John A. Schuster: Draft Articles for *Cambridge Descartes Lexicon* (Ed. Lawrence Nolan)

- **Hydrostatics**
- **Light**
- **Magnetism**
- **Physico-Mathematics**
- **Vortex**

"Hydrostatics"

Hydrostatics was one of a number of areas of "mixed mathematics" —including geometrical optics, positional astronomy, harmonics and mechanics—developed by Alexandrian authors in the Hellenistic era. Until the late sixteenth century the canonical work on hydrostatics was "On Floating Bodies" by Archimedes (c. 287 – 212 BCE). It deals in a rigorous geometrical manner with the conditions under which fluids are at rest in statical equilibrium, and with the equilibrium conditions of solid bodies floating in or upon fluids.

At the end of 1618, the twenty-two year old Descartes, working with **Isaac Beeckman**, addressed a number of problems in hydrostatics involving the "hydrostatic paradox." In 1586, Simon Stevin, the leading exponent of the mixed mathematical sciences at the time, brilliantly extended Archimedean hydrostatics. He demonstrated that a fluid filling two vessels of equal base area and height exerts the same total pressure on the base, irrespective of the shape of the vessel and hence, paradoxically, independently of the amount of fluid it contains. Stevin's mathematically rigorous proof applied a condition of static equilibrium to various volumes and weights of portions of the water. (Stevin 1955-66, vol. I, 415-7)

In Descartes' treatment of the hydrostatic paradox (AT X, 67-74) the key problem involves vessels B and D, which have equal areas at their bases, equal height and are of equal weight when empty. (see Figure 1) Descartes proposes to show that, "the water in vessel B will weigh equally upon its base as the water in D upon its base"—Stevin's hydrostatic paradox. (AT X, 68-69)

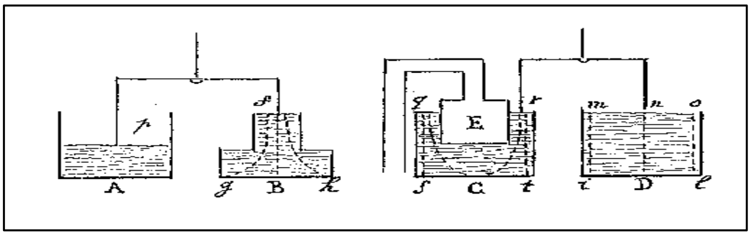

Fig. 1. Descartes, "Hydrostatics Manuscript" (AT X 69)

First Descartes explicates the weight of the water on the bottom of a vessel as the total force of the water on the bottom, arising from the sum of the pressures exerted by the water on each unit area of the bottom. This "weighing down" is explained as "the force of motion by which a body is impelled in the first instant of its motion," which, he insists, is not the same as the force of motion which "bears the body downward" during the actual course of its fall. (AT X, 68)

In contrast to Stevin's rigorous proof, Descartes attempts to reduce the phenomenon to corpuscular mechanics by showing that the force on each "point" of the bottoms of the basins B and D is equal, so that the total force is equal over the two equal areas, which is Stevin's paradoxical result. He claims that each "point" on the bottom of each vessel is serviced by a unique line of instantaneously exerted "tendency to motion" propagated by contact pressure from a point (particle) on the surface of the water through the intervening particles. But, while the surface of D is equal to and congruent with its base and posed directly above it, the surface of B is implied to have an area one-third its base. Hence, exemplary points i, D and l in the base of D are each pressed by a unique, vertical line of tendency emanating respectively from corresponding points, m, n and o on the surface. In contrast, point f on the surface of B is the source of three simultaneous lines of tendency, two being curved, servicing exemplary points g, B and h on the bottom of B. Descartes claims that all six exemplary points on the bottoms are pressed by an equal force, because they are each pressed by "imaginable lines of water of the same length" (AT X, 70); that is, lines having the same vertical component of descent. Descartes smuggles the tendentious three-fold mapping from f into the discussion as an "example" but then argues that *given the mapping*, f can provide a three-fold force to g, B and h. (AT X, 70-1)

The "hydrostatic manuscript" shows the young Descartes articulating the program that he and Beeckman at that time termed "**physico-mathematics,**" in which reliable geometrical results in the mixed mathematical sciences were to be explained by an embryonic corpuscular-mechanical matter theory and causal discourse concerning forces and tendencies to motion. Stevin's treatment of the hydrostatic paradox is within the domain of mixed mathematics, rather than natural philosophy. It does not explain the phenomenon by identifying its causes. Descartes' account falls within the domain of natural philosophy, attempting to identify the material bodies and causes in play. For Descartes fluids are made up of corpuscles whose tendencies to movement are understood in terms of a theory of forces and tendencies. This can explain the pressure a fluid exerts on the floor of its containing vessel. These moves imply a radically non-Aristotelian vision of how the mixed mathematical sciences, such as hydrostatics, should relate to natural philosophising. (Gaukroger and Schuster 2002, 549-550)

