spherical <u>boules</u> of the second element. Grosser particles of third matter are assumed to have existed all along. The first matter at the center of the vortex is highly agitated and forms "perfectly liquid and subtle round bodies", that is, stars, including the sun at the center of our vortex.[22]. It is the sun's presence in the center of the vortex that alters the first distribution of size and speed of particles in the vortex. This is absolutely crucial to the final theory, for the star's disturbing effect on the original size/speed distribution produces a second, quite different stable distribution of size and speed of the vortex particles, and it is this second distribution that allows the planets to maintain stable orbits.

The sun is made of up the most agitated particles of first element; their agitation communicates extra motion to parts of the vortex near the surface of the sun; that is to those spheres of second element in the vortex lying near the sun. This increment of agitation decreases with distance from the sun's surface and vanishes to nothing at a certain radius, labelled by Descartes in Figure 2 as K.[23] In **Fig 4** we represent Descartes' conception of the solar disturbance and its decrease with distance up to radius K. [47] The solar effect alters the original size and speed distribution of the spheres of second element in the vortex, below the K layer.[24]

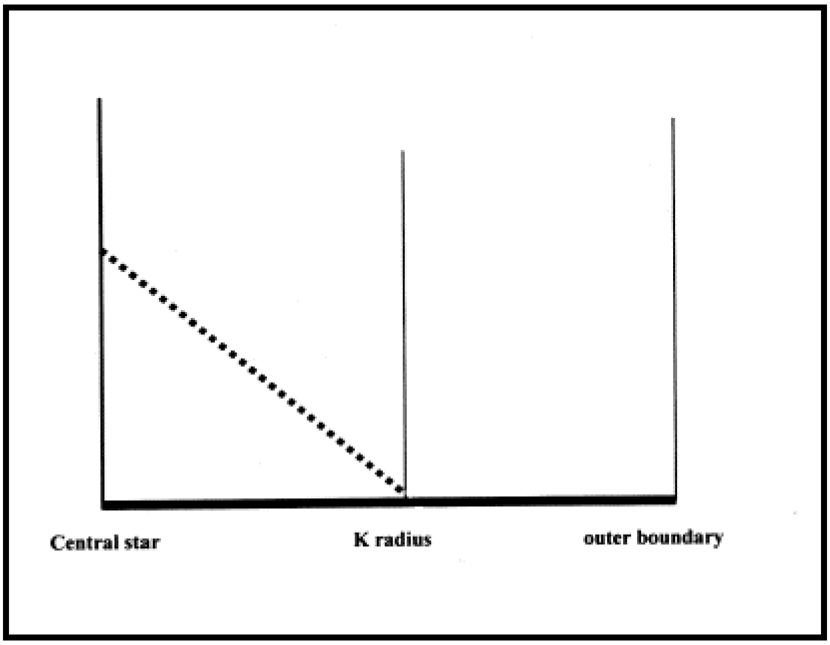

Figure 4. Agitation Due To Existence Of Central Star

We now have greater corpuscular speeds close to the sun than in the pre-sun situation. But the force-stability principle, of course, still holds, so the overall size/speed distribution must change, below the K layer.[25] Descartes description of this situation may be represented in **fig 5.** [48]

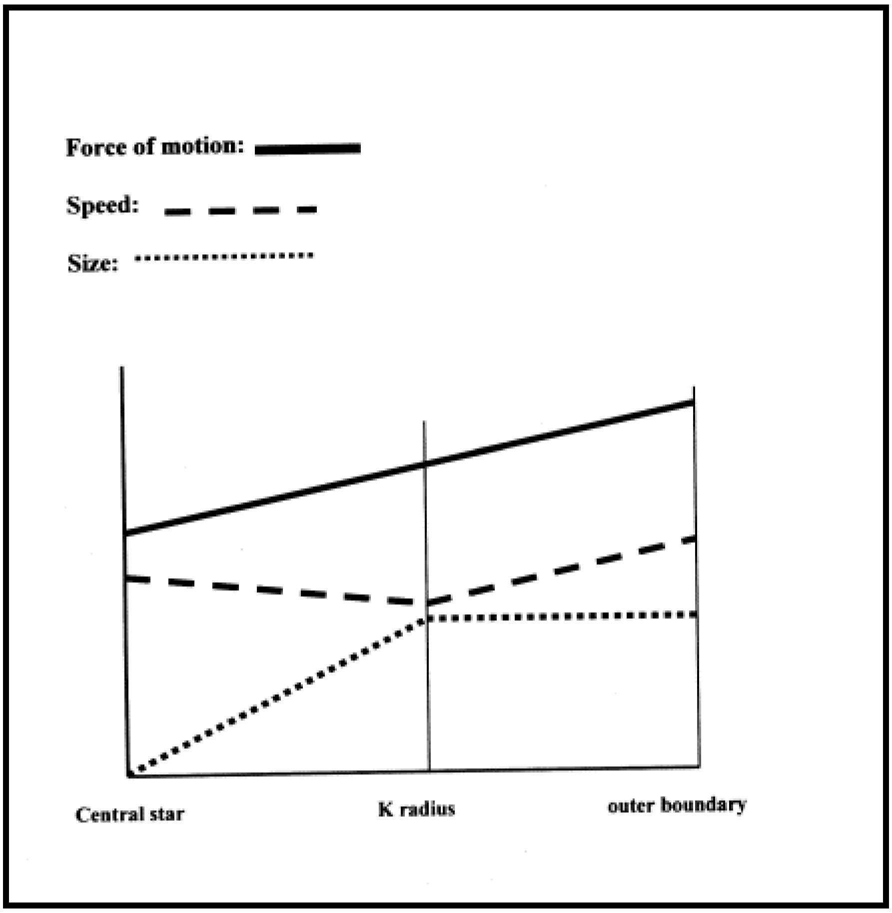

Figure 5. Size, Speed and Force of Motion Distribution Of Particles Of 2nd Element, In A Solar Vortex

In the solar vortex as one moves away from the sun the agitation (speed) of the boules decreases, reaching a minimum at the distance K (where Descartes will locate the planet Saturn). From K outward to the boundary of the vortex the agitation increases again. The size of the boules increases from a minimum near the sun to K; and from K outward the size remains constant or perhaps diminishes a little. From the sun to K the size of the boules of second element increases proportionately more than their speed decreases; from K outward the speed increases proportionately more than the size decreases. Thus we can draw a line of positive slope representing the force of motion of the boules (agitation X size) at each distance from the sun. [49]

Two points are crucial about Descartes' model, and they are particularly clear in our representations in **Figure 3** and **Figure 5**:

[1] It is the action of the sun that transforms the distribution of **Figure 3** into that of **Figure 5**. The presence of the sun not only shifts the distribution of agitation, but it also as a consequence induces a change in the relative size distribution of the particles. This is due to the theoretical requirement that when the speed curve shifts, the size distribution must change accordingly so that the force condition on the stability of the vortex is maintained. **[2]** The K radius is the critical distance. It marks the locus beyond which the sun's added effect vanishes. Beyond K we have the old, stable pattern of size/speed distribution; below K we have a new, stable pattern of size/speed distribution—we still have force of motion increasing continuously with radius, but that comes about because size increases more quickly than speed decreases. This new distribution permits the observed celestial motions to occur. In effect it turns the vortex into a special kind of machine—a machine that locks planets into their appropriate orbits below K and that extrudes them from inappropriate orbital distances.[26] All this occurs in what, to Descartes, seems a straightforward mechanical fashion.[27]

Descartes' approach focuses on the centrifugal tendency of planets and of surrounding particles of second element in the vortex. Remember that, according to Descartes' dynamics and his sling exemplar, as a body or corpuscle moves on a curve, it has a certain force of motion along the tangent at any moment in its translation. Because it is constrained to move along a curved path, part of its tangential tendency manifests itself as a centrifugal tendency to recede along the normal to the path at that point on the curve. So, all bodies moving along a curve generate a centrifugal tendency to motion proportional to their size, quantity of matter, and instantaneously manifested tangential force of motion.

The key question in Cartesian celestial mechanics now becomes this: when and why is centrifugal tendency actualised as centrifugal motion, and when and why does that not happen? In the vortex, what plays the role of the sling, constraining the planet into a curved path and thus generating centripetal tendency on its part? Well, it is of course the neighboring, superjacent particles of second element that do this job—they surround and penetrate the pores of every piece of third matter making up a planet.

Why then do planets maintain orbits and why at different distances—all within radius K—from the sun? This depends on the amount of resistance the superjacent second element can put up, and that is dependent upon how much second element [50] can surround and envelop the parts of the planet, as what we may term a 'surface envelope'—a term of hermeneutical art that greatly helps our explication of Le Monde and the Principles.[28] The more matter of second element in this envelope, the more resistance the envelope will present to its being shoved aside by the planet's tendency to recede from the sun. **Fig 6** is a schematic representation of this notion: a simple ball of third element in circular motion is surrounded by a smaller and a larger envelope of corpuscles of second element.

What determines how large a surface envelope is relative to a given planet? Well, obviously, the size distribution of the second element with distance from the sun. Descartes recognised that the size of a surface envelop is dependent upon the volume to surface ratio of the spheres of second element. That ratio is function of their radii. The greater the radius of a sphere, the greater the V/S ratio.[29] Imagine a [51] ball of third element in circular motion surrounded by an envelope of second element. As long as the spheres of second element are so small compared to the piece of first element that we do not reach the point at which only a few spheres of second element suffice to 'cover' its surface, we can get a great variation in overall, aggregate size of the envelop, hence its quantity of matter, and hence its resistance to being moved out of place by the centrifugal tendency of the piece of third matter.

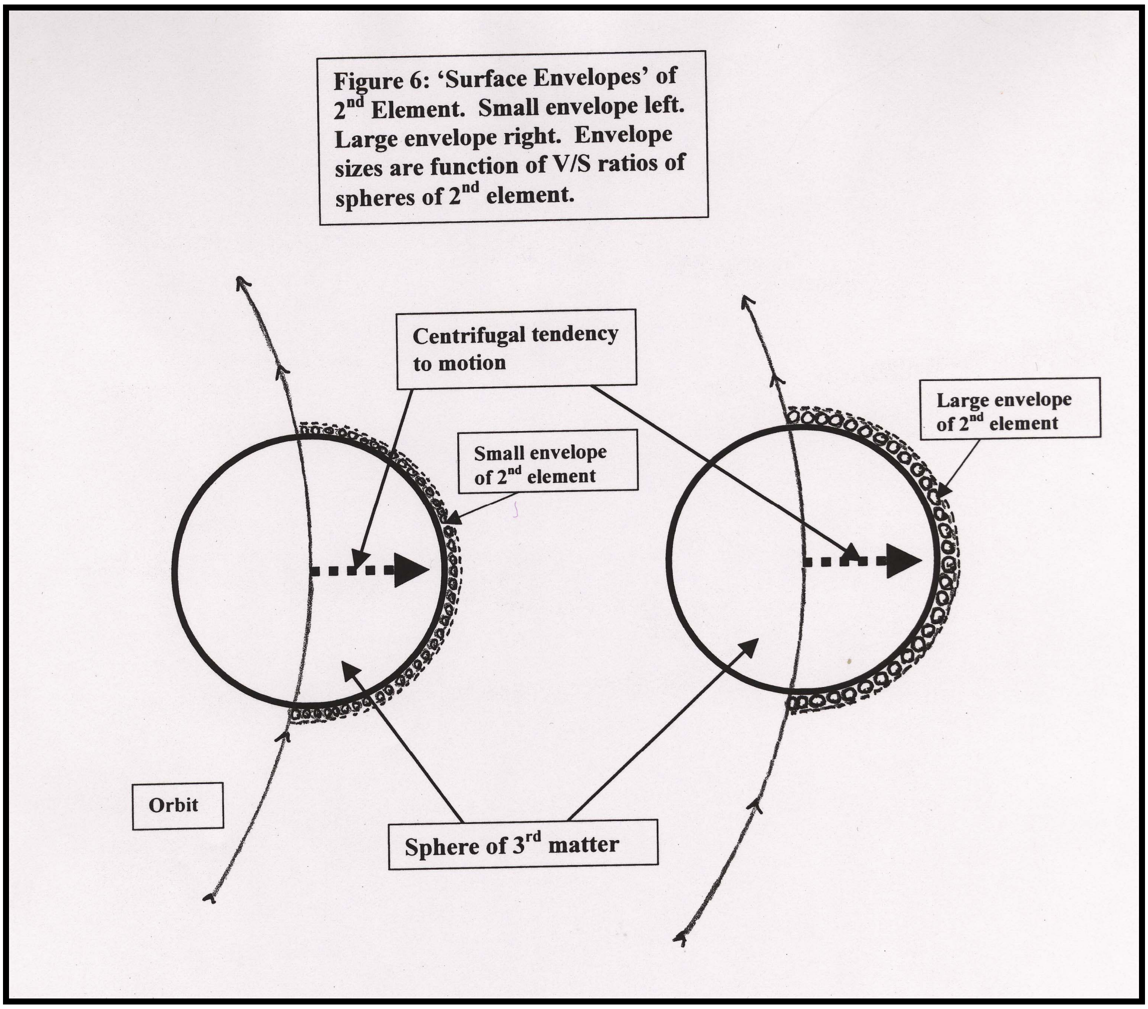

Figure 6: 'Surface Envelopes' of 2nd Element. Small envelope left. Large envelope right. Envelope sizes are function of V/S ratios of spheres of 2nd element.