Although Descartes never again directly considered hydrostatical problems, this early fragment is of tremendous importance for understanding his mature natural philosophy. Throughout his later career Descartes continued to use descendants of the concept of instantaneous tendency to motion analysable into its directional components (later termed "determinations"). (see **force)** These ideas are central to his "dynamics," the concepts that govern the behaviour of micro-corpuscles in the *Treatise on Light* and the *Principles of Philosophy.* (see **light** and **vortex**)

See: BEECKMAN, ISAAC; EARLY WRITINGS; FORCE; LIGHT; MECHANICS; PHYSICO–MATHEMATICS; *PRINCIPLES OF PHILOSOPHY*; *TREATISE ON LIGHT*; VORTEX

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“Light”

Descartes’ corpuscular-mechanical natural philosophy is intended to replace the Aristotelianism of the late medieval universities and the resurgent neo-Platonic natural philosophies, in which light is conceived as the intermediary between base **matter** and higher spiritual and immaterial entities. In the most simple version of his theory, Descartes explains light mechanically as a tendency to motion, an impulse, propagated instantaneously through continuous optical media. This has the very important implication that in Descartes’ theory, the propagation of light is instantaneous, but the magnitude of the force conveyed by the tendency to motion constituting light can vary—there can be stronger and weaker light rays, all propagated instantly. (Schuster 2000, 261)

Descartes’ theory of light cannot be understood in detail without his theory of corpuscular dynamics. (see **force**) Descartes holds that bodies in motion, or tending to motion, are characterised from moment to moment by the possession of two sorts of dynamical quantity: (1) the absolute quantity of the ‘force of motion’ and (2) the directional modes of that quantity of force, which Descartes calls “determinations.” As corpuscles undergo instantaneous collisions, their quantities of force of motion and determinations alter according to the **laws of nature**. Descartes focuses on instantaneous tendencies to motion, rather than finite translations in space and time. His exemplar for applying these concepts is the dynamics of a stone rotated in a sling. (see figure 1) (AT XI 45-6, 85; G 30, 54-5)

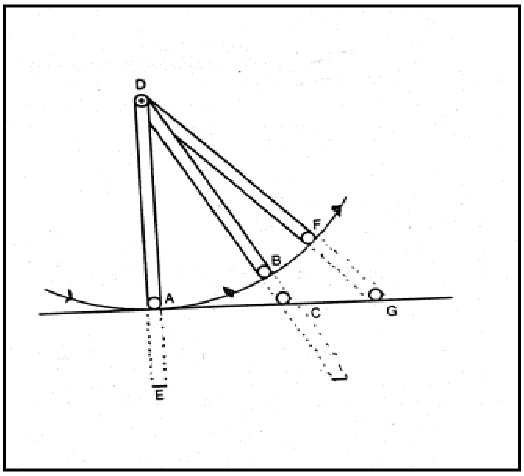

Figure 1: Descartes’ Dynamics of the Sling in *Treatise on Light*

Descartes considers the stone at the instant that it passes point A. The instantaneously exerted force of motion of the stone is directed along the tangent AG. If the stone were released and nothing affected its trajectory, it would move along ACG at a uniform speed reflective of its conserved quantity of force of motion. However, the sling continuously constrains what can be termed the 'principal' determination of the stone and, acting over time, deflects its motion along the circle AF. The other component of determination acts along AE, completely opposed by the sling, so that only a tendency to centrifugal motion occurs, rather than centrifugal translation. It is this conception of centrifugal tendency that Descartes uses when he articulates his theory of light inside his cosmological theory of vortices.

Each **vortex** consists of a central star, made up of the highly agitated and extremely small particles of first **element**, surrounded by a fluid which rotates around the star. The fluid consists of spherical particles of second element, which are significantly larger than first element particles, and which are tightly packed together, maintaining points of contact with each other. The remaining interstitial spaces are filled with first element particles. As the fluid rotates, the particles of second element generate centrifugal tendency to recede from the center, but since they are tightly packed together and every vortex is bounded by other vortices, this tendency cannot manifest itself as actual centrifugal translation of the second element particles. Descartes identifies light with the resulting lines, or rays, of centrifugal tendency to motion transmitted instantaneously outward through the vortex. (AT XI 87-90; G 55-8; AT VIIIA 108-116; MM 111-118)

In his **Optics** of 1637 Descartes presented the long sought after law of refraction of light, and attempted to demonstrate it (see figure 2) by use of a model of a tennis ball struck by a racket along AB towards refracting surface CBE. (AT VI 97-8; CSM I 158-9) Using Descartes' theory of light, and his corpuscular dynamics, one can analyse both his published "proof" of the law of refraction, and its underlying rationale in terms of his real theory of light as instantaneous tendency to motion transmitted through spherical particles of second element. The tennis ball's weight and volume are ignored. It moves without air resistance in empty geometrical space on either side of the cloth, which is taken to be perfectly flat and vanishingly thin. In breaking through the cloth, the ball loses, independently of its angle of incidence, a certain fraction (one half) of its total quantity of force of motion. Descartes applies two conditions to the motion of the ball: [a] the new quantity of force of motion is conserved during motion below the cloth; and [b] the parallel component of the force of motion, the parallel determination, is unaffected by the encounter with the cloth. Drawing a circle of radius AB around B, he assumes the ball took time *t* to traverse AB prior to impact. After impact, losing half of its force of motion, hence half its speed, it must take *2t* to traverse a distance equal to AB, arriving somewhere on the circle after *2t*. This represents condition [a]. Descartes writes that prior to impact the parallel determination "caused" the body to move towards the right between lines AC and HBG. For condition [b] he considers that after impact, the ball takes *2t* to move to the circle's circumference, so its unchanged parallel determination has twice as much time in which to act to "cause" the ball to move toward the right. He sets FEI parallel to HBG so as to represent that doubled parallel travel. At time *2t* after impact the ball will be at I, the intersection of FEI and the circle point below the cloth. It follows that (sin i/sin r) = (AH/KI) or 0.5 for all angles of incidence.

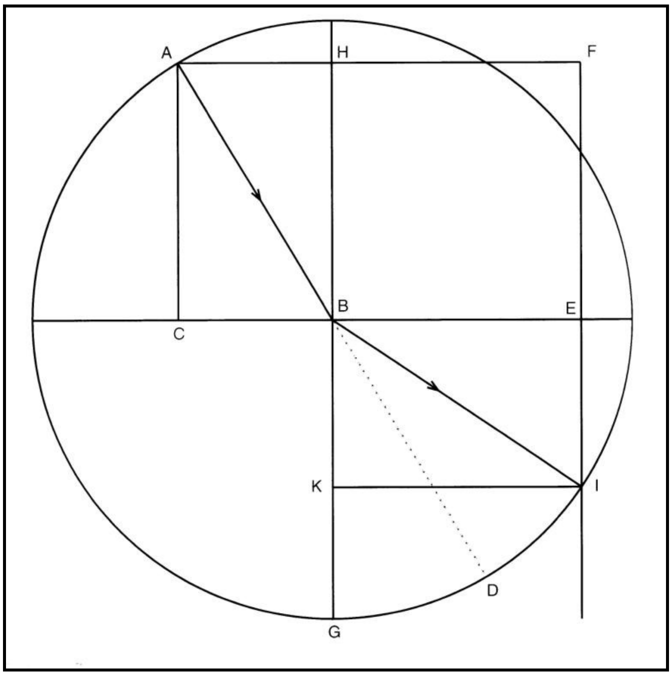

Figure 2: Descartes' Figure for Refraction of Light (Tennis Ball) in *Optics*

Descartes' published proof is superficially kinematic. But if we consider his corpuscular dynamics and the fact that his tennis ball is virtually a mathematical point in motion, we can translate Descartes' proof into the terms of his actual theory of light as instantaneously propagated tendency to motion. (see figure 3) Consider a light ray incident upon refracting surface CBE. Let length AB represent the magnitude of the force of the light impulse. The *orientation* and *length* of AB represent the principal determination of the ray. The force of the ray is diminished by half in crossing the surface. So, to represent condition [a] we draw a semi-circle below the surface about B with a radius equal to one half of AB. As for condition [b], the unchanged parallel determination, we simply set out line FEI parallel to HBG and AC so that AH=HF. The resulting intersection at I gives the new *orientation* and *magnitude* of the force of the ray of light, BI and the law follows, as a law of cosecants. The case of the light ray requires manipulation of unequal semi-circles, representing the ratio of the force of light in the two media. In the tennis ball case Descartes moves from the ratio of forces to the ratio of speeds and hence the differential times to cross *equal* circles. But, at the instant of impact, the same force and determination relations are attributed to the tennis ball and the light ray.