Next recall the size distribution of the second element **[Fig 5]** We can turn this into a curve of V/S ratios, which in turn indicates the magnitude of the surface envelopes made out of the second element at different distances as related to a given piece of third matter. **[Fig 7]** The K layer marks an inflection point. From there outward, the spheres of second element get smaller not larger, and hence, surface envelops made out of them are progressively less capable of resisting a centrifugally tending piece of third matter.

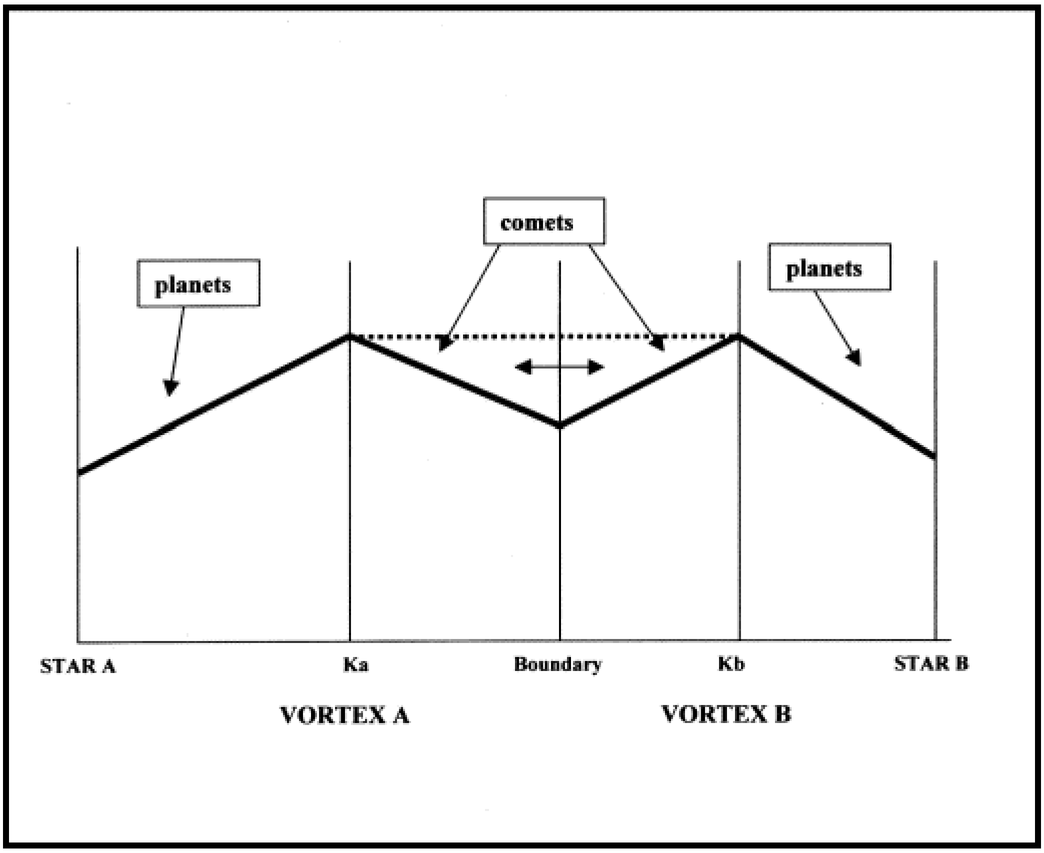

Figure 7. 'Resistance Curve': Derived from V/S Ratios Of Spheres Of 2nd Element

The bottom line is this: planets will always be locked into the vortex at a radius below the K layer. If you like, and Descartes speaks this way obscurely in Le Monde, more clearly in Principles, a planet will drift outward due to actualised centrifugal tendency, until it reaches a layer of the vortex where the spheres of second element have a V/S ratio sufficient to make the surface envelope they form resist any further centrifugal translation by the planet. The planet is locked in somewhere along the V/S curve of the spheres of second element. In his discussion of this part of the theory Descartes spoke of the 'massiveness' or 'solidity' of a [52] planet, meaning its aggregate volume to surface ratio.[30] This locking occurs at a radial distance from the sun at which the centrifugal tendency of the planet, a function of its massiveness, is exactly balanced by the resistance to centrifugal translation offered by the surface envelop in play at that location in the vortex. The greater a planet's V/S ratio or massiveness, the more distant that planet's orbit will from the sun.[31]

Imagine a planet, hypothetically finding itself not in its proper orbital place, literally too high up in the vortex given its degree of massiveness. It will not be able to develop sufficient centrifugal tendency and will be extruded downward by superjacent spheres of 2nd element. It will stop 'falling' when a balance is realised on the one hand between the centrifugal force of the subjacent second element at that radius in the vortex and the resistance offered by the planet (owing to its degree of massiveness), and, on the other hand, its own centrifugal tendency, conferred by its massiveness balanced by the resistance of the superjacent surface envelope at that layer in the vortex.[32]

Let us stop for a moment here and note that Descartes has constructed his mechanical heavens in such a way that mechanically efficacious stars are absolutely essential to the functioning of the celestial machine. If stars were inert, or if the [53] second element filled the centers of the vortices, then two sets of consequences would follow: first, light would be propagated only along radial lines from the axis of revolution of the vortex (a matter we cannot pursue in the present paper); and, more importantly for our present concern, the resultant distribution of size and shape of the boules would not be proper for the existence of planets in stable orbits.

Descartes' theory of comets now follows with a kind of mechanistic inevitability: We already know that according to this theory of celestial mechanics, the more distant a planet is from the sun, the greater its V/S ratio or massiveness. Now what if a planet is very massive, and it has the centrifugal tendency sufficient to overcome even the most highly resistant surface envelopes formed by second element at or near the K level? Well then, the object will pass by actualised centrifugal tendency beyond the K level, beyond the hump in the resistance curve. Beyond K it will meet second element with decreasing V/S ratios, and less resistance, so that this object will move right on out of the vortex and stream into a neighbouring one. The locking mechanism fails for these extremely 'solid' or 'massive' 'planets'.

When such an object of great 'solidity' is flung into the neighbouring vortex, it meets increasing resistance to its centripetal trajectory—as we can see by looking at the curve in **Fig 7**. The object picks up increments of orbital speed, until it starts to generate centrifugal tendency again, and again overcomes all obstacles—reading the curve in **Fig 7** backward—and gets flung back out of that second vortex. These, of course, are Cartesian comets, planets of high massiveness that oscillate between vortices, never penetrating any lower than the K level—trapped on our representation in **Fig 7** in the resistance depression between K levels of adjoining vortices.[33]

To summarise, then, each vortex is a locking and extrusion device. Its corpuscular make-up, size and speed distribution, given Descartes' theory of planet/comet make up, entails that planets are locked into orbits of differing radii. Comets are objects extruded from vortex to vortex, first 'falling' into a vortex and being extruded out. The existence and make up and mechanical behaviour of the central stars are crucial, not to the existence of vortices, but to the existence of planet locking/comet extruding vortices. Otherwise extrusion would be the universal rule. Multiple vortices are necessary, as each vortex is set in a container made of contiguous vortices, exerting a kind of centripetal backwash at its boundary.

We now explore three genealogical steps in Descartes' trajectory to the vortex celestial mechanics of Le Monde. As foreshadowed in Section 1, this exploration starts in 1619, and focuses on Descartes' activities in hydrostatics and physical and geometrical optics, and his relations, spanning a decade, with his mentor in [54] corpuscular-mechanism, Isaac Beeckman. The genealogy will illuminate a great deal about the structure of the vortex mechanics as we have just decoded it.

5. Genealogy Part A: 1619—From Hydrostatics to Dynamics; From Mixed Mathematics to Corpuscular-Mechanical Natural Philosophy[34]

In November 1618, Descartes met Isaac Beeckman. For two months they worked together on problems in natural philosophy, mechanics, theory of music, mathematics, and, hydrostatics. Descartes served a second natural philosophical apprenticeship with Beeckman. The Scholastic vision purveyed during his schooldays at La Flèche was overlaid with an incipient corpuscular mechanism, derived from Beeckman, but about to take on a uniquely 'Cartesian' character, even at this early date. Sometime during this period Beeckman set Descartes four problems in hydrostatics culled from the work of Stevin. Beeckman was probably curious about how Descartes would explain Stevin's fundamental but strange result, the hydrostatic paradox. This was to be an exercise in their self-proclaimed style of 'physico-mathematics', briefly mentioned above in Section 2, the agenda of which demanded that macroscopic phenomena be explained through reduction to corpuscular-mechanical models. Beeckman's questioning Descartes about Stevin's 'paradoxical' hydrostatical findings arguably sits squarely within this practice. This in general was what Beeckman and Descartes were envisioning when in 1618 they congratulated themselves on being virtually the only '*physico-mathematici*' in Europe.[35] What they meant was that only they unified the mathematical study of nature with the search for true corpuscular-mechanical causes.[36] Beeckman wanted to see what his new friend and fellow 'physico-mathematicus' could do about reducing Stevin's work to corpuscular mechanical terms, thereby fundamentally explaining it.

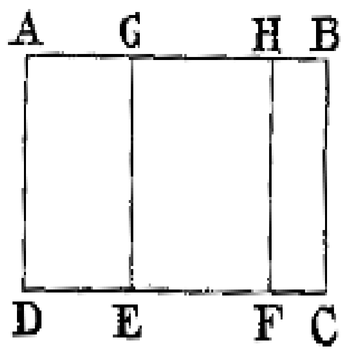

Figure 8 Stevin, <u>Elements of Hydrostatics</u> (1586) in <u>Principal Works of Simon Stevin</u> vol I p.415.

In his <u>Elements of Hydrostatics</u> 1586, Stevin demonstrated that a fluid can exert a total pressure on the bottom of its container many times greater than its weight. In particular, he showed that a fluid filling two vessels of equal base area and height exerts the same total pressure on the base, irrespective of the shape of the vessel and hence, paradoxically, independently of the amount of fluid contained in the vessel. Stevin's argument proceeds entirely on the macroscopic level of gross weights and volumes. The rigour of the proof depends upon the maintenance of static equilibrium, understood in terms of Archimedes' hydrostatics.

Stevin proves that the weight of a fluid upon the horizontal bottom of its container is equal to the weight of the fluid contained in a volume given by the area [55] of the bottom and the height of the fluid measured by a normal from the bottom to the upper surface.[37] He employs a reductio ad absurdum argument: ABCD is a container filled with water **[Fig. 8]**. GE and HF are normals dropped from the surface AB to the bottom DC, dividing the water into three portions, 1 [AGED], 2 [GHFE] and 3 [HBCF].

Stevin has to prove that on the bottom EF there rests a weight equal to the weight of the water of the prism 2. If there rests on the bottom EF more weight than that of the water 2, this will have to be due to the water beside it, that is water 1 and 3. But then, there will also rest on the bottom DE more weight than that of the water 1 ; and on the bottom FC also more weight than that of the water 3 ; and consequently on the entire bottom DC there will rest more weight than that of the whole water ABCD, which would be absurd. The same argument applies to the case of a weight of water less than 2 weighing upon bottom EF.[38]

Stevin then ingeniously argued that portions of the water can be notionally solidified, replaced by a solid of the same density as water. This permits the construction of irregularly shaped volumes of water, such as IKFELM, to which, paradoxically, the theorem can still be applied. **[Fig. 9]** [39]. [56]

That is, on bottom EF there actually rests a weight equal to that of a volume of water whose bottom is EF and whose height is GE. Stevin then

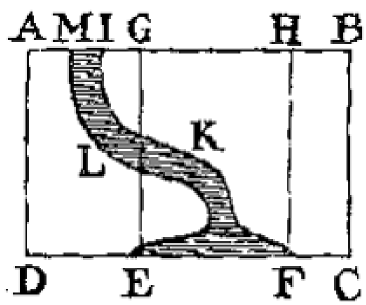

Figure 9 Stevin, Elements of Hydrostatics (1586) in Principal Works of Simon Stevin vol I p.417.

applies these findings to the sides of containing vessels.

Descartes' response to Beeckman's request that he explain Stevin's results is given in a report preserved in Beeckman's famous Diary, which we term, 'the hydrostatic manuscript'.[40] It involves an attempt to 'improve' upon Stevin's work; that is, to provide a deep natural philosophical explanation for his results, based on an incipient corpuscular-mechanical ontology. The hydrostatic manuscript amounts to an 'ultrasound scan' of Descartes' embryonic corpuscular-mechanical agenda, disclosing its fine, foetal anatomy. But in order to understand the hydrostatics manuscript and its place in the genealogy of Descartes' vortex celestial mechanics, we need first briefly to consider the work of Beeckman, his then thirty year old mentor.

Beeckman was one of the very first individuals in Europe to pursue consistently the idea of a micro-mechanical approach to natural philosophy. He conceived of a redescription of all natural phenomena in terms of the shape, size, configuration and motion of corpuscles, and he insisted that the causal register of this account, that is, the principles of all natural change, had to be derived from the transdiction of the presumed mechanical principles of macro-phenomena, in particular the behaviour of the simple machines. Beeckman offered on a first-hand basis an approach to natural philosophy which was not available to Descartes from any other contemporary source. [57]

Beeckman held a fundamentally atomistic view of nature. His atoms possess only the geometrical-mechanical properties of size, shape, and impenetrability (being absolutely hard, incompressible and non-elastic). Motion is conceived as a simple state of bodies, rather than an end-directed process which they undergo. Unlike previous advocates of atomism and prior to any of the great mechanists of the later seventeenth century, Beeckman sought to explain the behaviour of his atoms by applying to them a causal discourse modelled on the principles of mechanics.

By 1613 or 1614 Beeckman formulated a concept of inertia holding for both rectilinear and curved motions. Combining his principle of inertia with his atomic ontology, Beeckman was led to conclude that only corpuscular collision and transfer of motion can account for the initiation of motion of resting bodies or alternation of motion of moving ones. What he needed was rules of corpuscular collision. Since his atoms are perfectly hard, he formulated rules applicable to what we would term perfect inelastic collisions. He measured the quantity of motion of corpuscles by taking the product of their quantity of matter and their speed. Significantly, Beeckman linked his measure of motion to a dynamic interpretation of the behaviour of the balance beam. He evaluated the effective force of a body on a balance beam by taking the product of its weight and the speed of its real or potential displacement, measured by the arc length swept out in unit times during real or imaginable motions of the beam. Beeckman was able to build up a set of rules of impact by combining certain intuitively symmetrical cases of collisions with the dictates of the inertial principle and an implicit concept of the conservation of the directional quantity of motion in a system. His treatment of symmetrical cases of collision and his notion of the conservation of motion owed their form and their putative legitimacy to the model of the balance beam, interpreted in a dynamic rather than static fashion.[41] Indeed, Beeckman consistently demanded a dynamical [58] approach to statics, the theory of simple machines and mechanics in general, including hydrostatics.

It is important to realise that Beeckman's tactic here was following the classic model set forth in that sixteenth century best seller, the pseudo Aristotelian text Mechanical Problems or Mechanica, which took a dynamical approach to the problems of statics and the behaviour of the simple machines. That is, in the Mechanica one views equilibrium conditions on a lever or simple machine as a balance of forces, where force is defined as Weight times Speed. The basic principle behind this comes from Aristotle: the same force will move two bodies of different weights but it will move the heavier body more slowly, so that the velocities of the two bodies are inversely proportional to their weights. In the case of the lever, when these are suspended from the ends of a lever, we have two forces acting in contrary directions, and each body moves in an arc with a force proportional to its weight times the length of the arm from which it is suspended. The one with the greater product will descend in a circular arc, but if the products are equal, they will remain in equilibrium. So, the Mechanica makes statics simply a limiting case of a general dynamical theory of motion, a theory that is driven by Aristotelian dynamics, above all by the principle of the proportionality of weight and velocity.

It is significant that some earlier anti-Aristotelian mathematicians and natural philosophers, such as Tartaglia, Benedetti and the young Galileo, had tried to exploit the dynamical interpretation of the balance beam and simple machines in the Mechanica against Aristotelianism, despite the Aristotelian underpinning of that text. Beeckman was working in this general vein but being more radical about it, by placing at the basis of his corpuscular-mechanism rules of collision founded on the Mechanica-type—dynamical--approach to the simple machines We shall now see that in the 'hydrostatics manuscript' Descartes was to take an even more surprisingly radical turn in the search for a causal doctrine, a dynamics, through which to 'run' corpuscular-mechanical explanations.

Descartes takes as given the following conditions **[Fig. 10]**: A,B,C, and D are four vessels with equal areas at their bases, equal height and of equal weight when empty. B,C and D are filled to their tops. A is filled with water equal to the amount it takes to fill B. [59]

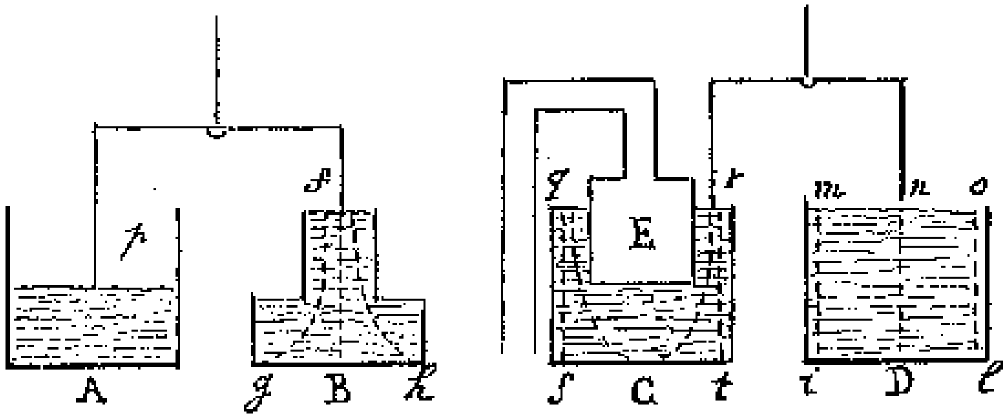

Fig 10. Descartes, Aquae comprimentis in vase ratio reddita à D. DesCartes, AT X 69

In the key problem Descartes proposes to show that, "the water in vessel B will weigh equally upon its base as the water in D upon its base"—Stevin's paradoxical hydrostatical result.[42] While Stevin's approach is geometrical, Descartes' analysis and explanation are based on an attempt to reduce the phenomenon to micro-mechanical terms. First Descartes tells us that of the various ways in which bodies may 'weigh-down' [*gravitare*], only two need be discussed: the weight of water on the bottom of a vessel which contains it, and the weight of the vessel and the water it contains.[43] By the weight of the water on the bottom of the vessel he does not intend the gross weight of the quantity of water measured by weighing the filled vessel and subtracting the weight of the container itself. He means instead the total force of the water on the bottom arising from the sum of the pressures exerted by the water on each unit area of the bottom. Next, and crucially, the term 'to weigh down' is explicated as 'the force of motion by which a body is impelled in the first instant of its motion'. Descartes insists that this force of motion is not the same as the force of motion which 'bears the body downward' during the actual course of its fall.[44] Finally, Descartes insists that we attend to both the 'speed' and the 'quantity of the body', since both factors contribute to the measure of the 'weight' or force of motion exerted in the first instant of fall. [60]

These three suppositions mark the first, embryonic appearance of some fundamental notions of Cartesian dynamics. Weight or heaviness reduces to the mechanical force exerted by a particle in its tendency to motion of descent. Weight is no longer an essential quality of bodies, but is jointly determined by the size of the body and its tendency to motion as conditioned by a given configuration of neighbouring bodies.

Descartes next solves the problem of accounting for the hydrostatic paradox. But, whereas Stevin offered an Archimedean argument from macroscopic conditions of equilibrium, Descartes manufactures a curious exercise in ad hoc micro-mechanical reductionism. He proposes to demonstrate the proposition by showing that the force on each 'point' or part of the bottoms of the basins B and D is equal, so that the total force is equal over the two equal areas.[45] He does this by claiming that each 'point' on the bottom of B is, as it were, serviced by a unique line of 'tendency to motion' propagated by contact pressure from a point (particle) on the surface of the water through the intervening particles. [See **Fig. 10**]

He takes points g, B, h; in the base of B, and points i, D, l. In the base of D. He claims that all these points are pressed by an equal force, because they are each pressed by "imaginable lines of water of the same length". That is, the same vertical component of descent. He says,

> ... line fg is not to be reckoned longer than fB or [any] other line. It doesn't press point g in respect to the parts by which it is curved and longer, but only in respect to those parts by which it tends downward, in which respect it is equal to all the others.[46]

This rather strange material requires some unpacking: Assuming the points on the bottoms are indeed served by unique lines of tendency transmitted from points on the surface; then, in so far as we are only concerned with the tendency to descend, we may compare the lines of tendency in respect to their vertical 'components'. What, then, about the mapping of the lines of tendency? Descartes is saying that when the upper and lower surfaces of the water are similar, equal and posed one directly above the other, then unique normal lines of tendency will be mapped from each point on the surface to a corresponding point directly below on the bottom. But, when these conditions do not hold, then some other unstated rules of mapping come into play.

So, in the present case the area of the surface at f in the basin B apparently is one-third that of the bottom, so each point or part on f must be taken to service three points or parts of the bottom. The problem, of course, is that no rules for mapping are, or can be, given. Descartes does not justify the three-fold mapping from f. He merely slips it into the discussion as an 'example' and then proceeds to argue that *given the mapping*, f can indeed provide a three-fold force to g, B and h. He proceeds to show by a syllogism, no less, that point f presses g, B, h with a force equal to that by which m, n, o press the other three i, D, l. [61]

> [1] Heavy bodies press with an equal force all neighbouring bodies, by the removal of which the heavy body would be allowed to occupy a lower position with equal ease.
>
> [2] If the three points g, B, h could be expelled, point f alone would occupy a lower position with as equal a facility as would the three points m, n, o, if the three other points i, D, l were expelled.
>
> [3] Therefore, point f alone presses the three points simultaneously with a force equal to that by which the three discrete points press the other three i, D, l. And so, the force by which point f alone presses the lower [points] is equal to the force of the points m, n, o taken together.[47]

In sum, there is a two-fold displacement away from what one might consider the original terms of the problem: [1] Descartes assumes an ad hoc mapping, and, [2] invokes a hypothetical voiding and consequent motion.[48] The proof of this 'example' is then taken as a general demonstration without any indication as to how the procedure is to be generalised to all the points or parts in the surfaces.

Strange as all this may appear to us today, Descartes himself was quite pleased. He continued to use descendants of these concepts the rest of his career. Here we have the key concept of instantaneous tendency to motion. Descartes' later mechanistic optics and natural philosophy will depend on the analysis of instantaneous tendencies to motion, rather than finite translations. Often Descartes will consider multiple tendencies to motion which a body possesses at any given instant, depending on its mechanical circumstances. There is evidence that even in 1619 Descartes was considering trying to systematise this set of new dynamical concepts to apply to corpuscular explanations, as he speaks in the 'hydrostatics manuscript' and surrounding correspondence of a treatise of "Mechanics" he is planning to write.[49]

This 1619 performance of Descartes was quite portentous, and it is worth pausing to crystallise precisely what we think was going on in his work at this stage: First of all, it is obvious that Descartes certainly was not denying the rigour or correctness of Stevin's strictly mathematical, Archimedean account. What he was after was proper explanation, meaning explanation in terms of natural philosophy. Stevin's treatment of the hydrostatic paradox fell within the domain of mixed mathematics. The account Descartes substitutes for it falls within the domain of natural philosophy: the concern is to identify what causes material bodies to behave in the way they do. Fluids are physical entities made up, on Descartes' account, of microscopic corpuscles. Their behaviour determines the macroscopic behaviour of the fluid. So, we need to understand the physical behaviour of the constituent corpuscles, if we are [62] to understand the behaviour of the fluid. Descartes therefore is saying that hydrostatics is no longer merely a discipline of mixed mathematics in the Aristotelian sense; rather it is an application of, and indeed illustration of, corpuscular-mechanical natural philosophy. This is deeply anti-Aristotelian, for it bids to shift hydrostatics from the realm of mixed mathematics unambiguously into the realm of natural philosophy.

Described in this manner, the 'hydrostatic manuscript' sounds quite in tune with a general notion of 'physico-mathematics' that he shared with Beeckman. There was, however, more going on in the manuscript, bespeaking some subtle differences between what Descartes was actually doing, and their otherwise shared general sense of what it meant to do physico-mathematics. As mentioned earlier, attempts to wring anti-Aristotelian natural philosophical conclusions out of the mixed mathematical sciences were hardly new. Most such attempts--as in the work of the young Galileo or in Beeckman himself-- depended on taking a dynamical approach to statics and the simple machines, following the lead of the pseudo-Aristotelian Mechanical Questions. But, in the hydrostatics manuscript, the young Descartes does not proceed via the Mechanical Questions dynamical account of the lever. Rather he plays the Archimedes/Stevin card—he starts from a mathematically rigorous hydrostatics of all things, which he *then* fleshes out in terms of the microcorpuscularian model he learned from Beeckman.[50] The young Descartes' hyper-radical program was this: he wanted to reduce Stevin's hydrostatics to an embryonic corpuscular mechanism in which discourse concerning causes or 'forces', elicited on the basis of that hydrostatics, would provide the basis for unifying the mathematical sciences. This difference, and the dynamic of research and concept formation it unleashed, was going to play out in his optical work in the 1620s, and crystallise in the program of Le Monde.