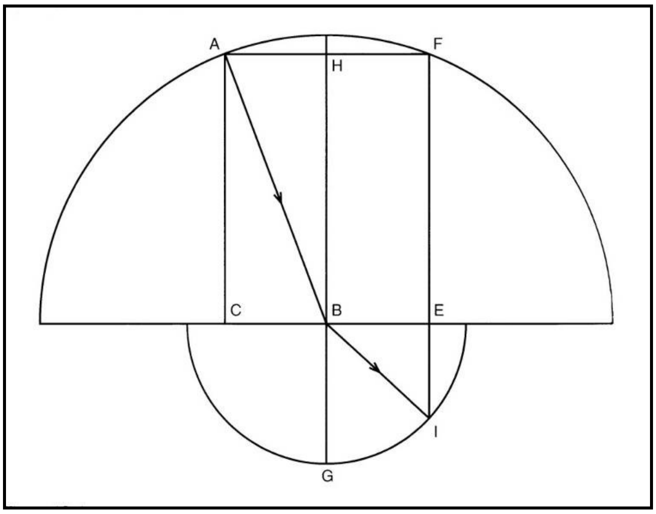

Figure 3: Refraction of Light Using Descartes' Dynamics and Real Theory of Light

The derivation of the long sought law of refraction using the principles of his dynamics of corpuscles marked the high point of Descartes' optical researches, along with his application of the law to the explanation of the telescope and to an ingenious solution of the equally long standing problem of explaining the **rainbow**. (also see **optics**) However, the full meaning of Descartes' optical triumph in relation to the overall development of his corpuscular-mechanical natural philosophy can only be grasped by looking at how his optical work unfolded over time, starting with his discovery of the law of refraction in Paris in 1626/7 while collaborating with the mathematician **Claude Mydorge**. This was accomplished independently of, but in the same manner as, Thomas Harriot, who first discovered the law around 1598. (see figure 4)

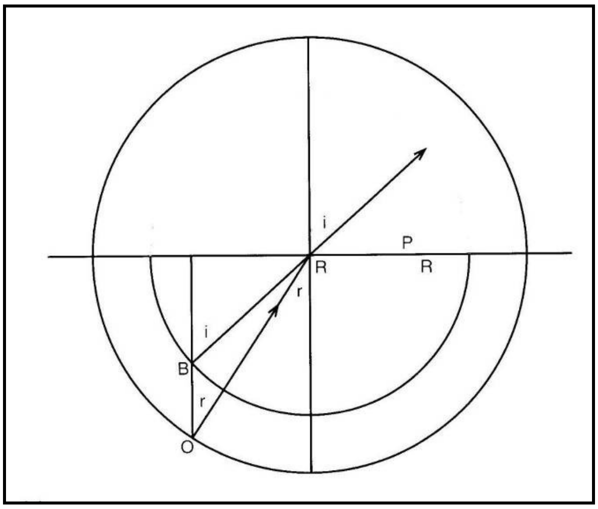

Figure. 4 Thomas Harriot's Key Diagram

Harriot used the traditional image locating rule to map the image locations of point sources taken on the lower circumference of a half-submerged disk refractometer. (Lohne, 1963; Buchdahl 1972) This yielded a smaller semi-circle as the locus of image points and hence a cosecant law of refraction of light. In a letter describing an identical cosecant form of the law, Mydorge presents a virtually identical diagram (see figure 5), but moves the inner semi-circle above the interface as a locus of point sources for the incident light. (Mersenne, 1932-88, I 404-15; Schuster 2000, 272-3)

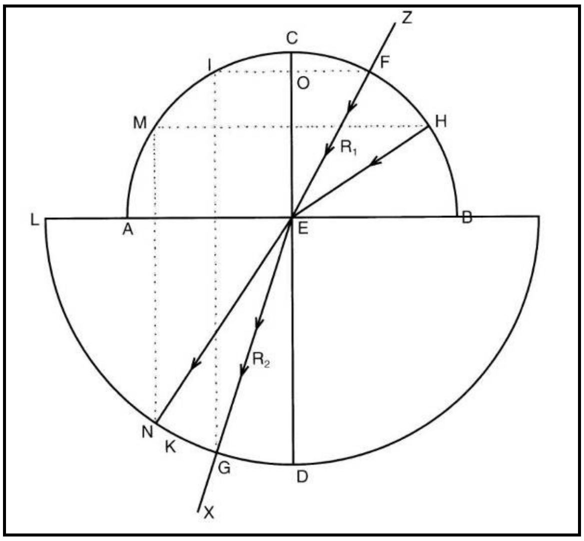

Figure 5 Mydorge's (and Descartes') Cosecant form of the law of refraction

Figure 5 closely resembles Figure 3, the derivation of the law of refraction using Descartes' conditions from the *Dioptrique* and his theory of light as instantaneously propagated tendency to motion. It is the key to unpacking the co-evolution of Descartes' theory of light and his dynamics of corpuscles.