6. Genealogy Part B 1627 The Laws of Light and the Laws of Nature:[51]

The next step in our genealogy of the vortex celestial mechanics of Le Monde involves perhaps the most important, successful and fruitful physico-mathematical [63] research Descartes ever attempted—his work in geometrical and physical optics in the 1620s. These endeavours climaxed with his discovery of the law of refraction of light around 1627; but, they did not end there, despite widely held views amongst scholars about this period in his life. Descartes immediately began to think about possible mechanical rationales or explanations for the law, and these attempts were intimately connected with a process by which he crystallised his emerging concepts of dynamics directly out of a 'physico-mathematical' 'reading' of his geometrical optical results. In short, his optical researches marked the high point of his work as a physico-mathematician transforming the 'old' mixed mathematical sciences and co-opting the results into a mechanistic natural philosophy: On the one hand his results confirmed his 1619 agenda of developing a corpuscular ontology and a causal discourse, or dynamics, involving concepts of force and directional 'determination' of motions or tendencies to motion. On the other hand, his results concretely advanced and shaped his concepts of light as an instantaneously transmitted mechanical tendency to motion, as well as the precise principles of his dynamics.

Around 1620 Descartes explored Kepler's speculations about the refraction of light, using his newly acquired physico-mathematical style of 'reading' geometrical diagrams representing phenomena for their underlying message about the causal principles at work. I have demonstrated elsewhere that Descartes, in one of his physico-mathematics fragments dating from around 1620, attempted to appropriate in physico-mathematical fashion a particular telling geometrical optical diagram he found in Kepler's Paralipomena ad vitellionem [1604].[52] [64]

In **figure 11** Kepler depicted the possibility that the density of the refracting medium and the inclination of the incident ray both exercise a geometrically representable physical effect on the refracted ray. Kepler takes AG incident upon a basin of water. The density of water is said to be twice that of air, so Kepler lowers the bottom of the basin DE to LK so that the new basin contains 'as much matter in the rarer form of air as the old basin contained in the doubly dense form of water'. Kepler then extends AG to I and drops a normal from I to LK. Connecting M and G gives the refracted ray GM. Its construction involves the obliquity of incidence and densities of media. Kepler then rejects this construction on empirical grounds.[53] But, [65] the young Descartes, fresh from his physico-mathematical foray into hydrostatics, played physico-mathematical games with it.

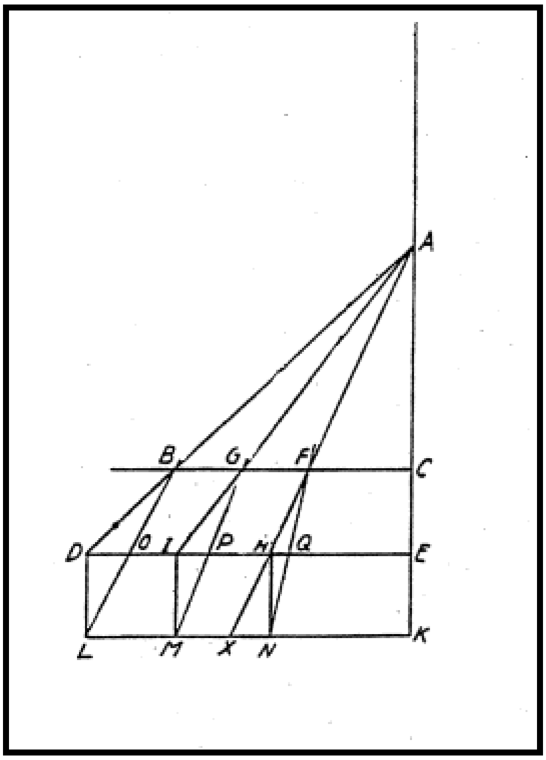

Fig 11. Kepler's Diagram Representing Possible Role In Refraction Of Light Of Density Of Refracting Medium And Obliquity Of Incident Ray. Kepler, Ad Vitellionem Paralipomena (1604), in Gesammelte Werke, ed. M. Caspar (Munich, 1938ff), vol.II p.85.

What Descartes, physico-mathematicus, saw here was the *physical–causal* speculation that denser media bend light toward the normal, and the *physico-mathematical* notion that you can represent this geometrically in order to construct refracted rays. Let us recall that opticians, Kepler included, treated light rays in these situations in terms of normal and parallel components. Descartes' manuscript fragment on this indicates that he saw the lower medium as acting to increase the normal component of the force of the ray in a fixed ratio. So, Descartes was reading Kepler the way he had read Stevin: as a physico-mathematician. That is, he was attempting to elicit some mathematicised physical theory from a compelling geometrical diagram for refraction presented by Kepler.[54] It must be noted, however, that this physico-mathematical exercise actually hindered Descartes' eventual attainment to the law of refraction of light, because on the likely concomitant assumption that the parallel component of the force of the incident ray remained constant, Descartes' 1620 speculation yields a law of tangents, rather than the law of sines (or in fact cosecants) which he achieved later.[55] Nevertheless, when he did succeed six or seven years later, his physico-mathematical style again came into play, with portentous results.

It was only in 1626/7 that Descartes, in collaboration with Claude Mydorge, discovered of the law of refraction. This discovery took place entirely with the confines of traditional geometrical optics, without the benefit of dynamical or corpuscular-mechanical theorising. My detailed reconstruction, published elsewhere,[56] involves Descartes and Mydorge having done in practice, or merely on paper, what we know Harriot earlier had done to construct the law of refraction—that is use the traditional image locating rule in order to map the image locations of point sources taken on the submerged circumference of a disk refractometer.[57] **[fig 12]** [66] Even using Witelo’s cooked data, one gets a smaller semi-circle,[58] and accordingly initially a law of cosecants, mathematically equivalent to the law of sines Descartes later published in the Dioptrics.[59]

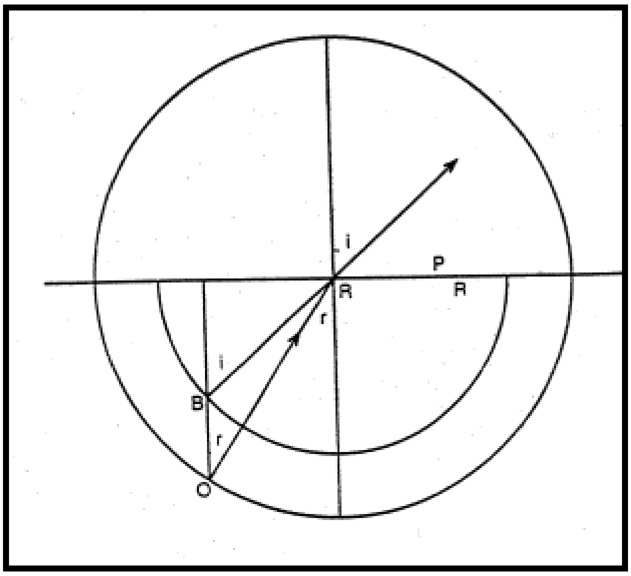

Fig. 12. Harriot’s Key Diagram: Half Submerged Disk Refractometer. Source Points On Lower Circumference. Image Points On Smaller Semi-Circular Locus. A Law Of Cosecants Results For Refraction of Light. J.Lohne, 'Zur Geschichte des Brechungsgesetzes', Sudhoffs Archiv 47 (1963), 152-72, p.160

In a key letter describing the cosecant form of the law and a resulting theory of lenses, Mydorge later drew this diagram as a refraction predictor, by flipping the inner semi circle up above the interface as the locus of point sources for the incident light. **[fig 13]**.[60] [67]

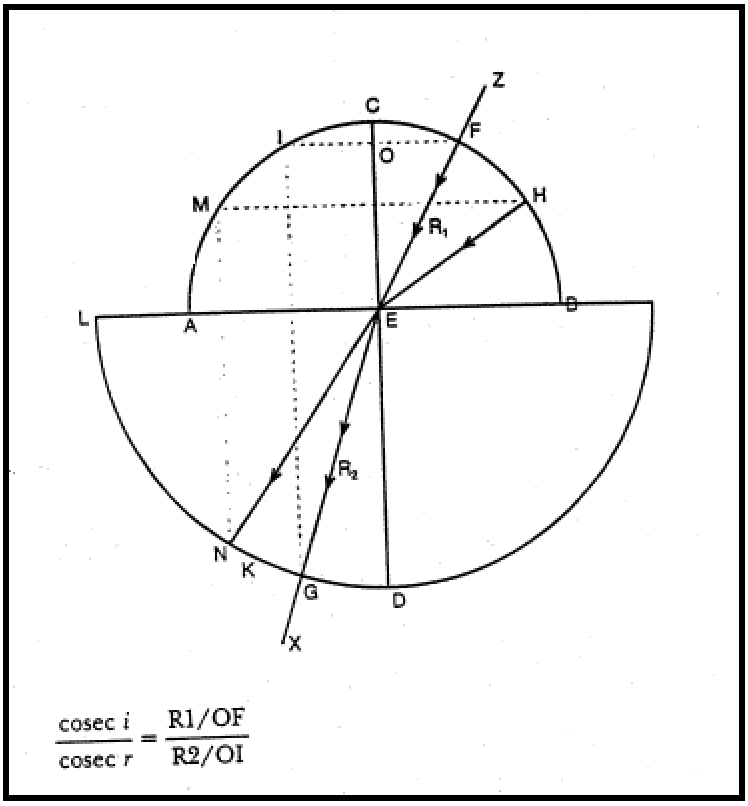

Figure 13. Mydorge's Refraction Prediction Device Mydorge to Mersenne in Correspondence de Marin Mersenne (ed. C. deWaard) (Paris 1945ff). I. 404.

After the discovery of the law of refraction by these purely geometrical optical means, and with this sort of geometrical representation available to him, Descartes looked for better conceptions of the dynamics of light by which to explain the law. Unsurprisingly, these he found by doing physico-mathematics in the style of 1619: that is, he transcribed into dynamical terms some of the geometrical parameters embodied in this diagrammatic representation of the law [Fig 13 above]. The resulting dynamical principles concerning the mechanical nature of light were: [1] the absolute quantity of the force of the ray was increased or decreased in a fixed proportion, whilst, [2] that the parallel component of the force of a light ray was unaffected by refraction. There is evidence dating from 1628 of Descartes using [68] these concepts in an attempted analogical proof of the cosecant law of refraction, by appeal to the behaviour of a bent arm balance.[61] His proof of the law of refraction in sine form in the Dioptrics, published in 1637 but inscribed sometime between 1629 and 1633, also deploys precisely these dynamical concepts, applied to light as an instantaneously transmitted tendency to motion. But, as I have shown elsewhere, to discern this one must peer below the surface of his superficially confusing tennis ball model for the motion, reflection and refraction of light.[62] There is no doubt, however, that these dynamical principles were constructed prior to the writing of Le Monde, since the text alludes to their existence in the as yet unpublished Dioptrics.[63] In sum, it may safely be concluded that the insights [1] and [2] above, abstracted from the original geometrical representation of the newly discovered cosecant form of the law, actually suggested the form of the two central tenets of his mature dynamics, described earlier in Section 3, when he came to consider the need for them in the course of composing Le Monde.[64] After all, to Descartes the physico-mathematician what could have been more revealing of the underlying principles of the punctiform dynamics of micro-corpuscles than the basic laws of light — itself an instantaneously transmitted mechanical impulse?

Descartes' dynamics, the causal register for talking about micro corpuscles, had eventuated from work in the mixed mathematical sciences—that was interesting and precisely in the agenda of physico-mathematics as conceived (and practised) in 1619. Even more interesting was that he focused on results and phenomena in which, paradoxically no motion of bodies took place at all—in hydrostatics, and in the exemplary refracting of instantaneously transmitted light rays. In these 'statical' exemplars, or phénoméno-techniques' Descartes found crisp, clean messages about the underlying dynamics of the corpuscular world and indeed about its laws. With these findings we are almost back at the text of Le Monde. But there was one final critical encounter in Descartes' trajectory of the 1620s from the early physico-mathematics to the dynamics and vortex celestial mechanics of Le Monde. Again, an encounter with Beeckman crystallised and shaped ensuing events. [69]

7 .Genealogy Part C: 1629 Cosmic Balancing Acts

In late 1628 after a gap of 10 years, Descartes reestablished contact with Beeckman.[65] He found Beeckman ploughing through the astronomical works of Kepler, seeking to evaluate instances in which Kepler had invoked immaterial celestial forces or powers. In each case Beeckman sought to re-write the "mechanisms" into corpuscular-mechanical terminology. As far as Beeckman was concerned, the key issues in astronomy did not involve the traditional activities of observation or even Kepler's work on elliptical orbits. Rather, Beeckman saw in Copernican astronomy, especially as transformed by Kepler, a broad, hitherto neglected field for natural philosophical explication, in particular corpuscular mechanical explanation. Beeckman specifically identified his celestial mechanical speculations as desiderata for a restitutio astronomiae.[66]

Similar concerns lay behind Descartes' celestial mechanics in Le Monde. Descartes, like Beeckman, avoided technical issues in observational astronomy, concentrating on plausible mechanical accounts of the causes of the motions of the planets in the Copernican system. Descartes and Beeckman were engrossed by the radical attempt to indicate how the latest conceptions in their 'physico-mathematics' might be brought to bear in explaining in a general way the causes of the motions of the planets in the Copernican system—allowing of course for the differences in content and trajectory in their respective versions of physico-mathematics, evident since 1619 and certainly quite further developed in Descartes' case by the late 1620s, as we have seen. In addition both Descartes and Beeckman sought to support their respective celestial physics by trading upon the suggestion that their celestial physics also explained the nature of light and thus was partially confirmed by its broad explanatory sweep.

Indeed Beeckman's review of Kepler starts with a penetrating mechanistic critique of Kepler's theory of light: light is corporeal, consisting in a type of heat particle emitted by stars. Kepler's law of illumination is explained by the way streams of light corpuscles spatially diverge from each other with distance from a source—an outcome impossible and unintelligible, he claims, on Kepler's own theory of light as an immaterial emanation.[67] This is crucial, because Beeckman's varied celestial mechanical speculations all play upon the idea of opposed, corporeally mediated forces that vary in strength with distance from source, hence constituting particular loci of equilibrium for the orbital placement of objects.

Beeckman then addresses a theory of lunar orbital placement: the moon is held in its orbit by a balance of attractive and repulsive actions delivered respectively, by rays of the sun reflected by the earth, and rays of the earth itself. The efficacy of the earth rays decreases with distance more quickly than that of the solar rays. The solar [70] rays are presumably Beeckmanian light rays—streams of corpuscles; the earth rays are rays of Beeckman's version of earth magnetism.[68] The 'attraction' and 'repulsion' attributed to these corporeal rays is unexplicated at the corpuscular level. (It is worth noting here in passing just how much better Descartes would later have judged his own constructions to be. In Le Monde he will have a plausible locking and extrusion mechanism deeply embedded in findings about hydrostatics, optics and a general dynamics of corpuscles.)

There were of course problems with the lunar theory,[69] which Beeckman detected (perhaps aided by his French friend), before he rushed onto a grander vision of the entire celestial mechanism: By substituting the fixed stars for the sun, and the sun for the reflecting earth, his moon theory could perhaps be applied to the entire solar system. Sometime between Oct 1628 and late Jan 1629 Beeckman boldly writes:

> [the same] thing can be said about all the planets (among which I also number the earth)... the light or corporeal virtue of the eighth sphere reflected by the sun draws the planets to the sun and the sun [itself] repels them. And thus each planet will be affected by each of the virtues according to its magnitude or rarity and therefore they will be located at different distances from the sun.[70]

This is indeed a striking speculation: the heavens are crisscrossed with the direct and reflected corporeal emanations of the fixed stars and the sun, leading the planets being located in the network of differential forces according to their "magnitude" and "rarity".[71] However, Beeckman then noticed that the sun's own emanations were now repulsive in nature, and so he quickly reverted to a simpler picture of paired sun-planet interactions, based, as before, on a balance of forces—in this case planetary magnetic attraction, corporeally mediated, working against the mechanical repulsion arising from impact of solar heat and light corpuscles.[72]

Continuing to jot in his Journal as his speculations wandered, Beeckman shift his ground again: He reverted to the fixed stars sending a flow of effluvia through the solar system. There are always more solar emanations immediately within the orbit of a given planet than immediately beyond it, thus fewer celestial emanations can make their way within the orbit and exert a back-pressure on the sunward side of the planet. Hence each planet suffers a pressure toward the sun arising from the incoming stellar rays which is to be balanced by the light/heat repulsion of solar [71] emanations.[73] This indeed was a mechanical picture of orbital equilibria of causes which was to be supplied in a much more elegant fashion by Descartes' vortices.

By mid-1629 Beeckman had not achieved a settled view and in typical fashion he unceremoniously dropped the matter. Beeckman's work just pre-dates and overlaps the period of renewed contact with Descartes in late 1628. Arguably, the interest Descartes evidenced after 1628 in the problem of celestial mechanics, as well as his mode of approach to it, grew from his acquaintance with Beeckman's speculation. Descartes would have been all the more confident in his union in Le Monde of a theory of light with celestial mechanics, if he recognised the advance in comprehensiveness, coherence and mechanical rigour achieved in his work as compared with these wranglings of Beeckman.

Had Descartes been quite a bit more charitable and magnanimous to his erstwhile mentor, he might just have acknowledged what is clear to us--the underlying spirit and structure of the argument of the celestial mechanics of Le Monde harks back to the notions behind Beeckman's shifting speculations of 1628-9. That was not Descartes' style, as is well known.[74] One can imagine him much more readily agreeing with an uncompromising technical judgment which we can now offer, recalling our discussion of the vortex mechanics in Section 4: Having been spurred by Beeckman highly interesting but inconclusive foray into a unified theory of light and celestial optics, Descartes was in a position to try to succeed where Beeckman was floundering, and he approached this by in effect cashing in the intellectual profit of his physico-mathematical endeavours since 1619. Instead of Beeckman's wandering and inconclusive jottings, Descartes elaborated his model of a celestial vortical locking and extrusion machine. He based himself on his principles of dynamics (the emergence of which was initiated in his hydrostatics of 1619 and articulated in his optical work of the 1620s); his theory of centrifugal tendency to motion; a theory of the make up of the stars and their surrounding vortices; and his notion of the massiveness of planets and comets.

A mechanistic theory of light as instantaneously transmitted tendency to motion could be fitted to this cosmic setting, providing the ultimate basis for the optical work and discoveries, and fulfilling the de facto challenge issued by Beeckman to render in corpuscular-mechanical and properly physico-mathematical terms the problematic of Kepler. Le Monde challenges Beeckman back by saying in effect: '*here is a physico-mathematical explanation of light in cosmic setting and of celestial 'physics'; causes are not multiplied; the same concepts of dynamics, applied to the nature of stars and vortices, explain everything!*' [72]

8. The Unified Theory of 'Weight', Orbiting, and Extrusion—The Case of Terrestrial Bodies

Having looked at the chief genealogical moments in its genesis, we have now arrived back at the vortex mechanics of Le Monde. Unfortunately, the scope of this essay precludes our now surveying the full range of detailed issues that fall into place, given this genealogy. Chief amongst these would be Descartes' theory of light in its cosmological setting, that is, in the context of his universe of vortices, each centered on a light giving star—a topic equally grounded in the genealogy and most enlightening about the status and aims of the vortex theory. Other topics would follow: the fall of heavy (third matter) bodies near the surfaces of planets; the motion of planetary satellites, and, for planets possessing both oceans and a moon, the nature and causes of the resulting tides. All of these articulations of the vortex mechanics will be addressed in my monograph on Descartes, 'physico-mathematicus'.

For the moment, we must limit ourselves to a brief discussion of Descartes' vortex theory of the fall of heavy bodies near the surface of the earth (or any planet). This theory, like the other articulations of the vortex mechanics listed above, contains quite a few conceptual and empirical problems, which are often emphasised in a kind of Whiggish perspective eager to move on to discuss Newtonian theory. Of course, every theory, indeed even Newton's, has its limits, its strengths, weaknesses, pointed difficulties, lacunae and, perhaps, contradictions. Descartes' bold, internally complex, and strategically thought out vortex mechanics is hardly an exception to this. But, in the spirit of this essay, I choose here to analyse the little appreciated coherences and strength of Descartes' account of local fall, as a token of my larger attempt to wring as much systemic cogency, and contextual and developmental 'reason' out of the vortex theory and its several articulations.

As we know, Descartes articulated his vortex theory by claiming that smaller, local vortices form around the planets orbiting stars. Using the case of the earth, Descartes attributes to its local vortex the explanation of the motion of the moon, the diurnal motion of the earth, the local fall of 'heavy' bodies, and, conjointly with the moon's motion, and the existence of oceans on the earth, the tides.[75] In very general terms the local fall of heavy bodies follows on this theory as a case of extrusion downward in the planetary vortex of bodies possessing less centrifugal tendency than the surrounding matter of the local vortex.

> Weight is the force... that makes all parts of the earth tend toward its center, each more or less according as it is more or less large and solid. That force consists the fact that, since the parts of the small heaven surrounding it turn much faster than its parts about its center, they also tend to move away with more force from its center and consequently to push the parts of the earth back toward its center.[76]

The analogy is to planets in the solar vortex which, located at the 'wrong' orbital distance, 'fall' (indeed spiral) downward toward the star until they pick up enough [73] centrifugal tendency to stabilise in an orbit—at a distance determined by the 'solidity' of the planet, and the speed, size distribution of balls of second element in the vortex, as we have explored in Section 4.

Descartes' account, as is well known, meets an immediate large objection, as obvious to contemporaries as to us: Because the local vortex spins on an axis coincident with that of the earth, fall on or near the earth should be in direction normal to the axis of rotation, not radially toward the center of the earth. Moreover, why do not falling bodies sweep laterally across the surface of the earth, spiralling downward, rather than apparently falling in straight lines normal to the local surface of the earth? Finally, to add to these commonly adduced puzzles, we are in a position to add a third one, grounded in our genealogy of the vortex theory: how in detail could Descartes consistently explain or reduce Stevin and Archimedes' rigorous, geometrical, and macro-descriptive hydrostatics to a full vortex theory of fall and weight? In 1619, let us recall, he had been inspired by such an hydrostatics, but had only asserted a piecemeal corpuscular-mechanical explanation limited to assertions about particles in the water, rather than a full vortex theory of weight, specific weight, the behaviour of air, water and circulating vortex particles of second element. Descartes had things to say about the first two issues, but never assayed the third, much more profound one. Be that as it may, I wish here to emphases the virtues of the theory, from Descartes' point of view, rather than explore further its deficiencies and its (arguable) anomalies.

In Le Monde Descartes furnishes an initial clue about what he thought was most striking about his theory of fall. It is precisely the fact that the theory of fall, on his view, *is completely consistent with his vortex theory of planets and comets at the level of basic explanatory machinery*. The very first issue Descartes discusses after his explanation of fall is the following likely misunderstanding on the part of the reader:

> You may find some difficulty in this, in light of my just saying that the most massive and most solid bodies (such as I have supposed those of the comets to be) tend to move outward toward the circumferences of the heavens, and that only those that are less massive and solid are pushed back toward their centers. For it should follow that only the less solid parts of the earth could be pushed back toward its center and that the others should move away from it.[77]

Descartes is directing us to his key concept of 'solidity' and to the fundamental theory of speed/size distribution of balls of second element in the star-centered vortex. He continues,

> But note that, when I said that the most solid and most massive bodies tended to move away from the center of any heaven, I supposed that they were already previously moving with the same agitation as the matter of that heaven. For it is certain that, if they have not yet begun to move, or if they are moving less fast than is required to follow the course of this matter, they must at first be pushed by it toward the center about which it is turning. Indeed it is certain that, to the extent that they are larger and more solid, they will be pushed with more force and speed. Nevertheless, if they are solid and massive enough to compose comets, this does not hinder them from tending to move shortly thereafter toward the exterior circumferences of the heavens, in as much as the agitation [74] they have acquired in descending toward any one of the heavens' centers will most certainly give them the force to pass beyond and to ascend again toward its circumference.[78]

Using precisely the conceptual terms of the larger vortex theory Descartes is distinguishing between: (a) the potential orbital distance of a body in a vortex as determined by its volume-to-surface ratio or solidity, and (b) the amount of force of motion the body possesses at any given time and radial place in the vortex. Now, although he is more precise in his expression about this later in the Principles, even in Le Monde, it is clear that comets or pieces of terrestrial matter have definite solidities, that ultimately determine their placement in a vortex, but only on condition that they have gradually acquired circulatory motion and have begun to translate centrifugally. A stone initially sharing only in the diurnal rotation will be forced down toward the center of the earth, and that is what we habitually observe. But, Descartes is also saying that if the earth's vortex were large enough and if the stone were released from a sufficiently great distance from the centre, it might acquire sufficient circulatory speed in the course of its descent to begin to rise as a result of the centrifugal tendency thus gained. It would rise through the vortex until it reached a level at which the resistance to centrifugal motion of the boules balanced its own centrifugal tendency. And such an object of terrestrial matter does exist in stable orbit in the terrestrial vortex—the moon.