After his discovery of the law of refraction by these purely geometrical optical means, issuing in the cosecant form of the law, Descartes sought to explain the law by use of a dynamics of corpuscles. Working in the style of his "**physico-mathematics**," he transcribes into dynamical terms some of the geometrical parameters embodied in the cosecant representation. The resulting dynamical principles concerning the mechanical nature of light are: [1] the absolute quantity of the force of the ray is increased or decreased in a fixed proportion, while [2] the parallel component of the force of a light ray is unaffected by refraction. These are effectively the conditions [a] and [b] which control the derivation of the law of refraction in the *Dioptrique*, and this is how he arrived at them, as is confirmed by the fact that by late 1628, Descartes used these concepts to explain the law of refraction to his friend **Isaac Beeckman**. (AT X, 336; Schuster 2000, 290-95)

So, principles [1] and [2] that control the proof of the law of refraction in the real theory of light and in the tennis ball model of light, were abstracted from the original geometrical representation of the newly discovered cosecant form of the law. Furthermore, these two mechanistic conditions for a theory of refraction suggested the two central tenets of Descartes' mature dynamics as he composed the *Treatise on Light* (1629-33). The first rule of nature in the *Treatise on Light* asserts the conservation of the quantity of the instantaneously exerted force of motion of a body in the absence of external causes. This rule subsumes and generalises [1]. The third rule of nature defines what we above called the principle determination of the instantaneously exerted force of motion of a body, along the tangent to the path of motion at the instant under consideration. This rule thus subsumes and generalises [2].

In sum, for Descartes the basic laws of light—itself an instantaneously transmitted mechanical impulse—immediately revealed the principles of the instant-to-instant dynamics of corpuscles. Not only is the theory of light central to the elaboration of Descartes' entire mechanistic system, but the principles of the dynamics of corpuscles governing that system arise from his research in geometrical and then mechanistic optics.

See BEECKMAN, ISAAC; COLOR; *DIOPTRICS*; FORCE; LAWS OF NATURE; MECHANICS; *METEOROLOGY*; MYDORGE, CLAUDE; OPTICS; PHYSICO–MATHEMATICS; PHYSICS; *PRINCIPLES OF PHILOSOPHY*; RAINBOW; *TREATISE ON LIGHT*; VORTEX

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"Magnetism"

Magnetism, long considered the exemplar of an occult, spiritual power, posed a challenge to mechanical philosophers like Descartes. William Gilbert's *De Magnete* (1600) offered an impressive natural philosophy, grounded in **experiments**, that could lead to interpreting magnetism as an immaterial power that possesses in its higher manifestations the capabilities of soul or **mind**. In his *Principles of Philosophy* Descartes accepts Gilbert's experiments, but he explains magnetism mechanistically, based on the movements of two species, right– and left–handed, of "channelled" or cylindrical screw-shaped particles of his first **element**. Descartes claims that magnetic bodies—naturally occurring lodestone, or magnetised iron or steel—have two sets of pores running axially between their magnetic poles: one set accepts only right–handed channelled particles; the other set of pores accepts only the left–handed particles. Descartes thus explained Gilbert's experiments, including his use of a sphere of loadstone, to demonstrate the properties of magnetised compass needles.

However, Descartes did more than appropriate and reinterpret Gilbert's "laboratory" work. Gilbert called his sphere of lodestone a *terrella*, a "little earth," arguing that because compass needles behave identically on the *terrella* as on the earth itself, the earth is, essentially, a magnet. Hence, according to his natural philosophy, the earth possesses a magnetic "soul", capable of causing it to spin. Magnetic "souls" similarly cause the motions of other heavenly bodies. In his *Principles*, Descartes, aiming to displace Gilbert's natural philosophy, focuses on the "cosmic" genesis and function of his channelled magnetic particles. Descartes argues that the spaces between the spherical corpuscles of the second element that make up his vortices**,** are roughly triangular, so that particles of the first element, constantly being forced through the interstices of second element spheres, become "channelled" or "grooved" with triangular cross–sections. Such first element corpuscles tend to be flung by centrifugal tendency out of the equatorial regions of vortices and into neighboring vortices along the north and south directions of their axes of rotation, thus receiving opposite axial twists. The resulting left– and right–handed screw shaped first element particles penetrate into the polar regions of central stars and then bubble up toward their surfaces to form, Descartes claims, sun spots. Stars are thus magnetic, as Gilbert maintained, but in a mechanistic sense.

Moreover, for Descartes, planets are also magnetic, as Gilbert claimed, but again the explanation is mechanical. Descartes describes how a star may become totally encrusted by sun spots. This extinguishes the star, its vortex collapses and it is drawn into a neighboring vortex to orbit its central star as a planet. But, such planets, including our earth, bear the magnetic imprint of their stellar origins, by possessing axial channels between their magnetic poles accommodated to the right- or left-handed screw particles. Descartes' explanation ranges from the cosmic production of magnetic particles, through the nature of stars and sunspots, to the birth and history of planets. He accepts the cosmic importance of magnetism, but renders the explanation mechanical, thus binding his natural philosophy into a cosmogonical and cosmological whole.