The greater the solidity of a body, whether in a solar or planetary vortex, the more difficult it will be for the surrounding second matter to impart motion to it. Thus upon, being released, a body of great solidity will yield more readily to centripetal extrusion than one of lesser solidity. In local fall near the earth, heavy bodies generally do not attain sufficient centrifugal tendency to begin their ascent. Viewing stellar and planetary vortices under a unified theory of vortex mechanics, we can formulate the following theorem, reflecting the essentials of Cartesian celestial mechanics—'*comets, planets in the wrong orbits and heavy bodies released near any planet's surface are all doing the same thing for the same reasons*'

Finally, as an explanatory conceit, let us imagine Descartes himself, brought back to discourse with us, commenting upon this unified theory, as well as other competing theories, including his post-mortem acquaintance with Newton's work. Perhaps such a revived, typically self-regarding and feisty Descartes might lecture us as follows:

> *I know all of you are, so to speak, in love with Newton—he's like you, or so you think. Well, for me, he is like Kepler, brilliant but ontologically unsound. Here is Newton's leading question-- orderly procedure starts with the right question. 'What single immaterial causal agency explains the motions of the planets, comets, satellites, the fall of bodies on earth, as well as the tides?' Very elegant, is it not? And to be sure, nobody ever posed that precise question: not Aristotle obviously; certainly not Copernicus, not even Kepler—he multiplied such unintelligible immaterial causal agencies, rather than look for one elegant one.*
>
> *Very well, my question, the methodologically appropriate one, was: 'What unique and certain set of dynamical principles applied to the vortex motion of corpuscles explains the motions of the planets, comets, satellites, the fall of bodies on earth, as well as the* [75] *tides?'. You have seen my dynamics and general vortex theory and can work out for yourselves why they constitute a unified general theory of the key phenomena in question. Newton pursued the same problematic. He had the benefit of my example. He grasped the aim or the problematic, but faltered badly on the issues of causation and ontology.*[79]

9. Conclusion: 'Waterworld'—A Crafty but poorly expressed gambit in the Natural Philosophical Agon:

In 1619 Descartes had begun to develop his conceptions of force and tendency to motion in a hydrostatical context: by 1633, having been crystallised in his profound work in optics, they sat at the centre of the corpuscular-mechanical 'hydro-dynamics' that ran 'the world', or as my friend and occasional collaborator, Stephen Gaukroger, incisively dubbed it, Descartes' 'Waterworld'.[80] The ambitious but embryonic physico-mathematical project of 1619 had borne some hefty dividends. Descartes, physico-mathematicus, was building a novel corpuscular-mechanical natural philosophy that would entrain new, non-Aristotelian relations between natural philosophising and the mathematically based physical disciplines. Indeed, once one grasps the underlying conceptual framework of Le Monde, and the genealogy of that framework, one sees that Le Monde was a work deeply symptomatic of a contemporary problematic in natural philosophy shared by certain bold, mathematically oriented anti-Aristotelian innovators, regardless of their own ontological differences. The vortex celestial mechanics was not just a fanciful and amusing advertisement for Copernican realism in infinite universe mode, nor was it just a representation of Copernicanism inside a proffered, alternative system of natural philosophy. Descartes' 'Waterworld' was in fact a post-Keplerian play for hegemony in the field of natural philosophising, in its particularly overheated and contested early seventeenth century state.

Le Monde, as Descartes would have seen it, was built in part on the basis of a concatenation of achievements in natural philosophising key chunks of the mixed mathematical sciences. He had come to terms with, competed with, and, in his view, surpassed Beeckman's natural philosophical strivings, themselves partially shaped in the shadow of Kepler. In the mixed mathematical sciences, Kepler's own master strokes had been the elliptical orbit of mars, and the laws of planetary motion in general. Descartes' competing jewels, in his view at least, were his corpuscular-mechanical reduction of hydrostatics, and his solution of the ancient and prestigious [76] refraction problem, and he too had a celestial mechanics, which followed Beeckman's critique of Keplerian spiritual neo-Platonic nonsense, but which out played Beeckman by being based on a coherent dynamics of corpuscles, itself the product of the same course of physico-mathematical research.

Consider this short list of the characteristics of Descartes' Le Monde program: Descartes was articulating Copernicus' claims; he was displaying what he thought was best dynamical practice, best causal discourse practice, to explain planetary motion and the dynamical role of stars; he was associating in the same problematic local gravity, the behaviour of satellites, orbital motions of planets, cometary motions, the nature and causal role of the sun, or of any star, in all this and in a theory of light in cosmic setting. Now, on each of these points, there are notable parallels to the enterprise of Kepler, allowing for complete difference of natural philosophical content (but not of aim). The problematic is the same in both cases: what Descartes unifies as explananda by virtue of his dynamics of vortices,[81] including the key role of stars within vortices, Kepler unifies by a theory of a set of hierarchically arranged causal forces, similar to each other in respect of their immaterial nature, and lawlike, mathematical functioning. Both natural philosophers attempted a unified set of explanations under the aegis of a new, alternative natural philosophy prominently advertising highly anti-Scholastic dynamical registers, or causal doctrines. In other words, and in conclusion, the vortices were serious business, and, as Aiton brilliantly showed, they remained serious business amongst a small committed crew of serious celestial mechanicians, such as Huygens and Leibniz, well into the eighteenth century, in competition with the Newtonian view.[82] [77]

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1 Kuhn (1959) pp. 240, 242. Note that references to the Adam and Tannery (1974-86) edition of Descartes' works will be in the following form: AT followed by the volume number in roman numerals and page in Arabic numerals, as AT x 32. References to Gaukroger's translation of Descartes' Le Monde, Gaukroger(1998), will be SG plus page number; References to Mahoney's translation of Descartes' Le Monde, Mahoney (1979), will be MSM plus page number.

2 Bachelard (1965) p.79, 'La métaphysique del'espace chez Descartes est la métaphysique de l'éponge.'

3 Aiton (1972)

4 Schuster and Watchirs (1990); Schuster and Taylor (1996); Schuster (2002)

5 Space constraints prevent discussion here of Descartes' 'cosmological optics', his theory of light in the context of the vortex universe. However, the development of the cosmological optics went hand in hand with that of the vortex mechanics, a relation to be treated at length in a monograph in progress, dealing with the development of Descartes as a physico-mathematician 1618 to 1633. In the present paper, it will at least be made clear that the genealogy of the vortex mechanics is entangled with the development of Descartes' work in physical optics and theory of light. See below, Sections 5 and 6 .

6 Readers of this volume may wish to compare the interpretation of Kepler and Descartes implicated here with they way they emerge in H.F.Cohen's interpretative essay on the causes of the Scientific Revolution, Chapter 1 above, pp.000-000.

7 The third rule of motion in Le Monde states: [AT xi. 43-44: SG 29-30] 'I shall add as a third rule that, when a body is moving, even if its motion most often takes place along a curved line and, as we said above, it can never make any movement that is not in some way circular, nevertheless each of its parts individually tends always to continue moving along a straight line. And so the action of these parts, that is the inclination they have to move, is different from their motion.[...leur action, c'est à dire l'inclination qu'elles ont à se mouvoir, est different de leur mouvement].' And, Descartes continues, 'This rule rests on the same foundation as the other two, and depends solely on God's conserving everything by a continuous action, and consequently on His conserving it not as it may have been some time earlier, but precisely as it is at the very instant He conserves it. So, of all motions, only motion in a straight line is entirely simple and has a nature which may be grasped wholly in an instant. For in order to conceive of such motion it is enough to think that a body is in the process of moving in a certain direction [en action pour se mouvoir ver un certain coté], and that this is the case at each determinable instant during the time it is moving.'

In the passages cited above, Descartes in his discussion of the third law defines 'action' as 'l'inclination à se mouvoir'. He then says that God conserves the body at each instant 'en action pour se mouvoir ver un certain coté'. This would seem to mean that at each instant God conserves both a unique direction of motion and a quantity of 'action' or force of motion. In other words the first law certifies God's instantaneous conservation of the absolute quantity of tendency to motion, the 'force of motion'. The third law specifies that as a matter of fact in conserving 'force of motion' or 'action', God always does this in an associated unique direction. The first law asserts what today one would call the scalar aspect of motion, the third law its necessarily conjoined vector manifestation. Just because he recognises that some rectilinear direction is in fact always annexed to a quantity of force of motion at each instant, Descartes often slips into abbreviating 'directional force of motion' by the terms 'action', 'tendency to motion' or 'inclination to motion', all now seen in context as synonyms for 'determination'.

8 The understanding of determination used here develops work of A.I.Sabra (1967) p.118-121; Gabbey (1980), pp.230-320; Mahoney, (1973); S. Gaukroger (1995); O. Knudsen and K.M Pedersen (1968) pp.183-186; Prendergast (1975),pp 453-62; and McLaughlin (2000).

9 These rudiments appear in the so-called hydrostatic manuscript of 1619. See Schuster (1977), pp.93-111; Gaukroger (1995).pp.84-9; and Gaukroger and Schuster (2002) It should also be noted that <u>Le Monde</u> itself contains a reference to the text of the <u>Dioptrique</u> attributing the distinction between force of motion and directional force of motion to that earlier text. AT xi. 9. cf Alquie (1963) p.321 n2. We shall see below that the key dynamical concepts probably did crystallise in Descartes' optical work of the 1620s, particularly his discovery of the law of refraction of light (cf Schuster [2000])

10 <u>Le Monde</u> AT xi, 45-6, 85

11 I have coined the interpretative concept of 'principal' determination to underscore this important concept, and differentiate this aspect of determination from the other determinations that can be attributed to the stone at that moment. I prefer this terminology to a perhaps too whiggish concept of 'inertial' determination.

12 <u>Le Monde</u> , AT xi. 85. Descartes argues from the first and third laws of nature that at the instant of time the body is at point A, it tends in and of itself along the tangent AC. The circular tendency along AB is that part of the tangential tendency which is actively opposed by the physical constraint of the sling and hence gives rise to the centrifugal tendency to motion along AE. For the sake of whiggish edification it can be noted that had Descartes dealt with the centrifugal constraint on the ball offered by the sling, instead of the circular tendency (which violates the first law in any case) he might have moved closer to Newton's subsequent analysis of circular motion.

13 <u>Le Monde</u>, AT xi. 85.

14 Indeed in oral presentations of this paper at seminars and conferences I have used, not unsuccessfully, the following conceit in synthetically presenting the vortex theory: that this is a pro-Cartesian university lecture in Cartesian natural philosophy circa 1660, assuming fairly widespread consensual acceptance of vortex mechanics. This allows the further conceit that the new diagrams and concepts I use below to explicate the vortex mechanics have actually become recognised parts of a Cartesian Scholastic tradition within a generation of his death. Perhaps if the remainder of this section is read in that spirit, the key points about the theory will come through, provided one remembers above all that I am not suggesting this was for anybody the explicit, publicly acknowledged version of vortices, but rather that this is very close, on a charitable reading, to Descartes' own best understanding of his vortex theory, as it related to his course of work and context of natural philosophical struggle up to the early 1630s.

15 A more textual critical approach to teasing the underlying theory out the literal sense of <u>Le Monde</u> was begun in Schuster [1977] and will be fully explored in my monograph on Descartes as a physico-mathematician. Amongst the inadequately or misleadingly expressed analogies and claims that--revised, criticised and explicated--will find their place the synthetic presentation of the theory below are [1] the appeal to the behaviour of

a large heavy boat compared to random flotsam in the confluence of two parallel rivers; [2] Descartes' mode of setting out the notion of a 'balance' of forces holding a planet in its orbit; and , [3] the articulation of the key concept of 'massiveness' or 'solidity' of an orbiting body.

16 In fact in the key analogy used by Descartes, in a strong river current boats behave like comets, and it is light flotsam that behaves on analogy to planets. So untutored intuition misleads as to Descartes' own preferred analogy (and hence misses the theoretical points he will be elucidating through the analogy).

17 Additionally, as we shall see, he was also interested in relating a theory of local terrestrial gravity to his vortex celestial mechanics—a nice trick, since on earth bodies of third element subjected to the local vortex fall down; but in the heavens, bodies of third element, subjected to the stellar vortex, find specific and stable orbital distances. Descartes thought there was a unified conceptual explication of these indubitable phenomena and he prided himself on designing it.

18 Let us call this the 'force-stability principle'. Strictly speaking, however, more is involved in Descartes' full conception of the orbital stability of the particles, or planets, orbiting at a given radial distance. Descartes' articulated version of the force-stability principle will be developed below, note 32

19 AT xi. 50-51

20 Note in relation to this figure, as well as figures 4 and 5 below that they of course do not exist in Le Monde and are interpretative tools of my own design, used to picture the relationships Descartes sets out verbally. Additionally, it should be remembered that Descartes had no way of assigning empirically meaningful dimensions to the sizes and speeds of the boules. Nor would it have occurred to him to insist on any specific relationship for the variation of size and speed with distance. He limited his discussion to notions of proportionately greater or lesser increase or decrease of variables, which the figures then represent.

21 Descartes adduces the elements at this stage in Le Monde in Chap 8 [AT xi 51-55], but he has already adumbrated their properties at the end of Chap 4. And, in Chapter 5 he writes in more detail that, "I conceive of the first, which one can call the element of fire, as the most subtle and penetrating fluid there is in the world….I imagine its parts to be much smaller and to move much faster than any of those other bodies. Or rather, in order not to be forced to imagine any void in nature, I do not attribute to this first element parts having any determinate size or shape; but I am persuaded that the impetuosity of their motion is sufficient to cause it to be divided, in every way and in every sense, by collision with other bodies, and that its parts change shape at every moment to accommodate themselves to the shape of the places they enter....As for the second, which one can take to be the element of air, I conceive of it also as a very subtle fluid in comparison with the third; but in comparison with the first there is need to attribute some size and shape to each of its parts and to image them as just about all round and joined together like gains of sand or dust. Thus, they cannot arrange themselves so well, nor press against one another, that there do not always remain around them many small intervals, into which it is much easier for the first element to slide in order to fill them. And so I am persuaded that this second element cannot be so pure anywhere in the world that there is not always some little matter of the first with it. Beyond these two elements, I accept only a third, to wit, that of earth. Its parts I judge to be as much larger and to move as much less swiftly in comparison with those of the second as those of the second in comparison with those of the third. Indeed, I believe it is enough to conceive of it as one or more large masses, of which the parts have very little or no motion that might cause them to change position with respect to one another." [AT xi 24-6; MSM 10-11]

22 AT xi 53, MSM 24

23 Descartes insists that a central star can agitate the surrounding particles of second matter of its vortex: "These spherical bodies] incessantly turning much faster than, and in the same direction as, the parts of the second element surrounding them, have the force to increase the agitation of those parts to which they are closest and even (in moving from

the center toward the circumference) to push the parts in all directions, just as they push one another." [AT XI 53 MSM 24]

24 The special radial locus at distance K is present in Descartes' own discussion. Here for expository purposes I introduce the term 'K layer' not used by Descartes. Note as well that the existence and location of the K layer are caused by the existence and action of the sun.

25 Descartes' final distribution of the size and speed of the particles of the second element is as follows: AT XI 54-6; MSM 24-5 (Fig.1): "Imagine....that the parts of the second element toward F, or toward G, are more agitated than those toward K, or toward L, so that their speed decreases little by little [as one goes] from the outside circumference of each heaven [vortex] to a certain place (such as, for example, to the sphere KK about the sun, and to the sphere LL about the star) and then increases little by little from there to the centers of the heavens because of the agitation of the stars that are found there....As for the size of each of the parts of the second element, one can imagine that it is equal among all those between the outside circumference FGGF of the heaven and the circle KK, or even that the highest among them are a bit smaller than the lowest (provided that one does not suppose the difference of their sizes to be proportionately greater than that of their speeds). By contrast, however, one must imagine that, from circle K to the sun, it is the lowest parts that are the smallest, and even that the difference of their sizes is proportionately greater than (or at least proportionately as great as) that of their speeds. Otherwise, since those lowest parts are the strongest (due to their agitation), they would go out to occupy the place of the highest."

26 It is crucial to notice this moment in Descartes' theorising—it is the fact that a star, made of first element, happens to inhabit the center of each vortex that transforms every vortex into an orbit-locking mechanism. This is Descartes' version of the Keplerian emphasis (compared to Copernicus himself) on the physical-causal role of the sun in orbital mechanics. Interestingly, and crucially, the central location, and physical behaviour of each vortex's star, are also essential to Descartes' theory of light in the cosmic setting—again it is the central star that completes the theoretical picture explaining the phenomena of light in the vortex universe.

27 The reconstruction that follows here skims over all the complexities of textual interpretation mooted above at the beginning of this section, including some hopefully non-whiggish appeals to clarifications in the utterances of the Principles eleven years later.

28 My notion of 'surface envelope' is a good example of a term of interpretative art belonging to my hermeneutical categories 2,3 and 5, discussed earlier in this section.

29 The second element, recall, is quite small compared to the pieces of third element, something Descartes goes out of his way to claim, in first describing the elements, as we saw above in note 21: "Its parts [third element] I judge to be as much larger and to move as much less swiftly in comparison with those of the second as those of the second in comparison with those of the third." We are about to see one important reason why he has done this.

30 In Le Monde Descartes did this somewhat confusedly, improving his explication of massiveness and its role considerably in the Principles. I am reconstructing the underlying model in Le Monde, using a crisp hermeneutics of 'solidity' as aggregate volume to surface ratio and meshing that concept with my analysis of the size/speed distribution of the boules in the vortex. By using the graphical representations of these ideas, mediated by my interpretive construct of 'surface envelopes', the resulting decoding of the underlying model emerges. Note that in this process of reading, the verbal descriptions of the size/speed ratios comes directly from the text, as does the concept of solidity. These are clarified and amplified graphically. The 'least Cartesian' notion used in this interpretation is that of 'surface envelopes', but even it has textual warrant in the overall direction of the theory, and in Descartes' various descriptions of the centrifugal tendency of planets (and comets) and the resistances they encounter at various levels of the vortex.

31 This articulates the simple notion of centrifugal tendency as a function of size (quantity of matter) and force of motion only. In this mature application of the dynamics to a 'real' fluid vortex, it is clear that centrifugal tendency is a function of size, force of motion and 'solidity' (or massiveness), the latter taken in relation to the solidity of the relevant, resisting surface envelope.

32 The condition for a piece of third matter to be in stable orbit in the vortex can thus be expressed as $\mathbf{F^m_b} < \mathbf{R_{mu}}$ **and** $\mathbf{F^m_{ml}} < \mathbf{R_b}$ Where $\mathbf{F^m_b}$ means Force of motion of the orbiting body; $\mathbf{R_{mu}}$ means resistance of superjacent layer of boules (upper medium) to being extruded downward by body; $\mathbf{F^m_{ml}}$ means Force of motion of subjacent layer of boules (lower medium) and $\mathbf{R_b}$ means resistance of orbiting body to being extruded downward by subjacent layer of boules. All these terms need to be taken in their full explication including the concepts of massiveness, surface envelopes, and the size/speed distribution of boules in the vortex. Note that the formula also expresses the conditions for a ball of second element to be in stable orbital motion as part of the total vortex, if we take $\mathbf{F^m_b}$ to mean the force of motion of the orbiting sphere, and $\mathbf{R_b}$ to denote the resistance of the orbiting sphere of second element to being extruded downward by the subjacent layer of spheres. This, then, would conduce to a fuller understanding of what we above termed the 'force-stability' principle for constitution of the vortex.

It must be reiterated that the systematic conclusions reached here constitute a charitable reading of the relevant passages in Le Monde, supplemented carefully by the somewhat more clear and cogent presentation in the Principles. There is no scope in this short paper for an explication of the construction of my reading, which will be reserved for a more copious discussion within the scope of a book length treatment of 'Descartes, physico-mathematician', dealing with all the matters touched upon in this and later sections of this paper, and other related topics as well.

33 There is of course much more to say about this theory of comets. It first of all makes some concrete empirical predictions, which could have stood unrefuted for at least a generation after 1633; to wit, comets do not come closer to stars than a layer K; they are 'more massive than planets, they move in spiral paths oscillating out of and into solar systems. In addition, in dealing with the phenomena of comets' tails, Descartes had to attribute odd optical properties to the K layer as part of his overall theory of cosmological optics—raising thereby issues quite telling about the origin and import of his theorising, but beyond the scope of the present essay. See Schuster (1977). The matter will taken up in more detail in my monograph on Descartes as a physico-mathematician.

34 Material in this section closely follows the argument of Gaukroger and Schuster (2002)

35 AT X 52

36 AT x. 52. In this regard Beeckman was to note in 1628 that his own work was deeper than that of Bacon on the one hand and Stevin on the other just for this very reason. Beeckman (1939-53) iii. 51-2, 'Crediderim enim Verulamium [Francis Bacon] in mathesi cum physica conjugenda non satis exercitatum fuisse; Simon Stevin vero meo judico nimis addictus fuit mathematicae ac rarius physicam ei adjunxit.'

37 Simon Stevin, De Beghinselen des Waterwichts (Leiden, 1586] in Stevin (1955-66), i. 415.

38 Ibid, i. 415.

39 Ibid, i. 417. "Let there again be put in the water ABCD a solid body, or several solid bodies of equal specific gravity to the water. I take this to be done in such a way that the only water left is that enclosed by IKFELM. This being so, these bodies do not weight or lighten the base EF any more than the water first did. Therefore we still say, according to the proposition, that against the bottom EF there rests a weight equal to the gravity of the water having the same volume as the prism whose base is EF and whose height is the vertical GE, from the plane AB through the water's upper surface MI to the base EF."

40 The text, *Aquae comprimentis in vase ratio reddita à D. Des Cartes* which derives from Beeckman's diary, is given in AT x. 67-74, as the first part of the *Physico-Mathematica*.

See also the related manuscript in the *Cogitationes Privatae*, AT x. 228, introduced with, ‘Petijt e Stevino Isaacus Midlleburgensis quomodo aqua in funda vasis *b*…’.

41 Beeckman’s rules fall into two broad categories: (1) cases in which one body is actually at rest prior to collision, and (2) cases which are notionally reduced to category (1). The concept of inertia and the stipulation that only external impacts can change the state of motion of a body provide the keys to interpreting instances of the first category. The resting body is a cause of the change of speed of the impacting body and it brings about this effect by absorbing some of the quantity of motion of the moving body. Beeckman invokes an implicit principle of the directional conservation of quantity of motion to control the actual transfer of motion. In each case the two bodies are conceived to move off together after collision at a speed calculated by distributing the quantity of motion of the impinging body over the combined masses of the two bodies. For example, in the simplest case, in which one body strikes an identical body at rest, ‘...each body will be moved twice as slowly as the first body was moved...since the same impetus must sustain twice as much matter as before, they must proceed twice as slowly.’ And he adds, analogising the situation to the mechanics of the simple machines, ‘...it is observed in all machines that a double weight raised by the same force which previously raised a single weight, ascends twice as slowly.’ (Beeckman (1939-53) i. 265-6) Instances of the second category of collision are assessed in relation to the fundamental case of collision of equal speeds in opposite directions [ibid, 266]. Being perfectly hard and hence lacking the capacity to deform and rebound, the two atoms annul each other’s motion, leaving no efficacious residue to be redistributed to cause subsequent motion. This symmetrical case, which was also generalised to cases of equal and opposite quantities of motion arising from unequal bodies moving with compensating reciprocally proportional speeds, derives from a dynamical interpretation of the equilibrium conditions of the simple machines. Instances in which the quantities of motion of the bodies are not equal are handled by annulling as much motion of the larger and/or faster moving body as the smaller and/or slower body possesses (Beeckman (1939-53) i. 266.) This in effect reduces the smaller and/or slower body to rest. The outcome of the collision is then calculated by distributing the remaining unannulled motion of the larger and/or swifter body over the combined mass of the two bodies (ibid). It is obvious that Beeckman viewed this case through a two-fold reference to the simple machines; for he first extracts as much motion as can conduce to the equilibrium condition for symmetrical cases, and then he invokes the principle cited just above in this note to determine the final outcome.

42 AT x. 68-9. ‘... the water in base B will weigh equally upon the base of the vase as does the water in D upon its base, and consequently each will weigh more heavily upon their bases than the water in A upon its base, and equally as much as the water in C upon its base.’ This is the second of the four puzzles posed in the text, the others are: ‘(First), the vase A along with the water it contains will weigh as much as vase B with the water it contains. … Third, vase D and its water together weigh neither more nor less than C and its water together, into which *embolus* E has been fixed. Fourth, vase C and its water together will weigh more than B and its water. Yesterday I was deceived on this point.’

43 AT x. 68.

44 AT x. 68. In the *Cogitationes Privatae* (AT x. 228) the inclination to motion is described as being evaluated ‘in ultimo instanti ante motum’.

45 Descartes consistently fails to distinguish between ‘points’ and finite parts. But he does tend to assimilate ‘points’ to the finite spaces occupied by atoms or corpuscles. Throughout we shall assume that Descartes intended his points to be finite and did not want his ‘proofs’ to succumb to the paradoxes of the infinitesimal.

46 AT x. 70.

47 AT x. 70-1.

48 There actually is a third displacement away from the original terms of the problem: Notice that Descartes implicitly solidifies parts of the fluid not involved in the first two steps. That is, in working out the hypothetical case of descent, Descartes imagines away the rest of the fluid, *qua* fluid. It is in effect hypothetically solidified, so that its behaviour

does not complicate the postulated mechanical relations between f and g, B and h. This sort of tactic, along with the first two, plays a key role later in his theory of light in the cosmic setting of vortices in Le Monde and the Principles.