See COSMOLOGY; ELEMENT; EXPERIMENT; *PRINCIPLES OF PHILOSOPHY*; SUBTLE MATTER; VORTEX

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"Physico-Mathematics"

In November 1618, Descartes, then twenty-two, met and worked for two months with Isaac **Beeckman**, a Dutch scholar eight years his senior. Beeckman was one of the first supporters of a corpuscular–mechanical approach to natural philosophy. However, it was not simply corpuscular mechanism that Beeckman advocated to Descartes. He also interested Descartes in what they called physico-mathematics. In late 1618, Beeckman wrote that "There are very few physico-mathematicians," adding, "(Descartes) says he has never met anyone other than me who pursues enquiry the way I do, combining Physics and Mathematics in an exact way; and I have never spoken with anyone other than him who does the same." (Beeckman 1939-53, I, p.244) They were partly right. While there were not many physico-mathematicians, there were of course others, such as **Kepler**, **Galileo** and certain leading Jesuit mathematicians, who were trying to merge mathematics and natural philosophy. (Dear 1995, 168-79)

Physico-mathematics, in Descartes' view, deals with the way the traditional mixed mathematical disciplines, such as **hydrostatics**, statics, geometrical **optics**, geometrical astronomy, and harmonics, were conceived to relate to the discipline of natural philosophy. In Aristotelianism, the mixed mathematical sciences were interpreted as intermediate between natural philosophy and **mathematics** and subordinate to them. Natural philosophical explanations were couched in terms of **matter** and **cause**, something mathematics could not offer, according to most Aristotelians. In the mixed mathematical sciences, mathematics was used not in an explanatory way, but instrumentally for problem solving and practical aims. For example, in geometrical optics, one represented **light** as light rays. This might be useful but does not facilitate answering the underlying natural philosophical questions: "the physical nature of light" and "the causes of optical phenomena." In contrast, physico-mathematics involved a commitment to revising radically the Aristotelian view of the mixed mathematical sciences, which were to become more intimately related to natural philosophical issues of matter and cause. Paradoxically, the issue was not mathematization. The mixed mathematical sciences, which were already mathematical, were to become more "physicalized," more closely integrated into whichever brand of natural philosophy an aspiring physico-mathematician endorsed.

Three of Descartes' exercises in physico-mathematics survive. The most important is his attempt, at Beeckman's urging, to supply a corpuscular-mechanical explanation for the hydrostatic paradox, which had been rigorously derived in mixed mathematical fashion by Simon Stevin. (AT X 67-74, 228; Gaukroger and Schuster, 2002) Descartes' physico-mathematical work on **hydrostatics** involves a radically non–Aristotelian vision of the relation of the mixed mathematical sciences to his emergent form of corpuscular mechanical natural philosophy. Descartes aims to shift hydrostatics from mixed mathematics into the realm of natural philosophy.

He believed that from crisp, simple geometrical representations of sound mixed mathematical results one can read out or "see" the underlying corpuscular mechanical causes.

Descartes and Beeckman's studies of **accelerated free fall** also belong to their physico–mathematical project. They did not achieve any agreed results, because they could neither settle on what the correct, geometrically expressed law of falling bodies is, nor discern firm clues about its underlying causes. (AT X 58-61, 74-78, 219-222) The failed physico-mathematicization of falling bodies reverberates later in Descartes' distrust of the scientific relevance of Galileo's announcement of a mathematical law of accelerated free fall.

Descartes' third physico–mathematical exercise showed more promise, stalling in the short run, although it yielded rich results later. In 1620 he attempted in a physico–mathematical manner to find the law of refraction of light by considering the geometrical representation of its likely causes. He based the endeavour on passages and diagrams in which Kepler suggests that light moves with more force in denser optical media and "hence" is bent toward the normal in moving from a less to a more dense medium. (AT X 242-3) On this occasion, Descartes found neither a law of refraction nor its natural philosophical causes. However, seven years later, while working with the mathematician **Claude Mydorge**, he found, by traditional mixed mathematical means, a simple (cosecant) version of the law of refraction. Descartes immediately set to work attempting, in a physico-mathematical manner, to read out of his key geometrical diagram the principles of a mechanical theory of light that would then subsume the new geometrical law that had prompted them. These developments in turn had large consequences for the system of corpuscular-mechanical natural philosophy he first developed in his ***Treatise on Light*** (1629-33): Descartes' ideas about mechanistic optics, themselves physico-mathematical in tenor, suggested key concepts of his dynamics of corpuscles, which in turn helped shape his theory of vortices.

See: BEECKMAN, ISAAC; EARLY WRITINGS; HYDROSTATICS; LIGHT; MECHANICS; VORTEX

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"Vortex"

The theory of vortical celestial **mechanics**, as presented in the ***Principles of Philosophy*** and ***Treatise on Light***, is the "engine room" of Descartes' system of natural philosophy. Descartes starts his vortex theory with an "indefinitely" large chunk of divinely created matter or **extension** in which there are no void spaces whatsoever. When God injects **motion** into this extension, it is shattered into micro-particles and myriads of "circular" displacements ensue, forming large numbers of gigantic whirlpools or vortices. This process eventually produces three species of corpuscle, or elements, along with the birth of stars and planets. The third **element** forms all solid and liquid bodies on all planets throughout the cosmos, including the earth. Interspersed in the pores of such planetary bodies are the spherical particles of the second element. The second element also makes up the bulk of every vortex, while the spaces between these spherical particles are filled by the first element, which also constitutes the stars, including our sun.