49 AT X p.72 and in correspondence with Beeckman early in 1619, AT X pp. 159, 162. For more discussion see Gaukroger and Schuster (2002)

50 We should note here a point that makes Descartes' moves all the more interesting: The Archimedean account, exploited by Stevin, comes without any dynamical, or more broadly speaking natural philosophical commitments. In the hands of Stevin, statics and hydrostatics, are hardly mixed sciences at all, since they really do make no physical or dynamical claims. Stevin was an arch Archimedean, and champion of the practical mathematical arts over against natural philosophical verbal wranglings. He pursued an ultra Archimedean program. So, he rejected the Mechanica, denying that the arcs through which bodies would move if they ceased to be in equilibrium have any bearing on the problem of the lever: You cannot deduce equilibrium conditions from the supposition that motion has or would occur—that is absurd, since if motion occurs the forces are not in equilibrium. This led Stevin to his famous reasoned denial that the study of motion, ie natural philosophy, could ever be pursued in a rigorous mathematical manner. How extremely interesting it is, then, that Descartes seemed able to make natural philosophical capital, indeed innovative natural philosophical capital, by recourse not to the Mechanica but to the purely statical, purely mathematical, equilibrium science of Stevin. See Gaukroger and Schuster (2002), pp.540, 545-9

51 For full details on claims in this section see Schuster (2000)

52 See Schuster (2000) The optical fragment of Descartes appears at AT x 242-3

53 Kepler, Ad Vitellionem Paralipomena, in Caspar (1938ff), vol. II pp.81-6 My analysis, Schuster (2000) pp.279-85 shows how this passage provides the source for Descartes speculation, which he further linked to two other passages in Kepler's optics, Caspar (1938ff).vol II Pp.89-90, 107

54 Schuster (2000) pp.281-2.

55 Schuster (2000) p.285 and note 69 thereto.

56 Schuster (2000)pp.272-7.

57 Lohne (1959) pp116-7, (1963) 160. Gerd Buchdahl provided a particularly clear statement of the methodological role played by the image principle in Harriot's discovery of the law Buchdahl (1972), 265-98 at p.284. Willebrord Snel's initial construction of the law of refraction also followed the type of path indicated by the Lohne analysis. See Vollgraff.(1913) (1936); deWaard, (1935-6)

58 Schuster (2000) p 276, referring to confirmation of the work of Bossha (1908)

59 For evidence on the movement from the original cosecant form of the law to the later sine form, based on Descartes' early work on lens theory, see Schuster (2000) 274-5.

60 On the important issue of the dating and content of Mydorge's letter containing this crucial diagram see Schuster (2000), pp.272-275.

61 Schuster (2000) pp.290-5.

62 Schuster (2000) pp.261-272

63 See above note 9. In discussing the distinction between the force of motion and its directional determinations, Descartes appeals to an already existing text on Dioptrics. AT xi 9.

64 Schuster [2000] pp, 302-3. The two principles read out of the optical diagram suggest that one may treat absolute quantities of force of motion (or force of tendency to motion) separately from their directional modes, or determinations. The diagram, read in this fashion, tells Descartes that a light ray is refracted due to the facts that [1] a change is affected in the absolute quantity of the force of motion (here force of tendency to motion] which is a constant for the two media in question, but that [2] the component of its

determination of tendency to motion parallel to the refracting surface is unaffected by the refraction. Later the first rule of nature in Le Monde will subsume [1] and the third rule of nature will subsume [2]. The results of the optical research directly parallel the two key dynamical concepts of Descartes as discussed above in Section 3.

65 Schuster (1977) 508-9.Beeckman (1939-53) iii p.114 note 3; Mersenne (1932-88) ii p.222, 217-8, 233-44; AT x 341-3. Beeckman (1939-53) iii p.103. Schuster (1977) pp.507-20.

66 Beeckman (1939-53) iii p.103. In the period July 1628 to June 1629 roughly twenty-one out of fifty-nine pages of Beeckman's journal deal with celestial mechanical and related matters. Material in this section is treated in more detail in Schuster (1977) 507-520.

67 Beeckman (1939-53) iii p.74

68 Beeckman (1939-53) iii pp.74-5.

69 One problem is that Beeckman realised that the unreflected rays of the sun would attract the moon to it. Beeckman (1939-53) iii 75.

70 Beeckman (1939-53) iii. p.100.

71 ibid. Two interesting queries arise in relation to Beeckman's model here: [1] Did Beeckman imagine this extended to a multi solar system universe of Cartesian type or was he thinking only of a one-off solar system and a chorus of fixed stars? We do not know for certain but it is indeed hard to see how any given star can play be in the attracting chorus and be a local attractor of its own planets. [2] Note Beeckman's emphasis on the magnitude and rarity (/density) of a planet. Beeckman was always acutely interested in how the volume to surface ratios of bodies, especially corpuscles, affected their mechanical interactions. The similarity in this respect to Descartes' later celestial mechanics is obvious.

72 ibid. p.101. Beeckman also applies this approach to the earth-moon problem, in which case he sees the earth as emitting both repulsive 'heat' and 'light' corpuscles and attractive magnetic 'virtue'. It is clear that he entertains a corporeal theory of magnetism, however, cf ibid. p.102. For Beeckman's corpuscular-mechanical theory of magnetism see also Beeckman (1939-53) I pp. 36, 101-2, 309, ii 119-20, 229, 339; iii 17,76.

73 Beeckman (1939-53) iii p.103. As Beeckman continued he also speculated about countervailing forces arising from impact of corpuscular emanations to explain, amongst other things, the eccentricity of orbits and precession of the equinoxes. ibid. pp.102, 108.

74 van Berkel (2000), Schuster (1977) pp.530-33.

75 AT xi pp.64-83

76 AT xi pp.72-3, 34; SG 47; MSM 123.

77 AT xi 73. MSM 125 SG 47

78 AT xi 73-4 MSM 125 SG 47

79 I am not advocating here history as mere literature or entertainment. Rather I believe that Descartes had intentions and conceptual structures reconstructable on the basis of textual and contextual evidence. My conceit is meant to motivate and focus proper historical scholarship on Le Monde and related texts, not to displace those texts or dissolve disciplined historical inquiry into more or less amusing creative writing. What 'Descartes' says here is also arguably an good heuristic guide to what to look for in post-Newtonian Cartesians.

80 The conceit arose out of Gaukroger's reflection on Gaukroger (2000) as well as issues arising in the composition of our joint study, Gaukroger and Schuster (2002). I have accordingly entitled the present chapter, as well as previous conference and seminar presentations of this argument, "Waterworld", in homage to Gaukroger's striking and amusing term.

81 Admittedly somewhat different types of vortices in detail—star centric and planet centric.

82 The rigorously contextual approach of this paper in regard to understanding the vortex mechanics and its genesis should not be taken to signal a denial of larger, long term, diachronic relevances of this inquiry or its findings. One important diachronic dimension immediately presents itself to the technical and internalist historian of classical mechanics: The natural philosophical contestation carried out by Descartes and Kepler was pursued with special attention to the subsumption of astronomy, ie Copernican astronomy, variously interpreted, and to its problem of celestial causation, in particular the function of stars. The nature of one's dynamics, the causal doctrine at the heart of one's system of natural philosophy, was thus focalised, and this drove both to contribute claims woven by later players in unintended and unforeseeable ways into what we recognise as the process of emergence of classical mechanics. Similarly, we should note the role of optical inquiries, in natural philosophical contexts, in the shaping the later crystallisation of classical mechanics, a matter hinted at in this paper and related work, and currently under serious study by Russell Smith of University of Leeds (personal communication). It would seem, as Stephen Gaukroger has expressed to me in discussion of themes of this and related work, that the long term genealogy of classical mechanics should be written, at least in part, in terms of the concatenation of unintended conceptual windfalls bequeathed to the emerging discipline by this and other nodes in the natural philosophical turbulence of the early and mid seventeenth century.

References:

Adam, C. and Tannery, P. Eds. (1974-86) Oeuvres de Descartes 2nd ed. 11 volumes. Paris.

Aiton, E.J. (1972). The Vortex theory of Planetary Motion. London.

Alquié, F. Ed.(1963). Oeuvres philosophiques de Descartes. t.1. Paris, Garnier Frères.

Bachelard, G. (1965, 1st ed. 1938), La formation de l'esprit scientifique, 4th ed. Paris, Vrin.

Beeckman, I. (1939-53). Journal tenu par Isaac Beeckman de 1604 à 1634. 4 vols. Ed. C. de Waard. The Hague: Nijhoff.

Bossha, J. (1908). 'Annexe note', Archives Neerlandaises des Sciences Exactes et Naturelles, ser 2 t. 13, pp.xii-xiv.

Buchdahl, G. (1972) 'Methodological Aspects of Kepler's Theory of Refraction', Studies in the History and Philosophy of Science 3, pp. 265-98

Caspar, M. Ed. (1938ff) Johannes. Kepler, Gesammelte Werke, Munich.

deWaard, C.(1935-6) 'Le manuscrit perdu de Snellius sur la refraction', Janus 39-40,pp. 51-73.

Gabbey, A. (1980) 'Force and Inertia in the Seventeenth Century: Descartes and Newton', in S.Gaukroger, Ed. Descartes: Philosophy, Mathematics and Physics. Sussex, Harvester: pp.230-320.

Gaukroger, S. (1995) Descartes: An Intellectual Biography. Chicago, University of Chicago.

Gaukroger, S. Ed and Trans.(1998) Descartes, The World and Other Writings. London, Cambridge University Press.

Gaukroger, S. (2000) 'The Foundational Role of Hydrostatics and Statics in Descartes' Natural Philosophy', in Descartes' Natural Philosophy. Eds. S. Gaukroger, J.A.Schuster and J. Sutton. London, Routledge:60-80

Gaukroger S. and Schuster, J.A. (2002) 'The Hydrostatic Paradox and the Origins of Cartesian Dynamics' Studies in the History and Philosophy of Science 33: 535-572

Knudsen, O. and Pedersen, K. M. (1968) 'The Link between "Determination" and Conservation of Motion in Descartes' Dynamics', Centaurus 13: 183-186

Kuhn T.S. (1959, 1st ed. 1957) The Copernican Revolution.. New York, Vintage.

Lohne, J. (1959) 'Thomas Harriot (1560-1621) The Tycho Brahe of Optics', Centaurus 6, pp. 113-21;

Lohne, J.(1963) 'Zur Geschichte des Brechungsgesetzes', Sudhoffs Archiv 47, pp 152-72

McLaughlin, P. (2000) 'Force Determination and Impact' in Descartes' Natural Philosophy. Eds. S. Gaukroger, J.A.Schuster and J. Sutton. London, Routledge: 81-112

Mahoney, M. (1973) The Mathematical Career of Pierre de Fermat 1601-1665. Princeton, Princeton University Press.

Mahoney, M. Trans. (1979) René Descartes, Le Monde ou Traité de la Lumière. New York, Abaris.

Mersenne, M. (1932-88) Correspondence du P. Marin Mersenne, 17 vols. Eds. C. de Waard, R.Pintard, B.Rochot and A. Baelieu. Paris, Centre National de la Recherche Scientifique.

T.L. Prendergast, 'Motion, Action and Tendency in Descartes' Physics', Journal of the History of Philosophy 13 (1975), 453-62

Sabra, A. I. (1967) Theories of Light from Descartes to Newton. London, Oldbourne.

Schuster, J.A. (1977) Descartes and the Scientific Revolution 1618-34: An Interpretation 2 vols. unpublished Ph.D. dissertation, Princeton University.

Schuster, J.A. and Watchirs G. (1990) 'Natural Philosophy, Experiment and Discourse in the18th Century: Beyond the Kuhn/Bachelard Problematic' in Experimental Inquiries: Historical, Philosophical and Social Studies of Experiment. Ed. H.E.LeGrand. Dordrecht, Reidel. pp.1-48.

Schuster, J.A. and Taylor, A.B.H. (1996) 'Seized by the Spirit of Modern Science', MetaScience, 9: 9-26

Schuster, J.A. (2000) 'Descartes' Opticien: The Construction of the Law of Refraction and the Manufacture of its Physical Rationales, 1618-29' in Descartes' Natural Philosophy. Eds. S. Gaukroger, J.A.Schuster and J. Sutton. London, Routledge:258-312

Schuster, J.A.(2002) 'L'Aristotelismo e le sue Alternative' in La Rivoluzione Scientifica. Ed. D. Garber. Rome, Istituto della Enciclopedia Italiana: 337-357.

Stevin S. (1955-66) The Principal Works of Simon Stevin, 5 vols Eds. E.Cronie et al. Amsterdam, Swets & Zeitlinger. [78]

van Berkel, K. ‘Descartes’ Debt to Beeckman: Inspiration, Cooperation, Conflict’, in Descartes’ Natural Philosophy. Eds. S. Gaukroger, J.A.Schuster and J. Sutton. London, Routledge:46-59.

Vollgraff, J.A. (1913) 'Pierre de la Ramée (1515-1572) et Willebrord Snel van Royen (1580-1626)', Janus 18, pp. 595-625

Vollgraff, J.A (1936). 'Snellius Notes on the Reflection and Refraction of Rays', Osiris 1, pp 718-25; [79]