The key to Descartes' celestial mechanics is his concept of the "massiveness" or "solidity" of a planet, meaning its aggregate volume to surface ratio, which is indicative of its ability to retain acquired motion or to resist the impact of other bodies. The particles of the second element making up a vortex also vary in volume to surface ratio with distance from the central star, as gathered from Descartes' stipulations concerning the variation of the size (and speed) of the second element particles with distance from the central star, illustrated in Figure 1. Note also the important inflection point in the size and speed curves at radius K. (Schuster 2005, 49) A planet is locked into an orbit at a radial distance at which its centrifugal tendency, related to its aggregate solidity, is balanced by the counter force arising from the centrifugal tendency of the second element particles composing the vortex in the vicinity of the planet—that tendency similarly depending on the volume to surface ratio of the those particular particles.

The most massive planet in a star system will be closest to, but not beyond the K layer—as Saturn is in our planetary system. Comets are planets of such high solidity that they overcome the resistance of the second element particles at all distances up to and including K. Such an object will pass beyond the K level, where it will meet second element particles with decreasing volume to surface ratios, hence less resistance, and be extruded out of the vortex into a neighbouring one. Entering the neighbouring vortex, the comet falls, and spirals, downward toward its central star, all the time meeting increasing resistance from the second element particles above that vortex's K distance. As it picks up increments of orbital speed, the comet starts to generate increasing centrifugal tendency, begins to rise and spiral upward, and eventually is flung back out of the second vortex.

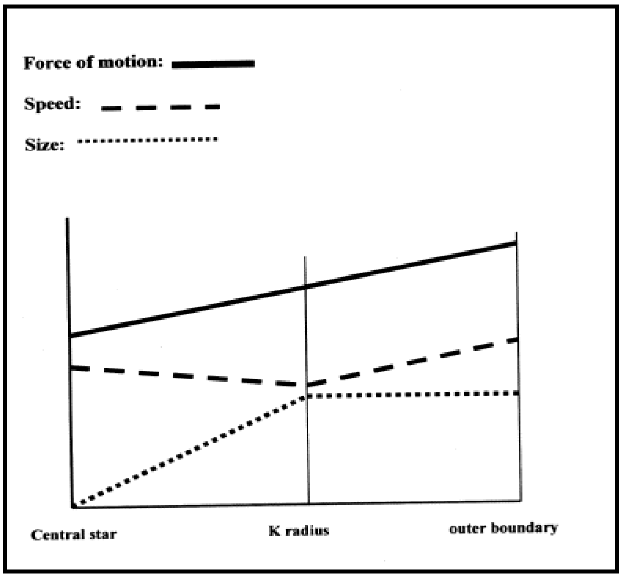

Fig.1 Size, Speed & Force of Motion Distribution Of Particles Of 2nd Element, In A Stellar Vortex

Also essential to Descartes' theory is a principle of vortex stability, which he introduces using his ideas about the dynamics governing the motions of corpuscles **(light)**. In the early stages of vortex formation, before stars and elements have evolved, the then existing vortical particles become arranged so that their centrifugal tendency increases continuously with distance from the center. (AT XI 50-1; G 33) As each vortex settles out of the original chaos, the larger corpuscles are harder to move, resulting in the smaller ones acquiring higher speeds. Hence, in these early stages, the size of particles decreases and their speed increases from the center out. But the speed of the particles increases proportionately faster, so that force of motion (size times speed) increases continuously. Figure 2 shows the distribution of size and speed of the particles in any vortex before a central star and the three elements have formed. (Schuster 2005, 46)

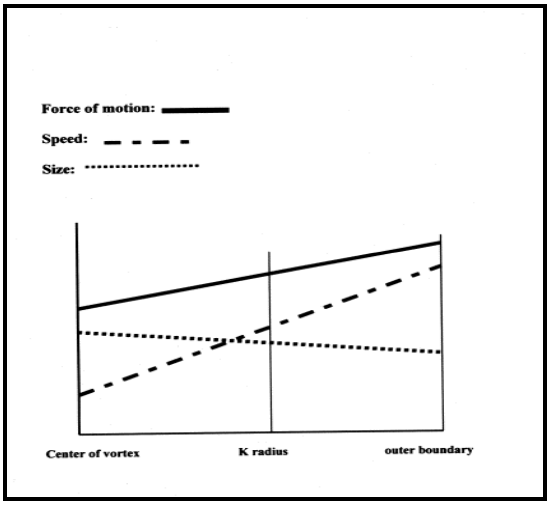

Figure 2. Size, Speed and Force of Motion Distribution of Particles of 2nd Element, Prior to Existence of Central Star

Stars do not exist in the early stages of vortex formation as described by Descartes. They form in the center of each vortex as part of the process leading to the emergence of the three final Cartesian elements. Every star alters the original size and speed distribution of the particles of the vortex, in a way that now allows planets to maintain stable orbits. Descartes explains that a star is made of up the most agitated particles of first element. Their agitation, and the rotation of the star, communicate extra motion to particles of the vortex near the star's surface. This increment of agitation decreases with distance from the star and vanishes at that key radial distance, called K. (Figure 3) (AT XI, 54-6; G 35-7; Schuster 2005, 48)

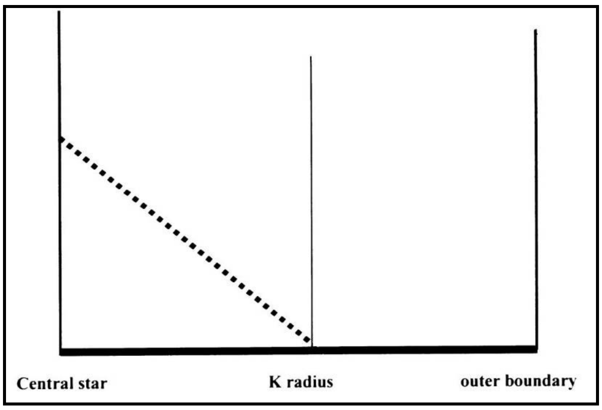

Figure 3 Agitation Due To Existence Of Central Star

This stellar effect alters the original size and speed distribution of the spheres of second element in the vortex, below the K layer. There now are greater corpuscular speeds close to the star than in the pre-star situation. But the all important vortical stability principle still holds, so the overall size/speed distribution must change, below the K layer. Descartes ends with the situation in figure 1, with the crucial inflection point at K. Beyond K we have the *old* (pre-star formation) stable pattern of size/speed distribution; below K we have a *new*, (post–star formation) stable pattern of size/speed distribution. This new distribution turns a vortex into a machine which, as described, *locks* planets into appropriate orbits below K and *extrudes* comets into neighboring vortices. In this way Descartes follows Johannes Kepler's lead in attempting to theorize about the physical role played in celestial mechanics by the sun, or any central star in a planetary system. Copernicus himself had never raised the issue of the sun's causal role in planetary motion.

In its intimate technical design Descartes' vortex mechanics is a science of equilibrium, resembling his work on **hydrostatics** in his early program in **physico-mathematics**. (Gaukroger and Schuster 2002; Gaukroger 2000; Schuster 2005) The forces at work upon a planet can only be fully specified when orbital equilibrium has been attained, although, of course no actual measurements are involved. The radial movement of a planet or comet (its rise or fall in a vortex) results from the breakdown of equilibrium and cannot be defined mathematically. Despite this limitation, Descartes intended that his theory of vortices qualitatively unify the treatment of celestial motions and the phenomena of local fall and of planetary satellites. A comet extruded from one vortex enters a neighboring one and falls toward its K layer before picking up centrifugal force and rising again out of the vortex in question. Similarly, Descartes makes it clear that a planet 'too high up' in the vortex for its particular solidity is extruded sun-ward, falling (and spiralling) down in the vortex to find its proper orbital distance. (AT XI 65-66; G 42; AT VIIIA 193; MM 169) In exactly the same fashion, Descartes' theory of local fall (AT XI 73-4; G 47), and theory of the orbital motion of the moon, when taken in their simplest acceptations, both also make use of the notion of falling in a vortex until a proper orbital level is found (assuming no other circumstances prevent completion of the process, as they do in local fall of heavy terrestrial bodies near the surface of the earth). However, Descartes' treatment of locally falling bodies and the motion of satellites both run into considerable difficulties when he attempts to explicate them in detail. (**gravity**)

Taken in both its technical details and its qualitative sweep, Descartes' vortex theory was a considerable achievement. The theory signaled Descartes' commitment to the truth of Copernicanism writ large, as an account of innumerable star and planet systems—interlocking sub-systems in the great machine of nature. He prefigured Newton in trying to bring planets, comets, satellites and locally falling bodies within one explanatory web, and his vortex theory persisted into the eighteenth century to compete with Newton's physics. (Aiton 1972)

See COSMOLOGY; EARTH, MOTION OF; FORCE; GRAVITY; PHYSICO–MATHEMATICS; LIGHT; HYDROSTATICS; MECHANICS; PHYSICS; *PRINCIPLES OF PHILOSOPHY*; *TREATISE ON LIGHT*

For Further Reading

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